Mirosław Szatkowski (Ed.)

Ontological Proofs Today
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The book *Ontological Proofs Today* is intended as a contribution to the vital research into the issue of rationality of theistic beliefs. Each of the twenty-one authors of the present volume has played an important role in this joint venture. As the editor, I would like to express my heartfelt gratitude to all of them for accepting the invitation to contribute to the volume. I would also like to thank them for the constant cooperation in the laborious task of proofreading and editing the texts. On the one hand, the preparation of a joint publication has the disadvantage of a long editorial process, which may weary some of the authors. On the other hand, it has the advantage of the extended deadline that some other authors badly need and are grateful for. This was exactly the case with our book.

We have decided to include in the volume the text by late Jerzy Perzanowski, who was a philosophy professor at the Jagiellonian University in Cracow. We are firmly convinced that if he were alive, he would have submitted a paper on ontological proofs, the subject he was greatly interested in, to be published in our volume. I would like to express my sincere gratitude to the Head of the Institute of Philosophy of the Jagiellonian University, Prof. M. Kuniński, who owns the copyright of Prof. J. Perzanowski’s works and has kindly agreed that the paper be published in our book.

Special thanks are directed to the anonymous reviewers, who have read the papers. It is thanks to their helpful suggestions and corrections that many of the submitted texts have been correctly edited.

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Mirosław Szatkowski
Authors of Contributed Papers

Anthony C. Anderson is Professor of Philosophy at the University of California at Santa Barbara. He earned his PhD in philosophy from University of California at Los Angeles under the direction of Alonzo Church. He also holds an M.S. in mathematics from the University of Houston, where he earned his undergraduate degree in physics and mathematics. Prof. Anderson’s work over the years has focused primarily in the philosophy of logic and the philosophy of language, although he also works in such areas as the philosophy of religion and has an interest in most areas of traditional philosophy. An example of his work that combines his skill at logic with his interest in the philosophy of religion appears in his papers on Gödel’s ontological proof. Among his publications, about 40 in number, the following five articles are more significant ones: Some New Axioms for the Logic of Sense and Denotation (1980), The Paradox of the Knower (1983), Russellian Intensional Logic (1989) - reprinted in Philosopher’s Annual as one of the ten best philosophical papers of 1989, Some Emendations of Gödel’s Ontological Proof (1990), Alternative: A Criterion of Identity for Intensional Entities (2001). During 2010-11 he was at Hertford College, Oxford University, on a Templeton Fellowship.


Sergio Galvan is Full Professor of Logic at the Catholic University of Mi-

**Stamatios D. Gerogiorgakis** teaches philosophy and religious studies at the University of Erfurt, Germany. He earned his PhD in philosophy from the University of Munich, and was habilitated at the University of Erfurt. Gerogiorgakis’ main interests are the philosophy of religion and the history of logic. Among others he has published three philosophical monographs: *Die Rolle des Schematismuskapitels in Kants Kritik der reinen Vernunft* (1998), *Necessary Being and Causal Determination* (2002 - in Modern Greek), *Zeitphilosophie im Mittelalter* (2006); and some ten articles which combine topics in logic with religion.

**Reinhard Hiltscher**, born in 1959 in Franconia, is “Außerplanmiger Professor”. He teaches at the Technische Universität Dresden philosophy. Main areas of research: Kant, German Idealism, Epistemology and Philosophy of Religion.

**Sре́cko Kovač** (PhD in Philosophy, University of Zagreb) is affiliated to the Institute of Philosophy (Zagreb, Croatia), leading the research project
Logical Structures and Intentionality (previous project Logic, Language and Modalities). Teaches basic and advanced courses in logic at the University of Zagreb. He is the author of Logical Questions and Procedures (with B. Žarnic, 2008), Logical-Philosophical Papers (2005), Logic (1994, 2nd ed. 2004), and Logic as a “Demonstrated Doctrine” (1992), in Croatian. He has published about 40 papers in international and Croatian journals and collections of papers. His current interests include logic of belief and knowledge, logic and ontology, logic of group decision-making, and history of logic.


Robert E. Maydole is an Emeritus Professor of Philosophy at Davidson College, Davidson, North Carolina. He specializes in logic and philosophy of logic, although he taught a wide range of courses ranging from the philosophy of mathematics and game theory to existentialism and the philosophy of religion. His current research interests are in the limits of reason, and in logical foundations of religion. Robert E. Maydole has published articles on logical paradoxes, and on ontological and cosmological arguments. He is currently working on a book about proving the existence of God. He is the author of several papers with new modal arguments for the existence of a supreme being.

Joason L. Megill received his Ph.D. (in Philosophy) from the University of Virginia in 2008. He previously has taught at the University of Colorado at Boulder and Old Dominion University. Currently, he is an Assistant Professor of Philosophy at Carroll College. From among his 11 published articles, 5 articles are in philosophy of religion. He also works in philosophy of mind, metaphysics and early modern philosophy.

Uwe Meixner is a professor of philosophy at the University of Augsburg,

**Edward Nieznański** studied philosophy and logic at the Academy of Ca-Theology in Warsaw. After graduating in 1962, he obtained his PhD in 1969 and was habilitated in 1977. He was appointed in 1979 as docent lecturer at the Academy of Catholic Theology in Warsaw. He was as a research fellow by Prof. Paul Weingartner at the Institut für Wissenschaftstheorie des Internationalen Forschungszentrums für Grundlagen der Wissenschaften in Salzburg (1979-1981) and as a Gastprofessor des Philosophischen Institutes der Geisteswissenschaftlichen Fakultät an der Salzburger Universität (1994-1995). He lectured at the University of Salzburg on *Spezielle Probleme der Logik* (*Logik der Gottesbeweise, Eine logische Theorie der Relationen, Logische Analyse philosophischer Texte, Formalisierte Wege zum Absoluten, Ideologiekritik: Leszek Kolakowski’s Religionsphilosophie and Wissenschaftliche Begründung*). He is a head of the Department of Logic at the University of Cardinal Stefan Wyszyński (since 1982) and of the Department of Theory of Law and Deontological Logic at the Lazarski’s University (since 1998) in Warsaw. His specialization primarily is formal logic, history of logic, metalogic, deontological logic, methodology of sciences and formalization of philosophical arguments. He has written over 100 publications including 10 books devoted to logic and the possibility of formalization of classical philosophy; he is also an editor
of the series *Miscellanea Logica* that is dedicated to applied logic.

**Graham Robert Oppy** is an Australian philosopher of religion. He is Professor of Philosophy and Head of the School of Philosophical, Historical and International Studies at Monash University, and Chair of Council of the Australasian Association of Philosophy. He serves on the editorial boards of: *Annales Philosophici, Australasian Journal of Philosophy, European Journal of Philosophy of Religion, International Studies for Philosophy of Religion, Oxford Studies in Philosophy of Religion, Philo, Philosophy Compass, Religious Studies,* and *Sophia.* He is the author of three books: *Ontological Arguments and Belief in God* (1996), *Philosophical Perspectives on Infinity* (2006), *Arguing about Gods* (2006); and co-author, with Michael Scott, of *Reading Philosophy of Religion* (2010). He is also co-editor, with Nick Trakakis, of the five volume *History of Western Philosophy of Religion* (2009), and of a number of works on the history of philosophy in Australia and New Zealand. He has published around 120 philosophical articles in books and journals, including papers in *Mind, Noûs, Philosophy and Phenomenological Research, British Journal for Philosophy of Science, Philosophical Studies, Philosophical Quarterly,* and the *Australasian Journal of Philosophy.*

**Jerzy Perzanowski** (April 23, 1943 - May 17, 2009) was a Polish logician and ontologist, Professor of Logic at the Jagiellonian University in Cracow and (simultaneously, from 1992 to 2004) at the Nicolaus Copernicus University in Torun. He was the originator of many ideas, including, inter alia, psychoontology, protophysics, and ontologic. A significant number of Professor Perzanowski’s works was devoted to ontology. Following the steps of Plato, St. Anselm, Descartes, Leibniz, Kant, Frege, Wittgenstein, Russel, and Ingarden, he developed combination ontology, together with the general theory of modality and combination semantics linked to them. In the field of ordinal ontology he introduced and explored locative ontology. Professor Perzanowski was also the author of noteworthy works on theo-logic and modal logic, including his general theorem on deduction and the topography of modal logics. He is also known for his fundamental texts on paraconsistent logic and logics of truth and falsehood. Perzanowski was the author or co-author of ten books, and more than eighty articles, the founder of the Polish journal *Logic and Logical Philosophy,* the co-founder and long-time editor-in-chief of *Reports on Mathematical Logic.* Besides, he was the member of editorial boards of the following journals: *Studia Logica, Filozofia Nauki, European Yearbook of Philosophy, Axiomathes,*
Alexander R. Pruss is Professor of Philosophy at Baylor University, USA. He earned his PhD in mathematics from University of British Columbia (1996) under the direction of John J. F. Fournier, and PhD in philosophy from University of University of Pittsburgh (2001) under the direction of Nicholas Rescher. His areas of specialization are: Metaphysics, Philosophy of Religion, Applied Ethics; and areas of competence are: Philosophy of Science, Ancient Philosophy, Aquinas, Leibniz, Truth, Philosophy of Probability. Alexander R. Pruss is the author of three books in Philosophy: The Principle of Sufficient Reason (2006), Actuality, Possibility and Worlds (2011), One Body: An Essay in Christian Sexual ethics (forthcoming); and the author or co-author of over 50 articles in philosophy and mathematics.

Amy Reagor studies Chemistry at Carroll College, and she is also interested in formal logic. She is working in tandem with Joason L. Megill.


Mirosław Szatkowski was affiliated as a professor of philosophy at the
Kazimierz Wielki University in Bydgoszcz, Poland, until 1 October 2011. At present he is associated with the Department of Philosophy, Theory of Science and Religious Studies at the Ludwig-Maximilians University in Munich, Germany. He earned his PhD in philosophy from the Jagiellonian University in Cracow under the direction of Andrzej Wroński, and was habilitated at the Ludwig-Maximilians University in Munich. Szatkowski’s main fields of research are: logic, the foundations of mathematics, and formal ontology. In logic, he has specialized in non-classical logics and published papers in Studia Logica, Zeitschrift für mathematische Logik und Grundlagen der Mathematik (Mathematical Logic Quarterly), Archiv für Mathematische Logik und Grundlagenforschung (Archive for Mathematical Logic), and Notre Dame Journal of Formal Logic. In formal ontology, his works concerning ontological proofs for the existence of God have been published in Studia Logica, Journal of Applied Non-Classical Logics and Journal of Logic, Language and Information. He is currently working on a book about Gödel-type proofs for the existence of God.


John Turri is Assistant Professor of Philosophy at the University of Waterloo. He currently holds an Early Researcher Award from the Ontario Ministry of Economic Development and Innovation, and has won research grants from the British Academy, the National Endowment for the Humanities, and the Social Sciences and Humanities Research Council of Canada. He is the author or editor of seven books, either published or forthcoming, as well as over fifty other publications in the areas of epistemology, philosophy of language, experimental philosophy, philosophy of mind, philosophy of normativity, and metaphysics.

Peter van Inwagen is an American analytic philosopher and the John Cardinal O’Hara Professor of Philosophy at the University of Notre Dame.

**Paul Weingartner** is an Emeritus Professor of Philosophy at the University of Salzburg, Austria. His fields of research include logic, philosophy of science, and philosophy of religion. He is author of nine books, among which are: *Basic Questions on Truth* (2000), *Different kinds of evil in the light of a modern Theodicy* (2003), *Laws of Nature* - together with P. Mittelstaedt (2005), *Omniscience. From a Logical Point of View* (2008), *God’s Existence. Can it be Proven? A Logical Commentary on the Five Ways of Thomas Aquinas* (2010); and editor or co-editor of thirty-six books. He also has published over hundred seventy articles. The places of his visiting professorships are: University of California at Irvine (1978, 1990-1991, 2000, 2001), National University of Canberra (1978), University of Seoul (1978), University of Pittsburgh (1984), University of Brasilia (1993, 1998), University of Manaus (1993), Universities of Santos, of Florianopolis, of Sao Paolo (1998); and his invited guest-lectures were at more than 100 foreign Universities (Europe, USA, Canada, Russia, Australia, Chile, Brazil, South Korea, Japan). Paul Weingartner is a member of: the New York Academy of Sciences, Academie Internationale de Philosophie des Sciences, Association of Symbolic Logic, International Union for the History and Philosophy of Science, Institut Internationale de Philosophie, Institut der Görresgesellschaft für Interdisziplinäre Forschung. In 1995 he received the honorary doctorate (Dr. h.c.) from Marie Curie Skłodowska University, Lublin (Poland).
Part I

Introduction
Guided Tour of the Book: 
*Ontological Proofs Today* 

Mirosław Szatkowski

1 Introduction

The issue of the existence of God is one of the oldest and most challenging problems in philosophy. Its theoretical appeal is reflected in the fact that it exerts fascination and inspires further research on the one hand, and raises objections and provokes criticism on the other. A wide range of arguments for and against the existence of God have been put forward, not only by philosophers. The issue of the existence of God is also the subject of vigorous cultural debate. The arguments pertaining to the existence of God give rise to a number of questions in such areas as epistemology, ontology, logic, theory of values (as the concept of God present in the arguments is often attached to the concept of positive value, perfection), and many more.

*Ontological proofs* – the phrase that appears in the title of the book – or *ontological arguments* are related to the issue of the existence of God. At this point, it should be stressed that the word ‘ontological’ is important in this context, as there are other proofs or arguments for the existence of God, for example, ‘transcendental’, ‘cosmological’, ‘teleological’ and ‘moral’ ones. Hence, there are two major questions that arise: (i). What is the difference, if any, between ontological proofs and ontological arguments?; and (ii). What is the difference between ontological proofs or ontological arguments on the one hand, and other proofs or arguments for the existence of God on the other.
Apart from first part – the present *Guided Tour of the Book* –, the book *Ontological Proofs Today* consists of six parts. The second part – *Interpretation of Old Ontological Proofs. God’s Attributes* – comprises five papers, each of which pertains either to historical ontological arguments; or to other, rather new, ontological arguments, but what makes them stand out from the other papers in this volume, is the fact that they all treat of the omniscience or the omnipotence of God. The third part – *New Ontological Proofs* – includes four papers, which introduce new ontological arguments for the existence of God, without referring to omniscience and omnipotence as the transparent attributes of God. The issue of the type of necessity with which ontological proofs work or may work is raised in the four articles of the fourth part – *Ontological Proofs and Kinds of Necessity*. In both articles from the fifth part – *Semantics for Ontological Proofs* – the semantics for some ontological proofs is defined. The sixth part – *Ontological Proofs and Formal Ontology* – consists of six articles. Even though they are quite different from each other in terms of content, they all explore some ontological issues, and formal ontology may be considered the link between them. The final part – *Maydole-Oppy Debate* – comprises two articles, by R. E. Maydole and G. Oppy, mutually controversial and different in their assessment of some ontological proofs. It should be remarked here that this categorization of the articles is not the only possible one: some papers could belong to more than one part.

The above division of the book into parts is reflected in the division of the present *Guided Tour of the Book* into sections. And so, in section two, *Understanding the ontological argument*, answers to questions (i) and (ii) formulated above are being sought. All the sections that follow have the same titles as the subsequent parts of the book. Each of them gives a brief outline of the papers from the part it refers to.

## 2 Understanding the ontological argument

Finding an answer to the question *What is the difference, if any, between ontological proofs and ontological arguments?* first of all requires defining the terms ‘argument’ and ‘proof’. Both are strongly rooted in everyday life, and, at the same time, are fundamental for mathematical practice. In everyday usage these terms mean something different for many people, whereas in mathematics they are usually precisely defined. However, regardless of whether we are developing a certain argument about a given subject, or an argument about a mathematical fact, each time we use some
kind of logic. In both these cases, the logic that is used determines the
same structure, the identification of which is relatively easy in mathematics, but may be quite complicated in natural language. One of the major differences between mathematical arguments or proofs and arguments or proofs in everyday life is that in mathematics they are not aimed at convincing someone of the correctness of the argument or the proof or of the truthfulness of the thesis they support. Bearing that in mind and accepting that the need to persuade and convince is absent from ontological proofs or arguments, it seems that it is justified here to concentrate solely on the mathematical understanding of the terms argument and proof.

Generally speaking, an argument is a list of statements, the last of which is called the conclusion, and the rest of which (the preceding ones) are called the premises. There can also be cases in which the list of premises is empty. It is said that an argument is valid (correct) if the conjunction of its premises implies its conclusions, or, in other words, if the truth of its premises implies the truth of its conclusion. It should be, however, stressed that the validity of an argument does not guarantee that its conclusion is true. It only makes it certain that the conclusion is true if the premises are true. The concept of proof is a more restrictive version of the concept of argument. A proof is a list of successive statements, the last of which – the one that is being proved – is a conclusion (thesis), and the rest of which are either axioms or the results of the application of one of the inference rules to the preceding statements. In a proof, every single step must be true if the preceding steps are true. This means that if a certain sequence of statements is a proof of a certain statement, let us call it $A$, then this statement is also an argument for $A$. The reverse does not hold true, as the fact that statement $A$ has some sequence of statements that support it, does not mean that $A$ is true. Similarly, the fact that both statements $A$ and $A \rightarrow B$ have some sequences of arguments that support them does not mean that there exists a sequence of statements that is the argument for $B$.

The next step should be to identify the factors which determine that certain proofs and arguments are ontological. It can be expected that the arguments or proofs that are called ontological in philosophical literature indicate the existence of something that makes them similar to each other. And so, according to P. van Inwagen [53], the term ‘ontological proof’ was introduced in philosophy by Immanuel Kant, who used it in reference to the line of reasoning presented by Descartes in Book V of his *Meditations*. The later authors started to use the term ‘ontological argument’ for the
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reasoning put forward by St. Anselm in Chapters 2 and 3 of *Proslogion*. At this point, it seems helpful, or even necessary, to provide a brief survey of the arguments which are described as ‘ontological arguments’ or ‘ontological proofs’. To do this, the works by the following authors will be referred to: E. J. Lowe [20], R. E. Maydole [24], G. Oppy [32] and P. van Inwagen [53].

There are three main periods in the history of ontological arguments. The first was in the 11th century, when St. Anselm of Canterbury came up with the first ontological argument. In the second period, from the middle 17th to early 18th century, Descartes sketched out a new kind of ontological proof, and Leibnitz modified it. Finally, the third period, which began in the 20th century and is still going on, has witnessed the rebirth of interest in ontological proofs. A number of arguments known as ‘modal ontological arguments’ have been formulated, the most significant of which are the ones proposed by N. Malcolm, Ch. Hartshorne, A. Plantinga, and K. Gödel.¹

Anselm’s argument has been reconstructed a number of times. One of the most successful reconstructions, let us call it Lowe’s reconstruction, goes as follows:

(1) God is, by definition, a being than which none greater can be conceived.
(2) A being than which none greater can be conceived exists at least in the mind.
(3) It is greater to exist in reality than to exist only in the mind.
(4) Therefore, God – a being than which none greater can be conceived – exists not only in the mind but also in reality.

In [20] E. J. Lowe comments upon the premises (1) - (3), and then proposes a modal reformulation of Anselm’s argument. According to Lowe, premise (1) implies that there are some grades of existence. However, many contemporary philosophers argue that being is ungradable, i.e. that something may either exist or not exist at all. Being close to the scholastic tradition, Lowe does not share this view and claims that there exist grades of being, because there exist grades of existential dependence. For instance, beings from the category of substances have a more basic mode of existence than beings that belong to the category of quality or mode. In other words, according to premise (2) it is possible to coherently think of the existence of such a being that has a thoroughly independent existence. Therefore, such

¹Ch. Hartshorne [13] is probably the first to formalize the modal substance of these proofs.
a being could exist, i.e. such a being has at least a possible existence, even if it does not actually exist. Lowe warns against such an understanding of premise (2) that would affirm the existence of something of a purely psychological nature: our thoughts or imaginations. The premise should not be thought of as referring only to a possible being, that is to one that does not exist actually and hence does not exist necessarily. For all contingent beings are dependent beings, and thus they are not ‘maximally great’ in terms of thoroughly independent existence. According to Lowe, premise should articulate these thoughts in a more definite manner. Taking all these remarks into account, Lowe proposes the following reformulation of Anselm’s argument:

\begin{align*}
(1^*) & \text{ God is, by definition, a } \text{maximally great being} \text{ that is, a } \text{being that is absolutely independent of anything else for its existence.} \\
(2^*) & \text{ A maximally great being could exist.} \\
(3^*) & \text{ A maximally great being could not have merely possible existence } \text{it would have to exist of necessity and so also in actuality.} \\
(4^*) & \text{ Therefore, God – a maximally great being – does actually exist.}
\end{align*}

Whereas Anselm’s argument as understood by G. Oppy [32] and P. van Inwagen [53] does not significantly differ from the reconstruction proposed in the points (1) - (4) above, the recollection by R. E. Maydole [24] is largely dissimilar to it. He claims that every reconstruction of an argument should clearly identify the assumptions that are hidden in the original text and that in some cases the reconstruction should add the philosophical rules which are not contradictory to the point of view of the author of the original text on the one hand, and strengthen the argument on the other. As far as Anselm’s argument is considered, R. E. Maydole reconstructs it informally in eight proof steps and formally in twenty-seven proof steps. The reconstruction will be presented in the last paragraph of the present paper.

Passing on to the Descartes’ ontological proof, it should be stated, as Oppy suggests in [32], page 21, that the source text of the argument is difficult to pin down. Lowe, however, claims that even though Descartes did not formulate the argument in the way Anselm had done, the above reformulation (1*) - (4*) by Lowe should be acknowledged as entirely faithful to Descartes’ intentions. As God, according to the definition, is a ‘perfect’ being, Descartes argues that eternal and necessary existence is a part of
its nature or essence. This brings Descartes to the conclusion that God must exist. In turn, Oppy [32] interprets Descartes’ text in two different ways, depending on how the sentence “I possess the idea of God” is to be understood, for Descartes himself understands the ideas of God as the ideas of a thing, and sometimes as the ideas of a certain nature, form, or essence. And hence, the two interpretations are as follows:

(1)  I possess the idea of supreme perfection. (Premise)

(2)  The idea of supreme perfection includes the idea of existence that is, the idea that supreme perfection is existent. (Premise)

(3)  The idea of supreme perfection is the idea of a true and immutable nature. (Premise)

(4)  Whatever belongs to a true and immutable nature may be truly affirmed of it. (Premise)

(5)  Existence belongs to the true and immutable nature of supreme perfection. (From 2, 3 and 4)

(6)  I may truly affirm of supreme perfection that existence belongs to it. (From 1 and 5)

(7)  (Hence) Supreme perfection is existent that is, God exists. (From 6)

And

(1)  I possess the idea of a supremely perfecti being. (Premise)

(2)  The idea of a supremely perfecti being includes the idea of existence that is, the idea that a supremely perfecti being exists. (Premise)

(3)  The idea of a supremely perfecti being is the idea of a being with a true and immutable nature. (Premise)

(4)  Whatever belongs to the true and immutable nature of a being may be truly affirmed of it. (Premise)

(5)  Existence belongs to the true and immutable nature of a supremely perfect being. (From 2, 3 and 4)

(6)  I may truly affirm of a supremely perfecti being that it exists. (From 1 and 5)

(7)  (Hence) Supremely perfecti being exists. (From 6)
R. E. Maydole [24] and P. van Inwagen [53] interpret Descartes’ argument in yet another way. These interpretations are not discussed right here, as the first of them will be discussed in the last paragraph of the present paper, and the other can be found in Inwagen’s article of the present volume. It seems important to supplement the deliberations on Descartes’ argument with Leibniz’s remarks; see, G. Leibniz [19]. He claims that Descartes’ ontological proof is promising but incomplete and argues that the argument would be correct if it demonstrated the coherence of a most perfect being, that is if it was shown that the existence of a most perfect being is possible. And this was exactly what Leibniz did. Oppy understands Leibniz’s line of reasoning in the following manner:

1. By definition, a perfection is a simple quality that is positive and absolute. (Definition)
2. A simple quality that is positive and absolute is irresolvable or indefinable. (Premise – capable of further defense)
3. A and B are perfections whose incompatibility can be demonstrated. (Hypothesis for reductio)
4. In order to demonstrate the incompatibility of A and B, A and B must be resolved. (Premise)
5. Neither A nor B can be resolved. (From 2)
6. (Hence) It cannot be demonstrated that A and B are incompatible. (From 3, 4 and 5 by reductio)

The multitude of ontological arguments that emerged in the 20th century and have appeared until today makes it impossible to discuss them in detail. Generally speaking, in most cases these arguments refer to the language of ‘possible worlds’ and to the contemporary modal logic. Hence they are called ‘modal ontological proofs’. From this multitude, E. J. Lowe selects only Plantinga’s ontological argument (see, [36]), let us say, as a representative of the others:

1. God is, by definition, a maximally great being and thus a being whose existence is necessary rather than merely contingent.
2. God, so defined, could exist; in other words, he does exist in some possible world.
3. Suppose that w is a possible world in which God, so defined, exists: then it is true in w, at least, that God exists there and, being God, exists there as a necessary being.
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(4) But a necessary being is one which, by definition, exists in every possible world if it exists in any possible world.

(5) Hence, the God who exists as a necessary being in w is a being that exists in every possible world, including this, the actual world.

(6) Therefore, God exists in the actual world; he actually exists.

P. van Inwagen, on the other hand, chooses as the representative of arguments of this kind the version presented below, which he considers the clearest and the most elegant one.

(1) A perfect being is a being that possesses all perfections essentially. (That is, a being is perfect in a possible world w if and only if it possesses all perfections in every world accessible from w.)

(2) Necessary existence is a perfection. (A being possesses necessary existence in a world w if and only if it exists in every world accessible from w.)

(3) Suppose that a perfect being is possible. (That is, suppose that there is a perfect being in some world w accessible from the actual world α.)

(4) Then, some being x that exists in α is a perfect being in w – since there is a perfect being (and hence a necessarily existent being) in w, w is accessible from α, and the accessibility relation is symmetrical.

(5) Might x exist only contingently in α? No, for in that case there is some world w₁ accessible from α in which x does not exist; and w₁ is accessible from w, since the accessibility relation is transitive.

(6) But is x a perfect being in α? Yes, for given the symmetry and transitivity of the accessibility relation, x will have a property essentially in α if it has it essentially in any world accessible from α.

(7) There therefore actually exists a being that has all perfections essentially, that is, there actually exists a perfect being.

R. E. Maydole does not choose one representative of ontological arguments formulated in the 20th and 21st century. In [24], he analyzes ontological arguments by N. Malcolm [22], Ch. Hartshorne [13], A. Plantinga [36] (in a version different than the one analyzed by Lowe), Dana Scott’s version of Gödel’s ontological proof, and two ontological arguments by Maydole himself that are different in several aspects. Finally, Oppy divides all ontological arguments into four kinds: ‘The first kind turns on the use that can be made of the ‘actually’ operator; the second kind turns
on the use that can be made of the ‘necessity’ operator; the third kind turns on a weak version of the principle of sufficient reason; and the fourth kind turns on the properties of incomprehensible beings.” [32], p. 65.

It could be asked now, as P. van Inwagen does in [53], whether the common name ‘ontological’ makes the above arguments identical to such an extent that finding a fault in one of them is tantamount to finding all of them faulty. To justify the negative answer, van Inwagen selects three arguments: Anselm’s ontological argument (he calls it Meinongian Argument), Descartes’ ontological argument (he calls it Conceptual Argument), and one modal argument (referred to above and considered a representative of all modal ontological arguments by van Inwagen), and then demonstrates that their ontologies are different. The first of them treats of two modes of being (or existence): a weaker one – existence in solo intellectu, and a stronger one – existence in re, i.e. it presupposes an ontology that is very similar to Meinongian ontology. The remaining two do not presuppose any kind of modes of being (or existence). In the second and the third one, existence is treated as a property, whereas in the first one there is no such assumption. And finally, the language of the third one is a modal language, presupposing the ontology of modality, whereas the first and the second one require no modal logic. Clearly, raising objections to one of these ontologies does not necessarily mean raising objections to the others. Neither need the objections raised against one of the four types of modal arguments identified by Oppy refer to the other types, cf., Oppy [32], Chapter 4.

In the literature on the subject, it would be difficult to find one background argument with all other ontological arguments being its versions. Moreover, as van Inwagen indicates in [53], to give a satisfactory answer to the question: What is it for argument A to be the version of argument B? would be an extremely challenging if not an impossible task. Without pointing to any single background argument, philosophical literature contains such statements as: “Ontological arguments are arguments which try to determine whether God exists by means of pure logic”, or “Ontological arguments typically start with the definition of God and conclude with statement about his (necessary) existence, using only or mostly a priori reasoning and no (or little) reference to empirical observation”. For instance Oppy in [32], p. 1, writes that:

“The distinctive feature of the arguments – at least according to the traditional Kantian method of classification – is that they proceed from premises which at least some defenders of the arguments allege can all be
known a priori. Consequently, it would be most appropriate to call these arguments ‘a priori arguments for the existence of God’. However, following Kant, it has been established practice to call these kinds of arguments “ontological arguments”, and I see no urgent reason to depart from this tradition."  

According to this definition, two features qualify an argument as an ontological argument: (i). Such arguments are only or mostly a priori reasonings without any (or with hardly any) reference to empirical observations; and (ii). Such arguments are concluded with the statement about the (necessary) existence of God. The first of these features is the feature that distinguishes ontological proofs from other proofs of the existence of God. Unfortunately, some people have reservations about this characterization of ontological proofs. For example, J. Pollock in [37] lays out a modal ontological argument for the non-existence of God. S. Sousedík in [43] and P. Hájek in [12] discuss the proof of non-existence of God contained in the

2 In the footnote 1 on the same page, Oppy writes: “In my view, a better characterization of ontological arguments than the traditional Kantian characterization given in the text is as follows: Ontological arguments are arguments that proceed from considerations that are entirely internal to the theistic worldview. Other theistic arguments proceed from facts, or pupative facts that are at least prima facie independent of the theistic worldview - for example, the presence of nomic, causal, or spatiotemporal order in the universe; the presence and nature of complex living structures in the universe; the presence and nature of conscious and intelligent agents in the universe. But ontological arguments are concerned solely with a domain or theory that is in dispute between theists and their opponents.”

G. Oppy in [32], p. 1 - 2, divides ontological arguments into six classes and he adds: “This decision will not be mutually exclusive; that is, some arguments may belong to more than one category. Moreover, it may not be exhaustive. However, I do not know of any a priori arguments for the existence of God that do not belong to at least one of these categories.” This six classes are as follows: “(i) definitional arguments, whose premises invoke certain kinds of definitions; (ii) conceptual arguments, whose premises advert to the possession of certain kinds of concepts or ideas; (iii) modal arguments, whose premises advert to certain possibilities; (iv) Meinongian arguments, whose premises invoke a distinction between different categories of existence; (v) experiential arguments, whose premises include the assumption that the concept of God is only available to those who have had veridical experiences of God; and (vi) “Hegelian” arguments, which, at least in my view, bear some relation to the philosophy of Hegel.”
book *Leptotatos Latine Subtilissimus* by Johann Caramuel von Lobkowicz (1606-1682), which they both classify as an ontological proof. It should be added that there are parodies of ontological arguments, the authors of which claim that if ontological arguments succeeded in proving the existence of God, a parodied argument would prove the existence of all kinds of things that are uncongenial to believe. For instance monk Gaunilo, who was Anselm’s contemporary, adopted Anselm’s argument to prove the existence of an island than which no greater island can be conceived. His intention was to demonstrate that the logical form of both arguments is the same.

To conclude this section, it should be remarked that there are no exact criteria for the classification of ontological arguments which would be widely accepted. Yet, although the concept ‘ontological argument’ remains vague, it has taken root in philosophical literature, and a vast majority of scholars familiar with the subject find it easy to distinguish these arguments from other types of arguments for the existence of God.

### 3 Interpretation of Old Ontological Proofs.

#### God’s Attributes

In this section van Inwagen’s paper *Three Versions of the Ontological Argument*, which was frequently referred to in the previous section, will not be discussed. The focus will be on the four other articles included in part two of the book. They will be subsequently analyzed below.

R. Hiltscher in [15] analyzes the proof of God’s existence known as Ratio Anselmi. According to him, Anselm’s argument consists of four parts. In the first part it is clarified that anyone who rejects the validity of the principle that thinking is the only form that makes any judgement possible contradicts himself. Building on the statement that ‘aliquid quo maius nihil cogitari potest’ (‘a being than which none greater can be conceived’), used in this part of Anselm’s argument, Hiltscher wants to resolve the meaning of the term ‘maius’, which would correspond to the meaning intended by Anselm. He rejects the interpretation that the term refers to perfection, but he thinks that it can mean the relationship between ‘principium’ and ‘principiatum’. The ‘principium’ in relationship to the ‘principiatum’ is ‘maius’, and the ‘principiatum’ in relationship to the ‘principium’ is ‘minus’. The second part shows that the principles of the mind are invariable principles that make the distinction between something
present both in the mind and in the reality, and something present solely in the mind. The highest principle of mind is the same as a being that is not formed by time or subject to it. In the third part Anselm returns to the theory of the finite mind of human beings. The human principle of the mind is only an image of the highest principle of the mind. Finally, in the final part it is argued that the absolute being does not depend on the contingent nature of thinking.

J. L. Megill in [28] formulates two arguments: the first one, with the conclusion that it is rational to think that an omniscient being exists in the actual world, and the second one, with the conclusion that it is necessary that an omniscient being exists. An omniscient being is defined as a being that knows as much as it is logically possible for a single being to know.

The first argument has the following form:

1. Consider a hypothetical world $W$ that is an exact duplicate of the actual world, though, for all we know, there might not be an omniscient being in $W$.
2. There is a logically possible world $W^*$ that is an exact duplicate of $W$, except that it definitely contains an omniscient being, i.e., a being that knows as much as it is logically possible for a being in $W$ to know.
3. It is rational to believe that $W^*$ is numerically identical to the actual world.
4. It is rational to think that an omniscient being exists in the actual world.

And the second argument has the following form:

1. An omniscient being in any random possible world $W$ would be a being in $W$ that knows as much as it is logically possible for a single being in $W$ to know.
2. It is logically possible for an omniscient being to exist in any random possible world $W$.
3. If it is logically possible for an omniscient being to exist in any random world, then it is logically possible for an omniscient being to exist in all worlds.
4. If it is logically possible for an omniscient being to exist in all worlds, then there is a possible world in which an omniscient being exists in all worlds.
If there is a possible world in which an omniscient being exists in all worlds, then there is a possible world in which an omniscient being is necessary, i.e., an omniscient being is possibly necessary.

If an omniscient being is possibly necessary, an omniscient being is necessary.

An omniscient being is necessary.

In the final part of his paper, Megill considers possible objections to both arguments, and refutes them.

Starting with the definition of ‘omnipotent being’: A being is omnipotent if it can make any logically possible, contingent state of affairs obtain, in [29] J. L. Megill and A. Reagor prove using modal logic S5 that such a being necessarily exists. Their proof, called the modal theistic argument, can be expressed as follows:

For any x and y: if x is an omnipotent being and y is a logically possible state of affairs, then x can make y obtain.

Any random logically possible world is a logically possible state of affairs.

For any x and y: if a being x is omnipotent and y is a random logically possible world, then x can make y obtain.

For any x and y: the conditions (i). If a being x is omnipotent and y is a random logically possible world, then x can make y obtain; and (ii). Possibly x is an omnipotent being, imply that x could coexist with y.

For a particular omnipotent being a and a particular random logically possible world b holds: the conditions (i). If a is a omnipotent being and b is a random logically possible world, then a can make b obtain; and (ii). Possibly a is an omnipotent being, imply that a could coexist with b.

The conjunction of the conditions (i). If a is a omnipotent being and b is a random logically possible world, then a can make b obtain; and (ii). Possibly a is an omnipotent being, is true.

For a particular omnipotent being a and a particular random logically possible world b holds: a could coexist with b.

If a could coexist with b, then it is possibly necessary that there is an omnipotent being.

It is possibly necessary that there is an omnipotent being.
It is necessary that there is an omnipotent being.

In the final section of their paper, Megill and Reagor consider seven possible objections against the reasoning presented above and reject them one by one.

Finally, M. Tkaczyk in [51] examines the proofs of God’s existence of Anselm of Canterbury, Thomas Aquinas, and Duns Scotus, from the point of view of contemporary modal logic and ontology (especially in terms of the difference between ‘necessary being’ and ‘contingent beings’), rather than from a historical perspective. His working hypothesis is that there is no very sharp difference between ontological proofs (Anselm’s proof) and non-ontological proofs (the proofs of Aquinas and Duns Scotus). According to Tkaczyk, the general framework of Anselm’s proof is the following: (i). God’s existence is possible; (ii). God’s existence cannot be contingent; (iii). Therefore, God exists. The premise (i) is grounded on two Anselm’s claims: (i.1). God is conceivable, and (i.2). God’s existence is conceivable. Tkaczyk thinks that the premise: God’s existence is possible would effectively communicate the requests which are present in (i), (i.1), and (i.2). The premise (ii) is grounded in the view: If God existed contingently, he would not be God. This statement, and consequently premise (ii), can also be read in two ways: either as (ii.1). It is not possible that the following conjunction holds: God exists and possibly God does not exist, which is logically equivalent to: Necessarily, if God exist then he necessarily exists, which is the view of Ch. Hartshorne [13]; or as (ii.2). It is not possible that the following conjunction holds: God possibly exists and God possibly does not exist, which is logically equivalent to: Necessarily, if God possibly exists then he necessarily exists, which is the view of N. Malcolm [22]. Tkaczyk argues that both sentences (ii.1) and (ii.2) are unacceptable.

The fundamental objection of Thomas Aquinas against Anselm’s argument is that there is no legitimate conclusion from a concept to existence in reality. Thomas Aquinas himself though is the author of five ways to prove God’s existence, for which Tkaczyk finds a common pattern: (i). Some existing objects are contingent; (ii). If contingent objects exist, God must exist; (iii). Therefore, God exists. He also claims that this proof would be most instructive if premise (ii) was replaced by the following two: (ii.1). If contingent objects exist, there must exist a necessary one; and (ii.2). God is the unique necessary object. And then it can easily be noticed that (ii.2) is actually identical with premise (ii) of Anselm’s argument, which means that Aquinas also legitimizes the transition from a concept to existence in
reality. According to Tkaczyk, the claim of contingency present in premise (i) can mean either (1). that the objects we know by acquaintance rise, change, and finally perish, unlike mathematical objects – which is a well grounded and truly empirical claim, or (2). that the objects we know by acquaintance have been brought into being by something other – which is ill grounded, because it assumes what pretends to be justified.

Duns Scotus, as Tkaczyk writes, finds Anselm’s and Aquinas’ arguments valid but not sound. In Anselm’s argument, he criticizes premise (i), and claims that the conclusion of the proof should be formulated as follows: God possibly exists. Each of Aquinas’ five ways should also finish with this conclusion. A proper proof of God’s existence, according to Duns Scott is: (i). Some existing objects are possibly contingent; (ii). If contingent objects exist, there must exist God; (iii). God could not exist contingently; (iv). Therefore, God exists.

Tkaczyk suggests that Duns Scotus’ argument contains some error. Premise (i) pertains to the nature of existing objects which are known by acquaintance. In turn, premise (iii) is a claim concerning God’s nature, and this premise is a tautology: either there is a nature of God or not. The possibility of God’s existence as premise (i) has it, which is a characterization of the content of the world, is different from the possibility of God’s existence as understood in premise (iii), which is a characterization of the concept of God. Hence, the conclusion about the nature of a necessary being is unjustified.

4 New Ontological Proofs
In this section, four papers are described.

And so, R. M. Gale in [7], after a brief critique of some traditional ontological arguments, discusses a new, more modest ontological argument, which he formulated together with A. Pruss; see, [8], and takes a stance towards some objections against this argument. Gale and Pruss’ argument is expressed in the language of possible world semantics. Possible worlds are interpreted as abstract entities comprised of maximal compossible conjunctions of propositions. And a proposition is: possible if it is true in some possible world, necessary if it is true in every possible world, and contingent if it is true in some but not all possible worlds. All the contingent propositions that are contained in some possible world are called a World’s Big Conjunctive Contingent Proposition. Calling the possible world whose Big Conjunctive Contingent Proposition is actually true by ‘p’, according
to Gale and Pruss, the question is whether \( p \) has an explanation and, if so, what kind of explanation it is. In the opinion of the opponents of the argument, the Principle of Sufficient Reason (PSR) is too strong to be such an explanation. Differentiating between the strong version (S-PSR) and the weak version (W-PSR) of the (PSR), namely:

(S-PSR)  \textit{For every contingently true proposition,} \( p \), \textit{there is a proposition} \( q \) \textit{and} \( q \) \textit{explains} \( p \);

(W-PSR)  \textit{For every contingently true proposition,} \( p \), \textit{there is a possible world} \( w \) \textit{that contains the propositions} \( p, q \), \textit{and that} \( q \) \textit{explains} \( p \);

Gale quotes the deduction, which is due to Pruss, about (S-PSR) on the basis of (W-PSR):

1. \textit{For every contingently true proposition,} \( p \), \textit{there is a possible world} \( w \) \textit{that contains the propositions} \( p, q \), \textit{and that} \( q \) \textit{explains} \( p \). (The principle (W-PSR))

2. \( p \) \textit{is a contingently true proposition and there is no explanation of} \( p \). (Assumption for indirect proof)

3. \textit{There is a possible world} \( w \) \textit{that contains the propositions: ‘}\( p \) \textit{and there is no explanation of} \( p \) ‘ \textit{and} \( q \), \textit{and that} \( q \) \textit{explains ‘}\( p \) \textit{and there is no explanation of} \( p \) ‘. (From (1) and (2))

4. \textit{In} \( w \), \textit{q explains} \( p \). (From (3), because explanation distributes over a conjunction)

5. \textit{In} \( w \), \textit{the proposition} \( p \) \textit{both does and does not have an explanation}. (From (3) and (4))

6. \textit{It is not the case that} \( p \) \textit{is contingently true and there is no explanation of} \( p \). (From (2) and (5))

7. \textit{It is not the case for any proposition} \( p \) \textit{that} \( p \) \textit{is contingently true and there is no explanation of} \( p \). (From (6))

8. \textit{There is a possible world} \( w_1 \) \textit{whose Big Conjunctive Contingent Proposition contains} \( p \) \textit{and} \( q \), \textit{and} \( q \) \textit{explains} \( p \). (From (7) and (W-PSR))

Hence, the only question that remains is what sort of explanation \( p \) can have. R. M. Gale thinks that it should take the form of an ontological argument.

In order to present Lowe’s new version of modal ontological argument for the existence of God (more precisely, for the existence of a necessary
concrete being), developed in [20], it seems necessary to quote the following six definitions: (D1). \( x \) is a necessary being iff \( x \) exists in every possible world; (D2). \( x \) is a contingent being iff \( x \) exists in some but not every possible world; (D3). \( x \) is a concrete being iff \( x \) exists in space and time, or at least in time; (D4). \( x \) is an abstract being iff \( x \) does not exist in space or time; (D5). \( x \) is a dependent being on \( y \) (\( x \) depends for its existence on \( y \)) iff necessarily, \( x \) exists only if \( y \) exists; and (D6). \( F \)'s depend for their existence on \( G \)'s iff necessarily, \( F \)'s exist only if \( G \)'s exist, where \( F \)'s and \( G \)'s are entities of different kinds. The argument goes as follows:

1. God is a necessary concrete being. (Definition of God)
2. Some necessary abstract beings exist. (Premise)
3. All abstract beings are dependent beings. (Premise)
4. All dependent beings depend for their existence on independent beings. (Premise)
5. All abstract beings depend for their existence on concrete beings. (From (3), (4), (D3) and (D4))
6. The only independent beings are concrete beings. (From (3), (D3) and (D4))
7. There is no possible world in which only abstract beings exist. (From (5))
8. There is no possible world in which no abstract beings exist. (From (2))
9. In every possible world there exist concrete beings. (From (7) and (8))
10. A necessary concrete being is possible. (From (9))
11. No contingent being can explain the existence of a necessary being. (Premise)
12. The existence of any dependent being needs to be explained. (Premise)
13. The existence of necessary abstract being needs to be explained. (From (2), (3) and (12))
14. Dependent beings of any kind cannot explain their own existence. (Premise)
15. The existence of dependent beings can only be explained by beings on which they depend for their existence. (Premise)
The existence of necessary abstract beings can only be explained by concrete beings. (From (14), (15), (3) and (5))

The existence of necessary abstract beings is explained by one or more necessary concrete beings. (From (13), (16) and (11))

A necessary concrete being exists. (From (17))

E. J. Lowe clearly states that he is unable to prove that there can be only one necessary concrete being, and that the necessary concrete being must have further properties, e.g., omniscience, omnipotence, and perfect goodness. It can clearly be noticed that Lowe’s argument is an a priori one, and that it focuses on the notion of necessary existence, rather than on the notion of actual existence.

The aim of Meixner’s paper [30] is to present a new version of the cosmological argument, which, according to the author, can also be considered an ontological argument, since it exclusively uses ontological concepts and principles. The argument also employs the achievements of modern physics and its ontology, and, in particular, the difference between event-causation and agent causation. The central concept of the paper is defined below:

\[\text{(D)}\] A first cause is a cause without a cause, in other, fully explicit words: a first sufficient cause is a sufficient cause (of some event), but a sufficient cause that itself has no sufficient cause.

And the structure of the Meixner’s argument is as follows:

(1) Effects, i.e., what is caused, are always events. (Premise)

(2) If agents are not events, then every agent that is a cause is a first cause. (From (1))

(3) Agents are not events, but substances. (Premise)

(4) Every agent that is a cause is a first cause. (From (2) and (3))

(5) If there are agents that are causes, then there are first causes. (From (4))

(6) Some physical events are causes, but there is no physical event that causes them. (Premise)

(7) Every event has a cause. (Premise)

(8) There are physical events that have a cause, though they are not caused by any physical event. (From (6) and (7))
(9) Every cause is an agent or an event. (Premise)

(10) Every first cause is an agent. (From (7) and (9))

(11) Every event that is caused by an event is also caused by an event that is not caused by any event. (Premise)

(12) For all $x$, $y$, and $z$: if $x$ causes $y$, and $y$ causes $z$, then $x$ causes $z$. (Premise)

(13) There is an agent that is a first cause. (From (4), (6), (7), (9) and (12))

Now, substituting (6) with:

(6*) The Big Bang (in symbols, BB) is a physical event that is a cause, but there is no physical event that causes it. (Premise),

with (6*) and the rest of premises, one can obtain:

(13*) There is an agent that is a first cause of the Big Bang. (From (4), (6*), (7), (9) and (12))

As U. Meixner says, “an agent that is a first cause of the Big Bang – that is: of the initial event of the Physical World – does seem to be godlike. By excluding the causation of the same event (any event) by several agents – which is a plausible theoretical step – we can even obtain that there is one and only one agent that is a first cause of the Big Bang.” (p. 200). In the final part of the article, Meixner discusses and refutes some potential objections against the particular premises.

Finally, Pruss’s paper [39] will be briefly discussed here. A. Pruss thinks that there are several ways of metaphysical understanding of the notion of a positive property: the excellence view – a positive property is the one that in no way detracts from its possessor’s excellence, but whose negation does; the limitation view – a positive property entails no limitation in its possessor, but its negation does; and Leibniz’s view – a positive property is the one that is a conjunction of basic properties, all subsets of which are mutually compatible. Pruss modifies Leibniz’s view to make it open to the possibility that some basic properties are not valuable, by saying that some basic properties are excellences, and a positive property is the one that is entailed by one or more basic properties that are excellences. In each of these accounts Gödel’s axioms (F1) and (F2) are plausible: (F1). If $A$ is positive, then $\neg A$ is not positive; and (F2). If $A$ is positive and $A$ entails $B$, then $B$ is positive. Adding two more axioms: (N1). Necessary existence
is positive; and (N2). Essential omniscience, essential omnipotence and essential perfect goodness are positive properties, where positive properties are strongly positive ones provided they are essential, Pruss has proved in [38] the following theorem:

**Theorem:** Given (F1), (F2) and (N1), if A is a strongly positive property, then there is a necessarily existing being that essentially has A.

From (F1), (F2), (N1) and (N2), it follows that there is a necessarily existing being that is essentially omniscient, that is essentially omnipotent, and that is essentially perfectly good. But Pruss could not show earlier that there is a being that has all these three essential properties. In [39] he remedies this defect by introducing four additional axioms: (N3). There is at least one unqualifying strongly positive property, where if A is a property such that it is impossible that there exist x and y such that x and y each have A, but x ≠ y, then A is called unqualifying; (N4). Essential omnipotence is unqualifying; (N5). Being essentially such that one is creator of every other being is a positive property; and (N6). If x is creator of y, then y is not creator of x. As a result, the following theorem and two corollaries are provable:

**Theorem:** Given (F1), (F2), (N1) and (N3), it follows that there is a necessary being that essentially has every strongly positive property.

**Corollary:** Given (F1), (F2), (N1), (N2) and (N3), there is a necessary being that is essentially omnipotent, essentially omniscient and essentially perfectly good.

**Corollary:** Given (F1), (F2), (N1), (N2), (N5) and (N6), there necessarily exists a unique God, where a God is a being that is essentially omnipotent, essentially omniscient, essentially perfectly good, and essentially creator of every other being.

A. Pruss also points out that the necessary existence of God can be proved using the notion of negative or limiting property as primitive instead of the notion of a positive property. Finally, he discusses Oppy’s parody of his argument and argues that this parody is not parallel to the parodied argument.
5 Semantics for Ontological Proofs

The two articles comprising this part of the book introduce the semantics of certain ontological arguments related to Gödel’s ontological argument. Thus, a brief explanation of Gödel’s argument seems necessary, along with pointing out some of its inherent lacks.

Informally speaking, Gödel uses a kind of modal language with a 2nd order notion of a positive property as a primitive, which he introduces with no elaborate clarification. However, his terse and sometimes cryptic explanations yield that he offers two readings of this notion: (1). positive in a moral - aesthetic sense. The positiveness in this sense is independent of the accidental structure of the world; and (2). positive in the sense of pure attribution. The positiveness in this sense is said to be opposed to privation. The additional, three more concepts are introduced by the following definitions: (D1). A God is any being that has every positive property; (D2). A property A is an essence of an object x if and only if A entails every property of x; and (D3). An object x has the property of necessarily existing if and only if its essence is necessarily exemplified.

These above concepts are characterized by the following axioms:

Axiom 1: Conjunction of positive properties is also positive;
Axiom 2: A property or its complement is positive;
Axiom 3: If a property is positive, then its complement is not positive;
Axiom 4: If a property is positive, then it is necessarily positive;
Axiom 5: The property of necessary existence is a positive property;
Axiom 6: Any property entailed by a positive property is positive.

The above set of definitions and axioms has been proposed by Gödel with a view to proving by means of an appropriate modal logic of the 2nd order that:

Theorem: A God necessarily exists.

Most probably due to Gödel’s recognition as one of the greatest mathematicians of the 20th century, his ontological argument has been particularly interesting for many researchers, which has led to the emergence of various versions of it, and even versions of those versions. Nevertheless,
it must be clearly stated that those researchers who see Gödel’s original argument as valid cause confusion in philosophical literature. In order to support this theory, we repeat the line of thinking already explicated in [46], p. 319:

“\textit{It is clear that the above axioms of K. Gödel leave a degree of freedom in interpreting the necessity symbol} \(L\). \textit{If, however, our theory meets the following rather natural condition:}

\begin{itemize}
  \item \textit{no new formula not containing the symbol} \(L\) \textit{can be proved if the axiom} 
  \[L\phi \leftrightarrow \phi\] 
  \textit{is added to the theory}
\end{itemize}

\textit{then the axioms: 1 – 6 are too weak to prove the much needed sentence:} 
\[\exists x G(x)\]. \textit{To see this, assume that variables of} 1^{st} \textit{order range over the set of natural numbers} \(\omega\), \textit{variables of} 2^{nd} \textit{order range over} \(2^{\omega}\) \textit{and} \(P\) \textit{is interpreted as a non-principal ultrafilter of the Boolean algebra} \(2^{\omega}\) \textit{containing all co-finite sets of natural numbers. Then, assuming that} \(L\phi \leftrightarrow \phi\), \textit{for every formula} \(\phi\), \textit{we get that the axioms: 1 – 6 are satisfied and the sentence} \(\exists x G(x)\) \textit{is false because the intersection of all co-finite sets is empty.”}

Another lack in Gödel’s original argument is related to the concept of \textit{necessary existence}. \textit{For there is an important question: What objects have the property of necessary existence? And unfortunately, the Gödel’s theory faces the fundamental difficulty: There exists no possible-worlds semantics in which the Gödel’s axioms and the formula} \(\Diamond \exists x NE(x)\) \textit{(Informally: \textit{It is possible that there exists an object which has the property of necessary existence}) could be valid. And so, it is not difficult to show that in such semantics every world-relative 2^{nd} order domain of quantification must contain the empty property. If now the extension of} \(\alpha\) \textit{in every world would be this empty property, then the formula} \(\Box \exists x \alpha(x)\) \textit{would be evaluated as false and the formula} \(\Box \forall y(\alpha(y) \rightarrow \beta(y))\) \textit{as true at every world. This entails the truthfulness of} \(\beta(x) \rightarrow \Box \forall y(\alpha(y) \rightarrow \beta(y))\) \textit{and} \(\forall \beta(\beta(x) \rightarrow \Box \forall y(\alpha(y) \rightarrow \beta(y)))\) \textit{for any object} \(x\) \textit{at every world, and consequently, the falseness of} \(NE(x)\) \textit{for any object} \(x\) \textit{at every world. From this, of course, it follows that} \(\Diamond \exists x NE(x)\) \textit{is also false, which means that} \(NE\) \textit{is the empty property, and this amounts to the validity of} \(\Box \forall x(NE(x) \rightarrow \neg NE(x))\). \textit{If now Gödel’s axioms were valid, then the formulas} \(P(\neg NE)\), \textit{i.e.,} \(\neg P(NE)\), \textit{and} \(P(NE)\) \textit{would be also valid – a contradiction. We think that D. Scott’s modification of the Gödel’s definition of essence is particularly suitable for overcoming the above difficulty.}
Not going into too much detail, a few words regarding semantic characterization of a theory, and of a modal theory in particular, seem to be beneficial for further analysis. And so, generally, a semantic characterization aims at giving a sharp criterion of truthfulness of formulas with the palette of tools that specify extensions for individual variables, individual constants and predicates, and interpret quantifiers – if the language contains these symbols at all. In the case of modal theories, there is a need for a reading or an interpretation of modal operators. In this sense it is possible to say that syntax and semantics are distinct and not reducible to one another – syntax produces thoughts, whereas semantics produces reality in its most general sense.

Usually, a semantics determines some ontology or, vice versa, it is determined by some ontology. Here, we omit the question: how should the link between a semantics and an ontology be understood? In particular, quantified modal theories have a strong ontological appeal, since they deal with: necessity and possibility, actual and possible objects, modal properties, existence, and trans-world identity of objects and properties. Ontological problems relevant to them are the choices between some controversial principles: classical and free principles of universal specification, the presence and absence of the principle of Barcan formulas and converse Barcan formulas, the presence and absence of the principle of exportation and converse exportation. Naturally, quantified modal theories can be studied and compared from different ontological perspectives, e.g.: from the perspective of actualism or possibilism, realism or anti-realism about possible worlds, or different theories on persistence conditions for material objects through change.

The starting point in S. Galvan’s paper [9] is a syntactical characterization of two logics of the 1st order: logic of existence and Leibnizian logic of existence. Both logics, as Galvan notes, are extensions of the S5 modal logic of the 1st order. More precisely, these logics extend the classical logic of the 1st order because the two axioms inherent in them: (T). \( \square \alpha \rightarrow \alpha \) and (NE). \( \alpha \rightarrow \Box \alpha \), provided \( E \) does not occur in \( \alpha \), cause the modal collapse for all formulas that do not contain the existence predicate \( E \). The presence of the existence predicate \( E \) in the language could suggest that these logics should reject: classical principles universal specification, Barcan and converse Barcan formulas, principles of exportation and of converse exportation – and on the contrary, all of them are the theorems of these logics. Moreover, it is easy to see that free principles of universal specification are also theorems of the two logics, i.e., \( \forall x \alpha(x) \land E(y) \rightarrow \alpha(y) \). However, the
axiom: $\forall x E(x)$, and axiom: $\forall x \Box E(x)$, typical of some free modal logics, do not function as the theorems of Galvan’s logics of existence.

The ontological frames (structures) embody the central term of semantics for both logics. There are two types of such frames – depending on the aspect of possible object which they incorporate. One type is related to Kant’s concept of possible object, the other to Leibniz’s concept of possible object. According to Leibniz, the source of possibility is the mere logical consistency of the notions involved, so that possibility coincides with analytical possibility. Kant also maintains that consistency is only a necessary component of possibility. But he adds that something is possible if there is a cause capable of bringing it into existence – consistency alone is not sufficient. Thus, while the Leibnizian notion of consistency is at the root of the concept of analytical possibility, the Kantian notion of possibility is the source of real possibility. This difference underlies the distinction between the Kant’s ontological structures for the logic of existence and Leibniz’s ontological structure for the Leibnizian logic of existence. Specifically, every ontological structure is formed by non-empty set of ontologically possible objects, which is constant for all worlds. The set of possible objects, which are simply analytically possible objects, is subdivided into two disjoint subsets: the subset of really possible objects and the subset of purely possible objects. This division is dictated by the function $E$, working from the set of all worlds into the power set of the set of possible objects – objects belonging to the sum of image sets of the function $E$ are really possible objects, whereas the remaining ones are purely possible objects. In the ontological structures for the logic of existence these functions work under two conditions: the existence condition (For every world, the image set of the function $E$ at this world is not empty) and the limited condition of exhaustiveness (It is not necessarily true that for every possible object there exists a world in which this possible object is actual). In ontological structures for the Leibnizian logic of existence, the limited condition of exhaustiveness is substituted by the unlimited (global) condition of exhaustiveness (For every possible object there exists a world in which this possible object is actual). Another key parameter for each ontological frame is the set of attributes, the elements of which are properties (= 1-ary relations) and, if necessary, $n$-ary relations, $n > 1$. For example, properties are subsets of the set of ontologically possible objects. But, what is worth noting, the same family of properties is related to every world. Moving on to the interpretation of individual variables and predicates, the former receive only rigid extensions in the set of ontologi-
cally possible object, while the latter receive rigid extensions in the set of properties and rigid intensions, which are constant functions from the set of all worlds into the set of properties. The predicate of existence is an exception, since it is the only one to receive non-rigid extensions.

Moving on to Leibniz’s formulation of the modal ontological proof and Gödel’s modified version of the ontological proof, Galvan characterizes the former only from the syntactic point of view, without going into too much detail, whereas he focuses more on the latter, analyzing it from both syntactic and semantic perspectives. Leibniz’s ontological proof is built on his logic of existence by adding: the strong Descartes’ principle (For every $x$ it is necessary that if $x$ is $G$ and $x$ really exists then it is necessary that $x$ is $G$ and $x$ really exists) or the weak Descartes’ principle (For every $x$ it is necessary that if $x$ is $G$ then it is necessary that $x$ is $G$), and the main premise (There exists $x$ such that it is possible that $x$ is $G$ and $x$ really exists). On the other hand, Gödel’s modified version of the ontological proof is obtained from the logic of existence by adding following three axioms: (Ax 1). Exactly one of a property or its complement is positive; (Ax 2). The properties entailed by positive properties are positive; and (Ax 3). The conjunction of any collection of positive properties is positive. Ontological frames for Gödel’s modified version of the ontological proof, called structures of pure perfections, contain the first order domain of possible objects and the second order domain of properties, just as in the case of ontological frames for the logic of existence, and they additionally include the third order domain of properties, which is a subset of the set of all subsets of the domain of properties of properties. Variables of the first, second and third order are valuated in these respective domains. Within the structures of pure perfections, a principal ultra-filter is singled out, defined over the domain of possible objects in which the predicate of perfection is interpreted.

Our paper [49] should be considered in the context of our previous work, namely [46] – [48]. In all these papers, the strong completeness theorems for different Anderson-like variants of Gödel’s theory with respect to classes of model structures are proved. They are called Anderson-like ontological proofs, because they adopt elements from Anderson’s theory (see, [2]): the definitions of a god-like being (God is any being that has necessarily all and only positive properties), essence (A property $A$ is an essence of an object $x$ if and only if and only if $A$ entails all and only the properties that $x$ has necessarily), and necessary existence (An object $x$ has the property of necessary existence if and only if its essence is necessarily exemplified), and
the three axioms: (i). If a property is positive, then its complement is not positive; (ii). The property of a god-like being is positive; (iii). Any property entailed by a positive property is positive.\(^3\) However, these three Anderson’s axioms are not sufficient to prove the statement: *God necessarily exists or God exists*, nor other ontological statements. They must be further supplemented with some non-modal and modal principles of the zero-, first- and second-order.

In all four papers, the following four criteria differentiating Anderson-like theories can be found: (i). treatment of an identity relation of the 2\(^{nd}\) order; (ii). treatment of the property of necessary existence; (iii). treatment of properties abstracted from expressions of the form \(I_x(y)\) defined by: \(I_x(y) \overset{df}{=} (x \overset{1}{\approx} y)\) and \((x \overset{1}{\approx} y) \overset{df}{=} \forall \alpha (\alpha(x) \leftrightarrow \alpha(y))\); and (iv). characterization of modal operators. In papers [48] and [49], an additional criterion is: (v). treatment of so called permanence, i.e., the formula: \(\forall x \Box E(x)\) (Every object of the domain of quantification is necessarily its element). An interesting quality of Anderson-like theories considered in papers [46] and [47] is the employment of classical principles of quantification of the 1\(^{st}\) and 2\(^{nd}\) order, whereas in papers [48] and [49] free principles of quantification of the 1\(^{st}\) order are employed, with classical principles of quantification of the 2\(^{nd}\) order in paper [48], and free principles of quantification of the 2\(^{nd}\) order in paper [49]. More specifically, theorems of Anderson-like theories in [46] and [47] are: classical principles of universal specification of the 1\(^{st}\) and 2\(^{nd}\) order, Barcan and converse Barcan formulas of the 1\(^{st}\) and 2\(^{nd}\) order, principles of exportation and converse exportation of the 1\(^{st}\) and 2\(^{nd}\) order. All of the 2\(^{nd}\) order principles were kept in Anderson-like theories in paper [48], while the classical principles of universal specification of the 1\(^{st}\) order were replaced with free ones. In comparison to [48], in Anderson-like theories in paper [49] the classical principles of universal specification of the 2\(^{nd}\) order were replaced with their free equivalents. The treatment of identity of the 1\(^{st}\) order is also important, in particular – modal Leibniz’s axiom schema: \(\Box (x \overset{1}{\approx} y) \rightarrow (\phi(z/x) \rightarrow \phi(z/y))\) where \(x\) and \(y\) are free for \(z\) in \(\phi(z)\), the axiom of necessity of identity: \((x \overset{1}{\approx} y) \rightarrow \Box (x \overset{1}{\approx} y)\), and the axiom of necessity of distinction: \(\exists \alpha (\alpha(x) \land \neg \alpha(y)) \rightarrow \Box \exists \alpha (\alpha(x) \land \neg \alpha(y))\) are theorems of all Anderson-like theories of [46], by contrast with the theories in [47] – [49].

The distribution of semantic “parameters” in specific types of semantics

\(^3\)The authorship of the axioms (i) and (iii) belongs to K. Gödel, and the authorship of the axiom (ii) belongs to Dana Scott (see, [1], [11]).
for Anderson-like ontological arguments are particularly interesting. Semantics of [46] and [47] adopt constant domains of the 1st order, whereas semantics of [48] and [49] adopt families of world-varying domains of the 1st order, in all four cases - with existing objects as their elements. All semantics, however, use families of world-varying domains of the 2nd order with existing properties as their elements. In addition, all these semantics share the common feature of being determined by conceptual properties, forming the following sets: in the case of semantics of [46] and [48], they are sets of all functions from the set of worlds to the union of all 2nd order domains, such that values of these functions at a given world belong to that world; and in the case of semantics of [47], they are some distinguished subset of the set of all functions from the set of worlds to the union of all 2nd order domains, such that values of these functions at a given world belong to that world; and, finally, in the case of semantics of [49], they are sets of all functions from the set of worlds to the family of all subsets of the union of all 1st order domains, such that if the value of a function at some world belongs to a 2nd order domain connected with this world, then every value of this function belongs to a 2nd order domain connected with its argument. The terms of the 1st order receive only rigid extensions in the set of existing objects in all semantics, whereas the terms of the 2nd order receive non-rigid extensions in the set of existing properties, and rigid intensions in the set of conceptual properties.

6 Ontological Proofs and Kind of Necessity

The notion of necessity, which we know from many expressions in natural language, is one of the core issues in contemporary philosophy, logic and other branches of science. There are many different types of necessity, although the exact criteria for the classification of these types are not widely agreed upon. Therefore, the statement that God necessarily exists gives rise to various puzzling questions concerning the necessity itself, including questions regarding its kind. The authors of the four papers in Part V point to different kinds of necessity with which ontological arguments can work.

In his paper [3], A. C. Anderson formulates the general form of the modal ontological argument, which we briefly present here. The argument rests on two premises: (i). It is possible that God exists; and (ii). It is necessary that if God exists then he necessarily exists. The process of reaching the conclusion: God necessarily exists from these premises, involves three
modal principles: (K*). If it is necessarily the case that if $\phi$ then $\psi$, and if $\phi$ is possible, then $\psi$ is possible; (B). If it is possible that it is necessary that $\phi$, then $\phi$ is true, and (T). If it is necessary that $\phi$, then $\phi$ is true. According to some philosophers, e.g. A. Church [4], M. Dummett [5], N. Salmon [41], and Y. Stephanou [44], principle (B) seems controversial. On the other hand, A.C. Anderson claims that “...whether or not this principle is correct depends of course on what concept of modality is being used” (p. 297). He introduces the notion of ‘conceptual modality’ as a certain relation between concepts. There are three types of concepts: propositions, attributes, and individual concepts. Propositions are “the kinds of abstract entities appropriate to be the meanings of sentences when such meanings are context independent”. And propositions “are taken to be concepts of truth-values, attributes – the sorts of things that can be attributed to something or other – are concepts of sets (or better: characteristic functions of sets) and individual concepts are concepts of, well, individuals of various kinds” (p. 298). In A.C. Anderson’s opinion, certain relations that happen between concepts stem from what these concepts are (with the respect, simply, of what the concepts are). Incompatibility or disparity is a primitive relation between concepts, according to Anderson. For example, being red excludes being blue, and these non-identities are conceptually necessary. Similarly, both statements: ‘$9 \neq 1+3$’ and ‘1 and 3 are the only smaller factors of 9’ are conceptually necessary. Furthermore, Anderson continues to add: “...let me just say that conceptual modality does not depend on form, generality, some specified list of “logical notions”, conceivability, semantical rules, definition, stipulation, or epistemic notions such as provability or deducibility” (p. 300). Having such an idea of conceptual necessity, Anderson sees the principle (B) as correct for conceptual necessity. He argues for the correctness of the principle, and refutes each of the objections raised by A. Church, M. Dummett, N. Salmon and Y. Stephanou against this principle.

In paper [10], S. Gerogiorgakis tries to answer the following question: Does the kind of necessity which is represented by $S5$ meet the traditional philosophical objections against ontological proofs? Yet, the author deliberately focuses on Kant’s criticism of the premise of the ontological argument that existence is a real predicate. According to Gerogiorgakis, there are entities the existence of which in the actual world necessarily follows from some property which is ascribed to them – an example of such entity is the second largest prime number calculated until now, namely $2^{42,643,801} - 1$. The same applies to God. Anderson-like ontological proofs,
modelled in S5, assert the existence of God as a consequence of His ultimate perfection (a property which is ascribed to God by definition). God, as an entity, is modelled in terms of logical necessity, therefore He is eo ipso actual. Gerogiorgakis writes: “The Kantian dictum “Existence is not a real predicate” does hold for entities like Sherlock Holmes, it does not hold however for entities whose existence is logically necessary.” (p. 313). Gerogiorgakis especially focuses on two counter-arguments to this view: (i). There are things – for instance, prime numbers – which exist by logical necessity in virtue of some property they have, a property other than existence that is; and (ii). Fictional entities have also some properties in the actual world. In his reply to (i), he says that (i) does not exclude God from being categorized among such things. And whether an alleged proof of the existence of God, which is supposed to be modelled in S5, is sound or not, depends only on logic alone. False or inadequate ontological proofs are satisfied with a certain version of the design argument which uses some natural necessity modelled on asymmetrical frames. With regard to (ii), Gerogiorgakis argues that even the assumption that fictional entities have some properties in the actual world does not mean that they do exist in this world. For example, the assumption that the sentence: ‘Sherlock Holmes is a human’ is actually true does not mean that the property of being a human implies the existence of Sherlock Holmes. In the case of God, His perfection implies His existence, which means that God is not a fictional entity but an ontologically real one.

Next, S. Kovač in [18] argues that the collapse of modalities, which is provable in Gödel’s ontological proof (precisely, in Dana Scott’s modification of Gödel’s ontological proof), should not be perceived as a defect of this proof, but as an intended consequence of Gödel’s ontology and a cosmological theory of the non-objectivity of the lapse of time. Kovač also shows that the concepts of modality and time should be derived in terms of cause, and he formalizes a causative version of Gödel’s ontology – a causative ontological proof, to be exact. For the sake of clarity, let us remind here that J. H. Sobel in [42] showed that Gödel’s axiomatic system is so strong that it implies that whatever is the case is so of necessity, \( \phi \rightarrow \Box \phi \), i.e. the modal system collapses. Therefore, it is crucial to ask: what causes this modal collapse? A few answers to this question can be found in the literature. C. A. Anderson argues in [2] that only the first (i) of the two components of the axiom: \( P(\alpha) \leftrightarrow \neg P(\neg \alpha) \), (i). If a property is positive, its complement is not positive; and (ii). If a property is not positive, its complement is positive, is acceptable, and the second component
(ii) is responsible for the collapse of modalities. C. M. Adams in [1] and A. P. Hazen in [14] claim that the collapse of the modalities can be eliminated by modifying the too broad concept of property. The too broad principle of abstraction, adopted by Sobel, also includes all the sentences which are seen as the properties of a given entity, even if one of these properties does not apply to this entity. With the use of a certain trick, Adam’s and Hazen’s suggestion may be proved to be false. M. Fitting in [6] proposes a different way of avoiding the collapse. He reformulates Scott’s modification of Gödel’s ontological proof by distinguishing between intentional and extensional properties, and using only the latter ones. Indeed, Fitting’s reformulation is free of modal system collapse. There are still other proposals of avoiding the collapse, for instance by a modification of the following axiom: the property of necessary existence is a positive property, or a rejection of the comprehension principle (\(\lambda\)-conversion schema), as argued by R. C. Koons in [16] and S. Kovač in [17]. In this context, Kovač approach to the modal collapse in Gödel’s ontological proof seems particularly original. Kovač refers to Gödel’s texts which supposedly show that the modal collapse was in fact Gödel’s intention, and that it is a part of his general philosophical view. Kovač suggests a very interesting way in which modality in Gödel’s ontological proof could be replaced by causality; see, his paper [18], pp. 333 - 335 – for philosophical basis, and pp. 335 - 340 – for a formalization of causative ontological proof.

Finally, moving on to Swinburne’s paper [45], certain introductory remarks should be made. Many competent judges, with Malcolm and Harschorne being chief advocates of this view, claim that God must be conceived as a necessary being, and that it is possible to demonstrate his existence a priori. According to the opponents of this view, the statement that God is a logically necessary being is encoded in it. But, whether or not a being can be logically necessary, is a quaestio disputata. The opponents argue that only propositions could be thought of as logically necessary, and, consequently, God should be defined as a physically or factually necessary being. And further, the sentence ‘a physical necessary being exists’ is not logically true, since the existence of a physically necessary being can be proven only by an a posteriori argument.

R. Swinburne in [45], on other hand, deals with the following question: Is it metaphysically possible that God is a metaphysically necessary being? He introduces some terminological distinctions. And so, a substance or an event is metaphysically necessary iff it is metaphysically necessary that it exists. Since there exists no discussable knowledge of whether it is
metaphysically necessary that a substance or an event exists, except (at least in part) by reflecting on features of the sentence which asserts this, Swinburne prefers to talk about necessity as belonging to the sentence. Among metaphysical necessities, he distinguishes the logical ones, that is, the ones that are discoverable a priori – by mere reflection on what is involved in the claim made by the sentence. More precisely, a logically necessary sentence is one the negation of which entails a contradiction, a logically impossible sentence is one which entails a contradiction, and a logically possible sentence is one which does not entail a contradiction. According to Swinburne, metaphysical necessity is the strongest kind of necessity, metaphysical impossibility is the strongest kind of impossibility, and metaphysical possibility is the weakest kind of possibility. And importantly, not all metaphysical necessities or impossibilities are logical necessities or impossibilities, respectively. For there exist metaphysically necessary or impossible sentences, the truth or falsity of which may be established only a posteriori, and not a priori. In Swinburne’s words, p. 353:

“What has made these necessary sentences a posteriori is that the sentence contains at least one rigid designator of which we learn the meaning by being told that it applies to certain particular things (especially substances and kinds of substances) having certain superficial properties, but where – we are told – what makes a thing that thing (that substance or a substance of that kind) is the essence (of which we may be ignorant) underlying those properties.”

Such rigid designators can be either informative or uninformative designators. A rigid designator is an informative designator iff someone who knows what the designator means knows a certain set of conditions necessary and sufficient for a thing to be that thing. To the contrary, a rigid designator is an uninformative designator iff these conditions are not satisfied. Having these terminological issues settled, Swinburne decides that God cannot be a logically necessary being. And God is not a metaphysically necessary being, because the very existence of any metaphysically necessary beings is logically impossible.

7 Ontological Proofs and Formal Ontology

R. E. Maydole in [25] reconstructs Grim’s Cantorian argument against the existence of an omniscient being in the following way:
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(G1) If a God of Theism exists, then something is omniscient.

(G2) If something is omniscient, then the class of all truths $\exists$ exists.

(G3) If $\exists$ exists, then the class of all subclasses $\mathcal{P}(\exists)$ (the power class $\mathcal{P}(\exists)$) of $\exists$ exists.

(G4) For every member $x$ of $\mathcal{P}(\exists)$, there is a truth $y$ such that $y$ is about $x$ and not about any other member of $\mathcal{P}(\exists)$.

(G5) If there is a truth $y$ that is about every member of a class $x$ and not about any other member of $x$, then $\exists$ has at least as many members as $x$.

(G6) $\exists$ has fewer members than $\mathcal{P}(\exists)$.

(G7) For any classes $x$, $y$ and $z$, if $x$ has fewer members than $y$, and $z$ has at least as many members as $y$, then $x$ has fewer members than $z$.

(G8) It is not the case that $\exists$ has fewer members than $\exists$.

Therefore, a God of Theism does not exist.

According to Maydole, the premise (G4) of this argument is false, and the premises (G3), (G6) and (G8) do not seem to be provable in Quine’s axiomatic set theory with classes of truths. Maydole also argues that because it has never been shown that the axiomatic set theory of Zermelo-Fraenkel that undergirds Grim’s reasoning is more plausible than Quine’s axiomatic set theory and all other set theories, therefore Grim’s Cantorian argument fails to falsify the existence of the God of Theism; cf. [25], pp. 373 - 375. The premise (G4) is false for two reasons: (i). Both premises (1) and (2) in the Grim’s argument for (G4) – (1). To each member of $\mathcal{P}(\exists)$ there corresponds a distinct truth; (2). For example, it is distinctly true that $t_1 \notin \emptyset$, that $t_1 \in \{t_1\}$, that $t_1 \notin \{t_2\}$, that $t_1 \notin \{t_3\}$, ..., that $t_1 \in \{t_1,t_2\}$, that $t_1 \in \{t_1,t_2\}$, ..., that $t_1 \in \{t_1,t_2,t_3\}$, ... ; (3). Therefore, there are as many distinct truths as there are members of the power set $\mathcal{P}(\exists)$ – are questionable, because (1) merely rephrases the conclusion, and (2) merely lists a finite number of members of $\mathcal{P}(\exists)$, which is surely infinite, with members that are impossible to even list or enumerate; and (ii). (G4) contradicts Cantor’s Theorem in the axiomatic set theory of Zermelo-Fraenkel.

In his paper [31], E. Nieznański settles the matter of sequitur in so-called “ways” ex gradibus perfectionis by Saint Anselm and Saint Thomas Aquinas by means of the method of formalization, and to examine its logical values.
Nieznański refers to certain texts of Anselm and Thomas Aquinas, and formalizes them in the first order language. The formalization of Anselm’s argument for the existence of God is as follows:

1. \( \exists_1 x \neg \exists y (x < y) \), read: There is exactly one object such that nothing greater - more excellent - is thinkable. (Premise)

2. \( \forall x (\neg E(x) \to \exists y (x < y \land E(y))) \), read: For each non-existent object \( x \) there is a certain existent object - being - \( y \) such that \( y \) is thinkable as a greater - more excellent - than \( x \). (Premise)

3. \( \neg \exists y (b < y) \), read: There exists no object that is greater than \( b \) (\( \text{df} \) God) (From (1))

4. \( \neg E(b) \to \exists y (b < y) \), read: If the one God does not exist, is not a being, then there is at least one object that is greater than he. (From (2))

5. \( E(b) \), read: God really exists. (From (3) and (4))

This deduction is free from the non sequitur and fallatium elenchi.

And the formalization of the Aquinas’s argument for the God’s existence is as follows:

1. \( \forall x \forall y (x > y \to \neg (y > x))) \), read: The relation > of a higher excellence is asymmetrical. (Premise)

2. \( \exists y \forall x (x \neq y \to y > x) \), read: There is a being more excellent than all others. (Premise)

3. \( \forall y \forall z (\forall x (y \neq x \to y > x) \land \forall x (z \neq x \to z > x) \to z = y) \), read: There is at most one highest being. (From (1))

4. \( \forall x (b \neq x \to b > x) \), read: God is a being more excellent than all others (the most excellent). (From (2) and (3))

5. \( \forall x (b \neq x \to b > x) \to \exists y (y = b) \), read: God defined as the only most excellent being actually exists. (It can be proved)

6. \( \exists y (y = b) \), read: A certain being is identical to God, thus God exists. (From (4) and (5))

This deduction is free from any formal or material fallacy, and it is not burdened with fallatium elenchi.

E. Nieznański compares the two deductions and says that if Thomas’s discourse were not limited only to beings, and Anselm’s universe of all objects were adopted, then two more premises, (1) and (2), should be added to Thomas’s:
With these additions, the final conclusion would be: $E(b)$, read: *God really exists*.

Perzanowski’s paper [35] may be summarized in the following way. Given a logical frame $\mathcal{U} = \langle U, M \rangle$, where $U$ is the universe of all objects (beings) and $M$ is a binary relation on $U$ ($xMy$ means: $y$ is better (is melior or is greater) than $x$), God is a co-atom in $\mathcal{U}$ defined as follows: $x$ is God (god-like) $\overset{df}{=} \exists y \text{ no object } y \text{ such that } y \text{ is different from } x$ and $y$ is greater than $x$. God*-like beings are also defined: $x$ is god*-like $\overset{df}{=} \exists y \text{ no object } y \text{ such that } y \text{ is greater than } x$. Perzanowski distinguishes six types of melioration: (i). Anselm’s melioration – a property $A$ is $M$-meliorated ($MA$) iff for every object $x$: if $x$ does not have a property $A$ then there exists an object such that it has the property $A$ and is greater than $x$; (ii). Augustine’s melioration – a property $A$ is $M^*$-meliorated ($M^*A$) iff for every pair of objects $x, y$: if $x$ does not have a property $A$ and $y$ has it then $y$ is greater than $x$; (iii). Overmelioration – a property $A$ is $M'$-meliorated ($M'A$) iff for every pair of objects $x, y$: if $x$ does not have a property $A$ then $y$ has the property $A$ and $y$ is greater than $x$; (iv). Cantor’s transmelioration – a property $A$ is $T$-meliorated ($TA$) iff for every object $x$: if $x$ does not have a property $A$ then there exists an object such that it has the property $A$ if and only if it is greater than $x$; (v). Nearly trivial melioration – a property $A$ is $T'$-meliorated ($T'A$) iff for every object $x$: if $x$ does not have a property $A$ then there exists an object such that if it has the property $A$ then it is greater than $x$; and (vi). Trivial melioration – a property $A$ is $T^*$-meliorated ($T^*A$) iff for every object $x$: if $x$ does not have a property $A$ then there exists an object such that if it is greater than $x$ then it has the property $A$.

It can be proved that the following relations hold: $M'A \Rightarrow M^*A \Rightarrow MA \Rightarrow TA \Rightarrow T^*A \Rightarrow T'A$. Perzanowski illustrates some of those meliorations with examples and characterizes them through proving relevant theorems. Anselm’s melioration is of special importance here. By using it, Perzanowski formulates his own version of Anselm’s ontological argument, which he, however, describes as cosmological argument based on the empirical assumption about the mind of the Fool to prove the consistency of the cosmo/onto/logical concept of Deity – in Perzanowski’s words.
The thesis of the Turri’s paper [52] may be formulated as follows: No ontological argument can succeed in reaching the conclusion that God exists, because ontological proofs attempt to achieve this non-empirically. Turri justifies this thesis in the following way:

(1) If you can non-empirically know that a certain person exists now, then you = that person. (Premise)
(2) If God exists, then God is a person. (Premise)
(3) So if you can non-empirically know that God exists now, then you = God. (From (1) and (2))
(4) If any ontological argument can succeed for you, then you can non-empirically know that God exists now. (Premise)
(5) So if any ontological argument can succeed for you, then you = God. (From (3) and (4))
(6) You ≠ God. (Premise)
(7) So no ontological argument can succeed for you. (From (5) and (6))
(8) If no ontological argument can succeed for you, then no ontological argument can succeed for any of us. (Premise)
(9) So no ontological argument can succeed for any of us. (From (7) and (8))

Turri also discusses some limitations of the above argument and voices his objections to it.

According to P. Weingartner [54], Anselm’s ontological argument consists of two parts: the first part – its aim is to show that the *something than which no greater can be conceived* (*QM*), the existence of which is to be proved, exists in the mind; and the second part – its aim is to show that the *QM* also exists in reality. Weingartner expresses the first part in three statements: (i). Religious believers understand by God a ‘something than which no greater can be conceived’; (ii). Even the fool i.e., also the non-believer understands the words ‘something than which no greater can be conceived’; and (iii). What the believers and the non-believers understand exists in their minds. These three statements are empirical ones, and – unlike interpretations of Anselm’s argument which use only the second part, Weingartner shows that Anselm’s argument can be interpreted with the help of intuitionistic logic in such a way that one of the premises of the argument must be empirical. This empirical premise says that a certain
a is that than which no greater can be conceived which exists in the mind. Next, Weingartner poses the following question: Are the premises of the (intuitionistically interpreted) Anselm’s argument self-evident in the sense of Aquinas? Where, according to Aquinas, a proposition is self-evident because the predicate is included in the essence of the subject, as ‘Man is an animal’, for animal is contained in the essence of man. And he distinguishes between self-evident in itself and not to us – if the essence of subject and predicate is known to all, the proposition is self-evident to all; and self-evident in itself and to us – if this is not the case, the proposition might be self-evident in itself, but not to us. Weingartner reflects only on the second meaning of the notion of self-evident and he comes to the conclusion that the premises of the (intuitionistically interpreted) Anselm’s argument are not self-evident to us, and consequently – the conclusion that God exists is also not self-evident to us. The other question that Weingartner asks is: Are the premises of the (intuitionistically interpreted) Anselm’s argument analytic in the sense of Kant? In [54], the author considers the following senses of analyticity: (1). Analytic truths (AT) are true in the sole virtue of the meanings of the terms they contain: (1a). AT’s are based on the definitions of terms, (1b). AT’s are based on the definitions of terms and their logical consequences, (1c). AT’s can be proved by means of logical laws and definitions (Frege), and (1d). AT’s are truths of logic and all truths which can be reduced to them by substituting synonyms for synonyms (Quine); and (2). Analyticity of arguments: (2a). An analytic argument cannot lead from the existence of an individual to the existence of a different individual (Kant), (2b). An argument step is analytic, if it does not introduce new individuals into the discussion, and (2c). An argument step is analytic iff it does not increase the number of individuals considered in their relation to each other. These differentiations are more or less connected to Kant’s ideas. Weingartner concludes that if an argument is considered to be analytic only if all of its premises are analytic, then (intuitionistically interpreted) Anselm’s argument is not analytic for sense (1a) and (1b). Also, none of the premises of this argument are analytic in the sense (1c) and (1d). Finally, Anselm’s argument in its intuitionistic version is analytic in the senses: (2a), (2b), and (2c), but its classical version is non-analytic in all these senses.
8 Debate Maydole-Oppy

A discussion between R. E. Maydole and G. Oppy regarding certain ontological arguments is presented in the present section. The starting point of the discussion is Maydole’s paper: The Ontological Argument; see, [24]. The author’s goal in this paper is to logically evaluate logical reconstructions of six ontological arguments, namely of: St. Anselm, Descartes with its reconstruction by Leibniz, Malcolm, Hartshorne, Plantinga, and Gödel. Maydole also presents and discusses two modal arguments of his own authorship.

As Maydole notes, p. 553:

“The logical evaluation of a logical reconstruction of an argument often requires that we explicitly identify assumptions that are only implicit in the author’s original presentation of the argument. And in some cases, it might involve the inclusion of “plausible” philosophical principles that are consistent with the author’s worldview, principles that strengthen the argument if we include them among the premises of the reconstruction. My modus operandi will be to make each of the arguments as strong as possible before critically evaluating them. Even though I shall try to remain reasonably faithful to the intent of the original author of each argument, my main objective will be logical instead of historical.”

The author shows the validity of particular reconstructed arguments, examines the truth of their premises, argues that these arguments are not question-begging, and tests their vulnerability to being refuted by some of the well-known parodies in the philosophical literature.

In the following, we briefly report on the discussion between Maydole and Oppy regarding the above-mentioned ontological arguments. Thus, Maydole reconstructs Anselm’s argument in eight steps and enriches his version with a formal deduction consisting of twenty-seven steps. These reconstructions are not cited here, since they were already presented – although in a slightly modified form, when it comes to the formal deductions – in Oppy’s paper Maydole on Ontological Arguments, included in our book, pp. 445 - 468. According to Maydole, this formal deduction is logically correct, i.e. it proves the validity of the argument, which is not questioned by Oppy. The authors do differ, however, in their evaluations of the truth of the premises. According to Maydole, all the premises are true: the first is true by definition; the second is introspectively and analytically true. For the third premise, Maydole develops a deductive argument (1.
Concepts are in that which conceives of concepts; 2. Whatever is in that which conceives of concepts has existence-in-the-understanding; 3. Therefore, the concept of whatever a definite description that is understood refers to has existence-in-the-understanding.) and an analogical argument (It is identical to the deductive argument, save for 2* in place of 2, where 2*. Having the property of existence-the-understanding is like having the property of being in that which conceives of concepts.) – evaluating the first one as valid and the second one as inductively strong. The fourth premise, according to Maydole, is intuitively obvious, while for the fifth premise he formulates a plausible argument sketch (1. Things that have existence-in-reality are ontologically complete. (Premise); 2. The property of being ontologically complete has the property of being great-making. (Premise); 3. For every property X and Y, X has Y if and only if everything that has X has a property that has Y. (Premise); 4. The property of being ontologically complete has the property of being great-making if and only if everything that has the property of being ontologically complete has a property that has the property of being great-making. (3, UI); 5. Everything that has the property of being ontologically complete has a property that has the property of being great-making. (2,4, Equiv, Simp, MP); 6. Hence, everything that has the property of existence-in-reality has the property of being great-making. (1, 5, UI, HS, UG); 7. The property of existence-in-reality has the property of being great-making if and only if everything that has the property of existence-in-reality has a property that has the property of being great-making. (3, UI); 8. Hence, the property of existence-in-reality has the property of being great-making. (6, 7, Equiv, Simp, MP.).) For the sixth premise, Maydole gives two arguments: neo-Platonic argument (1. \( \forall x (M(x) \rightarrow \circ E(x)) \); 2. \( \forall x (S(x) \rightarrow M(x)) \); \( \therefore \forall x (S(x) \rightarrow \circ E(x)) \)), where \( M(x) \equiv \) a mental replica of x exists-in-a-mind; \( S(x) \equiv \) the concept of x exists-in-the-understanding; \( E(x) \equiv x \) exists-in-reality; and \( \circ \ldots \equiv \) it is conceivable that..., and ultra-realistic one (1. \( \forall x (M(x) \rightarrow \circ E(x)) \); 2. \( \forall x (S(x) \rightarrow \circ M(x)) \); \( \therefore \forall x (S(x) \rightarrow \circ E(x)) \)); and finally, he shows that the seventh premise is true in a couple of ways. First, as he acknowledges, the following argument is valid and both premises are logical truths: 1. \( \forall Y \forall z (z = (ix)Y(x) \rightarrow Y(z)) \); 2. \( g = (ix)\neg \circ \exists y G(y,x) \); \( \therefore \neg \circ \exists y G(y,x) \), where \( G(x,y) \equiv x \) is greater than y. Second, using the Russell’s Theory of Descriptions to eliminate the definite description from the seventh premise, formally written, and assuming the equivalence of conceivability and possibility, one gets the formula: \( \neg \Diamond \exists x (\neg \Diamond \exists y G(y,x) \land \neg \circ \exists y G(y,x) \land \circ \exists y G(y,x)) \).
∀z(¬◊∃yG(y, x) → z = x) ∧ ∃yG(y, x)), which is logically true even in modal logics weaker than S5, which proves that the seventh premise is true. According to Oppy, Maydole’s reconstruction of Anselm’s argument begs the question. In [26], p. 470, Maydole comments on Oppy’s objection as follows:

“Oppy intimates that AOA begs the question because I allow the quantifiers to range over possibilia (all things that either might or do exist). That means, according to Oppy, that Anselm’s Fool would have to grant the existence of God if she/he accepted both modal logic S5 and the belief that God must have necessary existence. And it also means, he says, that if necessary existence is a great making property, then God cannot even refer or be understood (if understanding the term ‘God’ implies that it refers) unless God exists in reality (pp. 448 - 449).”

Maydole replies to Oppy’s objection and gets a response in Oppy’s [32]. Next, Maydole argues in [24] that his version of Anselm’s argument is not question begging. Strictly speaking, he exposes the faulty nature of W. L. Rowe’s argumentation [40], according to which Anselm’s ontological argument begs the question by granting what it tries to prove. As Maydole puts it, pp. 561 - 562:

“According to Rowe, Anselm’s ontological argument boils down to one that defines God as a greatest possible being, and also counts the property of existence-in-reality as great-making. These two things, he correctly argues, imply that nothing that fails to exist-in-reality can be a greatest possible being; but they do not alone imply that a greatest possible being actually exists-in-reality. However, if we also assume that a greatest possible being possibly exists-in-reality, we can then infer that a greatest possible being actually exists-in-reality because no such possible being could fail to exist-in-reality and still be a greatest possible being, given that the property of existence-in-reality is great-making. This means, Rowe then concludes, that the assumption that a greatest possible being possibly exists-in-reality is “virtually equivalent” to the concluding proposition that it actually exists-in-reality. “In granting that Anselm’s God is a possible thing, we are in fact granting that it actually exists … the argument begs the question: it assumes the point it is trying to prove” ([40], p.41).”

In Maydole’s opinion, Rowe’s argumentation is wrong due to two reasons: (i) He equivocates on the word ‘grant’, which can mean either ‘assume’ or ‘implies’. But, if we assume the premises which imply the
conclusion of an argument, then it does not mean that the argument begs the question; (ii). The proposition that a greatest possible being possibly exists-in-reality does not even imply the proposition that it actually exists-in-reality, unless we assume that the other premises are logical truths; however, such an assumption cannot be made since the premise that the property of existence-in-reality is great-making is not a logical truth. Even if such an implication between the two propositions existed, then “the argument would beg the question only if the latter were given as a reason for believing the former”. Oppy in [33], pp. 453 - 455, criticises Maydole’s argumentation and argues that this version of Anselm’s argument is question-begging. Neither Maydole in [26] and [27], nor Oppy in [34] return to this issue.

Moving on to the discussion of parodies of ontological arguments, what Maydole generally sees as a parody of an argument, is an argument that is structurally similar to the argument parodied but has an a absurd conclusion. According to Maydole, parodies can refute the argument parodied in two ways: (i). If the parody has true premises and the same logical form as the argument parodied, then the argument parodied must be invalid; (ii). If both the parody and the argument parodied are valid and the premises of the parody are not less justifiable than the premises of the argument parodied, then at least one of the arguments parodied must be unjustifiable. Having made these general remarks, Maydole formalizes Gaunilo’s parody of Anselm’s argument and concludes that this parody is valid but not sound, due to one of the premises – namely, the seventh premise: $\neg \exists x G(x, i)$ is false. Replacing ‘$G(x, i)$’ in this controversial premise with ‘$(I(x) & G(x, i))’$, he admits that even though the premises of the new parody are true, provided that Anselm’s premises are true, the parody is invalid. Moreover, by conducting essentially the same replacement in the fourth premise, Maydole sees the newly-emerged parody as valid but not sound, because the new fourth premise: $\forall x \forall Y (P(Y) & \neg Y(x) & \circ Y(x) \rightarrow \circ \exists y (I(y) & G(y, x))$ is false. Maydole proposes two more modifications of the fourth premise, namely: $\forall x \forall Y (P(Y) & \neg Y(x) & \circ Y(x) \rightarrow \circ \exists y (L(y, x) & G(y, x))$ and $\forall x \forall Y (P(Y) & \neg Y(x) & \circ Y(x) \rightarrow \circ \exists y (K(y, x) & G(y, x))$, where ‘$L(y, x)$’ is short for ‘$y$ is exactly like $x$ except for having $Y$ in place of the negation of $Y$’ and ‘$K(y, x)$’ is short for ‘$y$ is the same kind of thing as $x$’. Both emergent parodies are assessed by Maydole as valid, since $\circ \exists y (L(y, x) & G(y, x))$ and $\circ \exists y (K(y, x) & G(y, x))$ entail $\circ \exists y (I(y) & G(y, x))$. In [33], pp. 449 - 453, Oppy puts all of the parodies discussed by Maydole into one form,
which he, on the one hand, finds valid, provided that Anselm’s argument is valid, and the other hand, he says that there is no evident reason to see the premises of the parody as less plausible than the premises of the original argument. Both authors defend their stands in the following papers: the first in [26], pp. 470 - 472, and [27], pp. 501 - 502, and the second in [34], pp. 488 - 489.

Maydole’s version of Descartes’ ontological argument contains four premises followed by the final conclusion; see Maydole [24], pp. 566 - 567, and also Oppy [33], Theorem, p. 457. In a 17-step formal deduction, Maydole proves the validity of this argument. With regard to the truth of the premises, Maydole claims that the first is analytically true, and the second and the fourth are synthetic a priori metaphysical truths. The validity of the third premise can be proved by a simple argument. Leibniz disagrees with this line of thinking and proposes his own argument, reconstructed by Maydole in five steps; see [24], p. 569, and Oppy [33], Lemma 2, p. 457. Except for the first premise, the validity of which is proved by Leibniz in one of his arguments and also reconstructed by Maydole in nine steps; see [24], p. 569, and Oppy [33], Lemma 1, pp. 456 - 457, the three remaining premises are seen by Maydole as self-evident.

Next, Maydole discusses the three main critiques of the Descartes-Leibniz ontological argument: (i). Kant’s critique of the fact that Descartes and Leibniz include existence as a property in the concept or essence of a supremely perfect being, which, according to Kant, is not a property at all; (ii). the critique that the argument begs the question; and (iii). the critique that the argument is easily parodied. Regarding (i), Maydole argues the following; [24], p. 570:

“Kant is half right and half wrong. He is right that existence is not a property in the usual sense of being includable in the concept of a thing. ... But, contrary to Kant, I think that we do predicate something new of a thing when we say that it exists.”

Regarding (ii), he carefully admits that perhaps the argument would beg the question if existence was really included (by definition) in the concept of a supremely perfect being, and he adds that this is not the case with his reconstruction of the argument. Finally, regarding (iii), he considers two parodies of the argument; the first one – the parody of Caterus, Descartes’ contemporary author, and the second – a parody constructed by himself through a replacement of ‘supremely perfect’ in his reconstruction of Descartes’ argument with ‘a non-supreme necessary being’. The first of
these parodies is invalid, while the second one is valid, but its premises are not as justifiable as the corresponding premises of Descartes’ ontological proof. Hence, both parodies fail to refute the argument parodied. Otherwise, Oppy in [33], pp. 457 - 460, argues that the Descartes-Leibniz ontological argument is susceptible of parody, though not of the kinds of parodies that Maydole discusses, and that these parodies suffice to show that the parodied argument is unsuccessful. Maydole, in turn, analyzes in [26], pp. 473 - 476, Oppy’s parody of Descartes-Leibniz ontological argument and shows in two ways that the fourth premise of his parody – namely: *Necessarily, supremely Q-perfect beings are necessarily supremely Q-perfect*, where Oppy defines a supremely Q-perfect being as one that has all and only Q-perfections essentially – is false. Oppy replies to Maydole’s argumentation in [34], to which Maydole answers in [27], pp. 502 - 503.

As already mentioned in Section 1, in the process of comparing ontological arguments Maydole finds the modal arguments of N. Malcolm [22], Ch. Hartshorne [13], A. Plantinga [36] particularly important. Maydole reconstructs them in [24], but he does not devote too much attention to them in his analyses. In fact, he focuses mainly on Dana Scott’s version of Gödel’s ontological argument; see, [42]. He claims that these three modal ontological arguments are valid, accounting for the reasons of their validity, yet he doubts their soundness. According to Maydole, “the first premise of each of these arguments, which effectively says in each case that it is possible for God to exist”, is questionable. According to him this is also the case with the 7th premise in Malcolm’s argument (*Anything that neither begins nor ceases to exist exists necessarily if it exists at all, and fails to exist necessarily if it exists at all*), “especially if the modalities are constructed logically, because an eternal being that does not exist in all possible worlds certainly is possible”. As for Scott’s version of Gödel’s ontological argument, Maydole evaluates it as valid, because, in his opinion, axioms imply that something is God-like. In order to prove that Scott’s version of Gödel’s ontological argument is not sound, Maydole chooses the so-called modal collapse argument of J. H. Sobel ([42]) as the starting point of his argumentation; see, Section 6. In their further works, neither Oppy nor Maydole engages in discussions related to the three modal ontological arguments mentioned above, or to Gödel’s ontological argument and its versions.

The final two sections in Maydole’s paper [27] are devoted to his own ontological arguments called: *the modal perfection argument* and *the temporal-
contingency argument, which were repeated by Oppy in [33]. The author shows the validity of the first of these arguments in two steps: firstly, he shows that the premises imply that it is possible that a supreme being exists, and secondly, he proves that the possibility of a supreme being implies the existence of a supreme being. In his previous paper [23], he already proved that all the premises are true. He also says that it can be easily shown that an Oppy-style parody based on the idea of an almost supreme being does not refute his argument. There is also no evidence, in Maydole’s opinion, that would indicate that the modal perfection argument begs the question. With regard to the second argument, Maydole presents a very long proof of its validity and evaluates all the premises of this argument as true. Also, in his view, there is no evidence showing that the argument begs the question and it should be resistant to being parodied. Criticizing these views, Oppy in [33] claims that the modal perfection argument can be refuted by ‘the supreme island’ parody (pp. 462 - 463), and he argues that this argument begs the question (pp. 463 - 464). As far as the temporal-contingency argument is concerned, Oppy contrasts two different theories: Theism and Naturalism. Since it is impossible that both these theories are true, any successful argument for Theism that requires the assumption that Naturalism is false simply begs the question against Naturalism. Both authors defend their stances, providing additional new arguments and counter-arguments in the following papers: Maydole [26] and [27], and Oppy [34].

Bibliography


[18] ————, “Modal collapse in Gödel’s ontological proof”, in [50], 323 - 343.


[33] ————, “Maydole on Ontological Arguments”, in [50], 445 - 468.

[34] ————, “Response to Maydole”, in [50], 487 - 499.


[39] ————, “A Gödelian ontological argument improvised even more”, in [50], 203 - 211.


[41] Salmon N., “The logic of what might have been”, in *Philosophical Review*, 98 (1989), 3 - 34.


[52] Turri J., “Doomed to fail: the sad epistemological fate of ontological arguments”, in [50], 413 - 421.


Part II

Interpretation of Old Ontological Proofs. God’s Attributes
Aristoteles was the first to introduce the elenctic method as a defined method of argumentation and inquiry in philosophy. It is based on the principle of contradiction. One can only reject the principle of contradiction if one makes use of this same principle of contradiction. If one has to make use of the same principle in order to prove that this principle is not valid then rejection of this principle is invalid and the argument is falsified, proves itself wrong by implication.

The elenctic method in philosophical argumentation has been used extensively throughout the history of philosophy. Elenctic methodology plays a prominent role in the philosophy of Johann Gottlieb Fichte. The “Tathandlung” is the embodiment of all principles of knowledge and is explained in the first “Grundsatz der Wissenschaftslehre”. This “Tathandlung” can be rejected only by implicitly contradicting oneself: if one rejects the validity of the first “Grundsatz” one either states there is no knowledge possible or one states knowledge has no principles.

Both alternatives are non-sensical. If one rejects the statement, that knowledge is based on reason, one is forced to implicitly accept reason as the base of all arguments made in order to give specific reasons as to why knowledge is not based on reason.

Elenctic methods are also employed in Hegel’s philosophy. Reinhold Aschenberg identified such elenctic elements in Hegel’s *Phänomenologie des Geistes* explicating Hegel’s *Phänomenologie des Geistes* is an introduction developing the standpoint of the absolute knowledge as highest form of knowledge using elenctic mode of argumentation throughout.

\(^1\)See [4], p. 388.
It is not apparent that elenctic methods play a key role in Anselm’s argument. Anselm’s argument is foremost proof of the highest being’s existence, named god. Elenctic methodology is not well fit to prove the existence of transmental beings. Elenctical methodology is clearly restricted and is applicable to the principles of thinking itself only. Nonetheless one can make an argument that elenctic methodology plays an essential role in Anselm’s argument in “Proslogion II”.

Anselm’s argument encompasses two integral components, that one can distinguish: an elenctical moment and a regressive moment. In “Proslogion II” [3] the “insipiens” is asserting that the “aliquid quo maius nihil cogitari potest” (p. 84) is not a being in mind and in reality. The “insipiens” is in conflict with the rules of thinking, because this implies the proposition.

“Si ergo id quo maius cogitari non potest, est in solo intellectu: id ipsum quo maius cogitari non potest, est quo maius cogitari potest. Sed certe hoc esse non potest.” ([3], p. 84)

It is very important to understand, what Anselm really refers to, when he uses the term “maius”. It is unlikely, that Anselm takes reference to a “stairway” to perfection. Anselm rather means to unfold the relationship between ‘principium’ and ‘principiatum’. The ‘principium’ in relationship to the ‘principiatum’ is “maius”, and the ‘principiatum’ in relationship to ‘principium’ is “minus”.2 When one has one’s mind on “Aliquod quo maius nihil cogitari potest”, one is thinking the highest principle of mind.3 But what means thinking in Anselm’s terminology. Thinking is the ability to distinguish presence in thought and as anything it relates to (=being in mind and in reality) from presence, exclusively in thought, without reference to anything other than itself (=being exclusively in mind).4 The highest principle is what makes it possible for whoever is engaged in thinking to distinguish between being in mind and reality and being exclusively

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2 A. Schurr points out: “Stehen zwei Glieder in einem Grund-Folge-Verhältnis, so wird dem einen ein maius esse, dem anderen ein minus esse zugesprochen. Da dem einen Glied ein Moment abgeht, nämlich dieses Grundsein, wodurch es gesetzt ist und ohne welches es nicht wäre, kommt der Folge ein minus esse zu. Andererseits liegt beim Grund ein maius esse deshalb vor, weil es die Folge um das Moment des Grundseins überragt.” ([9], p. 89)  
3 “Quidquid enim per alius est magnum, minus est quam id, per quod est magnum.” ([2], p. 48)  
4 See Riesenhuber [8], pp. 45 - 48, and p. 50; [5], pp. 22 - 58; and [6], pp. 21 - 34.
in mind.\textsuperscript{5} Utilizing the elenctic mode of argumentation in “Proslogion II” Anselm shows that one can reject the validity of the highest principle only if one contradicts oneself. The unanswered question now is: how is the highest principle characterized as best? One has to keep in mind: The capacity to distinguish being, present in mind and in reality from being, present solely in mind is the highest principle of thinking. The highest principle, which makes that difference possible, cannot itself be subject to this difference. Thus the highest principle must be either an invariable (!) entity, present in mind and in reality (proposition 1) or an invariable (!) entity, present solely in mind (proposition 2).\textsuperscript{6} Proposition 1 and proposition 2 are mutually exclusive.

If the embodiment of principles of thinking is the highest principle, the highest principle would end up as an entity, valid even as mere thought in mind, if one follows the argument to the end. The logical rules and laws are “beings in mind and in reality”. The logician does not create logical laws, the logician identifies logical laws as something that are rules of thinking. Logical laws are valid irrespective of being identified by the logician and are not subject to time.

As such logical laws are very much subject to the being in mind and in reality and being solely in mind. Each act of thinking ends regardless of how long that takes and is finite. The logical principles are “beings in mind and in reality” only during the act of thinking, and end up, after the act of thinking the principles has ended, as entities solely in mind. As such they do not exist independent of the act of thinking. The act of thinking and its product, the thought, would not be subject to any rules,

\textsuperscript{5}Klaus Riesenhuber points out: “Sofern aus dem verstandenen Begriff die Frage nach der Wirklichkeit, dem Sein des in ihm gedachten Sinngehalts entspringt, vollzieht sich in ihm ein Vorgriff über den bloßen Begriff hinaus auf Sein selbst. Denn der Sinngehalt als solcher ist als Sinngehalt von etwas (res, aliquid) gemeint, dieses Etwas aber im Denken als der Intention des Sinngehalts vorausgesetzt problematisch angezweifelt. Begrifflicher Sinngehalt ist also ein solcher nur in Beziehung auf jenes, das nicht in gedachtem Sinngehalt aufgeht, auf Sein. Im Denken als solchem zeigt sich so die Differenz zwischen der Intention eines begrifflichen Sinngehalts und der diese Sinnintention als solche ermöglichenden Intention auf Wirklichkeit; [...]. Seinsverständnis wird also nicht aus dem Standpunkt eines Dritten, der außerhalb von Intellect und Wirklichkeit stünde und ihre Beziehung beschrieben, erschlichen. Vielmehr ist das Verständnis von Sein ursprünglich, schon für das Verständnis des Begriffs grundlegend, dem Denken als solchem gegeben.” ([8], p. 46 f.)

\textsuperscript{6}“Quod utique sic vere est, ut nec cogitari possit non esse” ([3], Proslogion III)
and, implicitly, as the only real essence left, for good.

Let us now look at the alternative that an extramental being is the highest principle. This alternative is not conclusive, either. An extramental being would end up with a defined beginning and end in time and be subject to it. This would render extramental beings subject to either being in mind and in reality or of being solely in mind. At the same time, the highest being exists whether it is thought or not: it exists independent from that. An extramental being, which is not thought obviously cannot be a being in mind and in reality. How could these inconsistencies be reconciled? In order to be invariable the highest being has to be a principle, that is an extramental principle and principle of thinking. Both alternatives as to what the highest principle represents (either an isolated principle of thinking or an isolated extramental being) would be subject to the alternative of being in mind and reality or being solely in mind. The highest principle has to be of such quality, that gives thinking the capacity to distinguish the difference between being in mind and in reality and being solely in mind. But the highest principle also has to be a being, which is not formed by time or subject to it. Only a principle not formed by time or subject to it can be thought as radically invariable.

In chapter 67 of the “Monologion”, Anselm introduces the term “imago dei” for the human mind. The “memoria” of the finite human mind is the image of godfather, the “intelligentia” is the image of the son, the “amor” is the image of the holy spirit.

“Aptissime igitur ipsa sibimet esse velut speculum dici potest, in quo speculetur, ut ita dicam, imaginem eius, quam facie ad faciem videre

7 Klaus Riesenhuber points out: “Das Argument zielt daher zwar auf die Behauptung eines absolut Seienden, stützt sich dabei aber nicht auf den Begriff eines [...] ens necessarium [...] oder ens perfectissimum, [...] sondern auf Denken als solches. [...] Wäre nämlich das Wesen des ‘aliquid’ als ‘notwendig existent’ bestimmt, so würde dem Denken ‘notwendige Existenz’ als eine objektive Denkbestimmung zu denken aufgegeben. Das Denken wäre damit von außen, nämlich durch einen im Denken als solches nicht unmittelbar vorfindlichen, also für das Denken als solches unmittelbar beliebigen Begriffsinhalt bestimmt. [...] Aus sich selbst müßte also das Denken diesen Begriff wie jeden anderen, der ihm innerhalb seines Intentionsfeldes als beliebiges Objekt gegeben wird, der Doppelmöglichkeit von [...] Existenzbehauptung und [...] möglicher Existenzverneinung unterwerfen. Daher bedürfte das Denken gegenüber einem solchen für es beliebigen Objekt eines von sich selbst verschiedenen Kriteriums, um über seine Wirklichkeit entscheiden zu können.” ([8], p. 50)
nequit. Nam si mens ipsa sola ex omnibus quae facta sunt, sui memor et intelligens et amans esse potest: non video cur negetur esse in illa vera imago illius essentiae, quae er sui memoriam et intelligentiam et amorem trinitate ineffabile consistit.” ([2], p. 194)

The capacity to reflect ensures that the finite mind is invariable. The finite mind is meant to turn away from the world and to refer to or reflect on itself (Memoria godfather). Human mind is obliged to acknowledge the identity of its own self (intelligentia son) and to acknowledge the highest principle of its unity (Holy spirit).

Human mind as an image of the invariable trinity can be thought of as a derivative invarialblity. For the human mind, the structure of validity of any statement, derives from its essence: as representation of the very image of trinity. Judgement formed in accordance to such an image is either “true” or “not true”. Validity of knowledge means in this original and genuine sense that it necessarily is of such character that it is subject to the distinction of being true or false. This also implies that knowledge is not exclusively “true” knowledge but also can be “wrong” knowledge. In the chapter 2 of Anselm’s “De veritate” ([1], p. 42) the following quote indicates that Anselm very much was aware of the implications of my line of interpretation and is not an artefactual one, when the pupil asks the teacher:

“At si quod debet significando, recta et vera est, sicut ostendisti: vera est oratio, etiam cum enuntiat esse wuod non est.” The teacher answers the pupil: “Vera quidem non solet dici, cum significant esse quod non est; veritatem tamen et rectitudinem habet, quia facit quod debet. Sed cum significat esse quod est, dupliciter facit quod debet [...] Alia igitur est

8：“Im ‘Prologus’ des Monologions macht Anselm eine methodische Anmerkung, die für das Gesamtwerk bestimmend geblieben ist. [...] Die Überlegungen verstehen sich nicht bloß als phänomenologisch-deskriptive Erhellung des jeweils Gesehenen; es wird zurückgefragt auf etwas, das im ursprünglichen geistigen Lebensvollzug schon mitgesetzt ist, wenn auch nicht so, daß es dem unmittelbaren Hinsehen präsent wäre. [...] Die Fragestellung kommt auf das Sehen selbst zurück, um die Bedingung der Möglichkeit von Sehen überhaupt zu ermitteln. Anselm schreitet bereits im ersten Kapitel vom Faktum des Sehens fort zu dessen Genesis aus seinem Ermöglichungsgrunde.” ([9], p. 31)

9See [7], p. 74; and [9], p. 38.

10See [7], p. 74; and [9], pp. 31, 34 - 38.
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rectitudo et veritas enuntiationis, quia significat ad quod significandum facta est, alia vero, quia significat quod accepit significare. Quippe ista immutabilis est ipsi orationi, illa vero mutabilis.”

Anselm apparently is aware of the two aspects of validity. An original disposition of valid thinking implies a distinct structure that thinking is subject to in order to be true or not true. The pursuit of being true, the judgment is obligated to, forms the second derivative aspect.

The highest being is not confined to or defined by the structure of the finite mind. The highest being is timeless and not subject to the rules of finite entities. “Aseitas” of this highest being exists even if no thought is formed as an image of it. Thus the absolute principle is the highest being.

“Considera quia, cum omnes supradictae rectitudines ideo sint rectitudines, quia illa, in quibus sunt, aut sunt aut faciunt quod debent: summa veritas non ideo est rectitudo, quia debet aliquid. Omnia enim illi debent, ipsa vero nulli quicquam debet; nec ulla ratione est quod est, nisi quia est.” ([1], p. 70)

“Ergo, Domine, non solum es quo maius cogitari nequit, sed es quiddam maius quam cogitari possit.” ([1], Prologion XV, p. 110)

In contrast, the finite principle of human thinking can only rule individual, discrete thoughts of the thinker and only under the condition that they are formed as representation of the image of the highest being (“imago dei”).

Anselm’s argument consists of 4 parts. The first part clarifies “modo elencticco” that anyone, who rejects the validity of the principle that thinking is the only form that makes any judgement possible, contradicts himself, as the conclusion implicitly applies these same principles to come to that conclusion. The second part shows, that the principles of thinking are the same invariable (!) principles that make a distinction possible between something, present in mind and in reality and something, present solely in mind. In a third part Anselm returns to the theological theory of the finite mind of human beings that represent the image of trinity (imago dei). Only as an image of the trinity finite mind can be an invariable principle, which makes the difference for valid thinking (= the difference of being true or not true) possible. In the last part Anselm teaches that the absolute being does not depend on the contingent nature of thinking.
Bibliography


Two Ontological Arguments for the Existence of an Omniscient Being

JASON L. MEGILL

1 Introduction

I formulate two ontological arguments. The first argument claims that (i) the definition of “omniscience” and (ii) the claim that it is rational to hold that there is only one concrete possible world entails that (iii) it is rational to believe that at least one omniscient being exists in the actual world. The second argument claims that given (i) the meanings of “omniscience”, “possibility” and “necessity”, and (ii) modal axiom S5, (iii) it is necessarily the case that an omniscient being exists.

2 Argument One

2.1 Preliminaries

In this section, I argue that it is rational to believe that at least one omniscient being exists in the actual world. But some preliminaries are necessary.

First, the argument is modest in a number of ways. For example, the argument will not try to establish that an omniscient being exists with complete certainty; rather, it merely claims that it is rational to believe that one does. Furthermore, theists generally believe a number of things about God aside from the claim that God is omniscient. For example, some theists believe that God is omnipotent; God is omnibenevolent; God is omnipresent; God is eternal; God is the creator of the universe; God
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cares about us; God is immutable; that there is only one God and so on. I will not argue for any of these claims. My conclusion is simply that it is rational to believe that at least one omniscient being exists in the actual world. Nevertheless, this claim has interest; of course, many would deny it. Second, “omniscience” is defined here as having maximal knowledge. “Maximal knowledge” is defined as knowing the greatest amount that it is logically possible for a single being to know. So, an omniscient being is a being that knows as much as it is logically possible for a single being to know. This definition entails that an omniscient being cannot know truths that are impossible to know, if there are any such truths. It might be that it is logically possible for an omniscient being to know all truths or it might be that there are some truths that not even an omniscient being could possibly know; the definition of omniscience used here is consistent with both possibilities. Limiting a divine attribute in this manner is nothing new: omnipotence is often restricted so that an omnipotent being cannot perform logically impossible actions.1 Third, posit a hypothetical or putative world – call it “W” – that has the following two characteristics: (i) this world is an exact duplicate of the actual world, (ii) though, for all we know, there might not be an omniscient being in W. So, for example, if we take the set of truths that are true of the actual world, and subtract any truths from that set that might involve the presence of an omniscient being, the resultant set is the set of truths that we think are true about W. In other words, we hold, at least for now, that the facts that obtain in and about W are all and only those facts that obtain in and about the actual world, minus any possible facts involving an omniscient being. World W will play a role in the argument below.

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1 Aquinas [2] is one thinker who limited omnipotence in this way. Descartes [4] is one example of a thinker who didn’t. Like other aspects of this argument, this definition of “omniscience” is modest too, insofar as it allows for the possibility that there are truths that not even an omniscient being could know. Often, omniscience is defined in the following manner: “S is omniscient df for every proposition p, if p is true then S knows p” (Wierenga [12], section 1); but again, the definition of “omniscience” used throughout does not claim that an omniscient being knows all truths, but only those that are logically possibly knowable. Just as one cannot expect God to be able to do the logically impossible, one cannot expect God to know what it is logically impossible to know.
2.2 The Argument

The argument is four steps: three premises and a conclusion. The first premise is,

(1) **Consider a hypothetical world W that is an exact duplicate of the actual world, though, for all we know, there might not be an omniscient being in W.**

This is simply the world W discussed above. Again, world W is exactly like the actual world in all respects, but, at least initially, we leave aside any issues involving the existence or nonexistence of an omniscient being. By hypothesis, there might not be an omniscient being in W. Indeed, (1) does not even claim that W is a possible or impossible world. It simply asks one to consider a hypothetical or putative world.\(^2\)

Premise (2) is,

(2) **There is a logically possible world W* that is an exact duplicate of W, except that it definitely contains an omniscient being, i.e., a being that knows as much as it is logically possible for a being in W to know.**

While (1) posited a world W that was a duplicate of the actual world, though we claim that it might lack an omniscient being, (2) posits a world W* that is an exact duplicate of W but that contains an omniscient being, and claims that such a world is logically possible. That is, there is a logically possible world W* that contains a being that has the maximum amount of knowledge that it is logically possible for a being in W to have.

The third premise is,

(3) **It is rational to believe that W* is numerically identical to the actual world.**

In short, it is rational to think that W* is the actual world; one can rationally believe that we exist in W*. So, while (2) posits the existence of a world W* that contains an omniscient being, and claims that W*...
is logically possible, premise (3) claims that it is rational to think that this possible world is in fact the actual world. Premises (2) and (3) need defended. But if (2) and (3) are true, then,

(4) It is rational to think that an omniscient being exists in the actual world.

The conclusion follows from (2) and (3). Note that if (a) there is a possible world $W^*$ that contains an omniscient being and (b) $W^*$ simply is the actual world, then clearly (c) the actual world contains an omniscient being. That is, (a) and (b) entail (c). Furthermore, if we know that (a) and (b) entail (c), and we do, and we know that (a) is true, and we also know that it is rational to believe (b), then it is rational to believe (c) (because it is rational to trust modus ponens). The argument is valid and premise (1) is unproblematic, so the soundness of the argument hinges on the truth of (2) and (3). I defend (2) and (3) in successive sections.\(^3\)

**Premise Two**

Premise (2) claims that, “there is a possible world $W^*$ that is an exact duplicate of $W$ except that it definitely contains an omniscient being, i.e., a being that knows as much as it is logically possible for a being in $W$ to know”. Premise (2) can be established with a fairly simple argument. Recall world $W$. This world is an exact duplicate of the actual world, though we do not suppose that it contains an omniscient being. Furthermore, recall the definition of “omniscience”. An omniscient being has maximal knowledge, where “maximal knowledge” means knowing as much as it is logically possible for a single being to know. So, an omniscient being in $W$ would be a being that knows as much as it is logically possible for a single being in $W$ to know. But then, by definition, it is logically possible for there to be an omniscient being in $W$; again, such a being would know as much as it is logically possible for a single being in $W$ to know. That is, it must be logically possible for there to be a world that is just like $W$ but that contains an omniscient being. But if it is logically possible for

\(^3\)Sometimes trying to categorize an argument can be tricky, but the argument above is arguably a type of ontological argument. As we’ll see below, it argues that it is rational to believe that an omniscient being exists because that claim is entailed by the very meaning of “omniscience” (among other things), just as some ontological arguments argue that the existence of a greatest conceivable being is entailed by the very meaning of “greatest conceivable being” etc.
there to be a world that is just like $W$ that contains an omniscient being, then there is a logically possible world that is just like $W$ except that it contains an omniscient being; if something is logically possible, then this simply means (in possible worlds semantics) that there is a logically possible world in which it is the case. Call this logically possible world "$W^*$". And since this argument shows that there is a logically possible world $W^*$, premise (2) is true.

In case the argument was too quick, here is a reformulation. Posit world $W$, a world just like the actual world; though we do not assume that there is or is not an omniscient being in $W$. But the question is: could there be an omniscient being in $W$? Is it logically possible for there to be an omniscient being in $W$? The answer is that it must be logically possible. An omniscient being in $W$ would be a being that knows as much as it is logically possible for a single being in $W$ to know. But, trivially, it is logically possible for there to be a being in a world that knows as much as it is logically possible for a single being in that world to know. But then it is logically possible for an omniscient being to be present in $W$. That is, there is a possible world $W^*$ that is just like $W$ but contains an omniscient being. And this is merely premise (2). The soundness of the argument now hinges on the truth of premise (3).

Premise Three

Premise (3) claims that, “it is rational to believe that $W^*$ is numerically identical to the actual world”. The argument for (3) relies on the claim that one can rationally believe that there is only one concrete possible world. Of course, some philosophers have defended the idea that there are multiple concrete possible worlds; Lewis’s [9] modal realism is the most prominent example. But Lewis’s modal realism is and has been extremely controversial. It faces a host of objections. To offer some examples: philosophers have questioned some of Lewis’s fundamental metaphysical principles (such

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4Of course, some physicists have put forth theories that posit a plurality of concrete universes, i.e., a multiverse. For instance, there is the many worlds interpretation of Quantum Mechanics. Another multiverse theory is formulated in Tegmark [11]. Though note that it might be that there is a multiverse in the actual world, and the actual world is the only concrete possible world. That is, even if we suppose that these multiverse theories are true, this would not necessarily entail modal realism. Furthermore, there is – at least currently – no empirical evidence that there is a multiverse.
as the principle of recombination (see Forrest and Armstrong [6]); some argue that modal realism entails paradoxical claims (see again Forrest and Armstrong [6]; also see Davies [3]); others think it entails an unacceptable inductive skepticism (Forrest [5]) or that it entails that there are no moral obligations (Adams [1]); others cannot get past how counterintuitive the view appears *prima facie*. Whatever one thinks of the debate surrounding modal realism, it is fair to say that the case for modal realism has not been *conclusively* made. So, for example, one can rationally reject modal realism; indeed, most who work in modal metaphysics do. In short, it is *rational* to claim that there is only one concrete possible world, i.e., ours.\(^5\)

Given that, here is an argument for premise (3). Note that, by hypothesis, every fact that obtains in and about the actual world that does not involve the existence or non-existence of an omniscient being will obtain in and about \(W\), and so in and about \(W^*\). \(W\) is an exact duplicate of the actual world (after we leave aside issues concerning the existence or non-existence of an omniscient being), and \(W^*\) is an exact duplicate of \(W\) (except that it definitely contains an omniscient being). But of course the actual world is actualized; it is a concrete possible world. And that is an undeniable fact about the actual world. But then it must be a fact about \(W\) and \(W^*\) as well, because again, all facts that hold in and about the actual world will hold in and about \(W\) and \(W^*\) as well. So \(W^*\) must be concrete as well. But recall that it is rational to claim that there is only one concrete possible world. And if there is only one concrete possible world, but the actual world is concrete (as we know), and \(W^*\) is concrete as well, then it follows that the actual world and \(W^*\) must be numerically identical. But then it is *rational* to believe that the two worlds are numerically identical; premise (3) is true. Also note that the same holds of \(W\) as well. *Initially*, e.g., in step (1) of the argument, we held that – for all we know – \(W\) might not contain an omniscient being. But we see *now* that it *must* contain an omniscient being too. \(W\), like the actual world and \(W^*\), will be concrete. Again, all facts that obtain in and about the actual world will obtain in and about \(W\); and one of these facts is that the actual world is concrete; so \(W\) will be concrete. But we already know that the actual world and \(W^*\) are one and the same world, and are concrete. So, if there is only one concrete possible world, and the actual world and \(W^*\) are concrete, and \(W\) is as well, it follows that \(W\) will be numerically iden-

\(^5\)And to be clear, I am not claiming that modal realism is definitely false. I am not even claiming that it is probably false. I am merely claiming that one can rationally refuse to endorse it. This should be uncontroversial.
tical to the actual world and $W^*$, and so will also contain an omniscient being. But the key point, of course, is that it is rational to believe that world $W^*$ and the actual world are numerically identical. Premise (3) is true, and therefore, the argument is sound. It is rational to believe that an omniscient being exists in the actual world.

In sum, we begin with the assumption that there is a world $W$ that is just like the actual world; we claim that, for all we know, this world might lack an omniscient being. But then we see that – trivially – it is logically possible that $W$ contains an omniscient being, and so there is a logically possible world $W^*$ that is just like $W$ except that it contains an omniscient being. But then we realize that the actual world, $W$ and $W^*$ will all be concrete, since, by hypothesis, all of the same facts (aside from those that involve an omniscient being) that hold in and about the actual world (which is concrete) will hold in and about $W$ and $W^*$. And since it rational to believe that there is only one concrete possible world, it is rational to hold that all three worlds are in fact numerically identical. The assumption that there is a world that might lack an omniscient being eventually leads to the claim that the actual world contains an omniscient being.

3 A Second Argument

One can formulate an argument that is stronger than the one given above, stronger because it claims that it is necessarily the case that an omniscient being exists as opposed to claiming that it is merely rational to think that one does. Step (1) is,

(1) An omniscient being in any random possible world $W$ would be a being in $W$ that knows as much as it is logically possible for a single being in $W$ to know.

But then, trivially,

(2) It is logically possible for an omniscient being to exist in any random possible world $W$.

An omniscient being in any random logically possible world would know as much as it is logically possible for a single being in that world to know; but of course, a being that knows as much as it is logically possible for a single being to know is logically possible. And,
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(3) If it is logically possible for an omniscient being to exist in any random world, then it is logically possible for an omniscient being to exist in all worlds.

This follows from general conditional proof: if one can show that something holds of a random element in a domain, then one can infer that it holds of all elements in the domain. General conditional proof is a basic pattern of inference used all throughout logic and mathematics. So, we can safely infer that if an omniscient being possibly exists in any random world, an omniscient being possibly exists in all worlds. Furthermore,

(4) If it is logically possible for an omniscient being to exist in all worlds, then there is a possible world in which an omniscient being exists in all worlds.

This follows from the meaning of “possibility” in possible worlds semantics: if something is logically possible, then this simply means that there is a logically possible world in which it is the case. This is uncontroversial. Also,

(5) If there is a possible world in which an omniscient being exists in all worlds, then there is a possible world in which an omniscient being is necessary, i.e., an omniscient being is possibly necessary.

This follows from the meaning of “necessary” in possible worlds semantics: in possible worlds semantics, something is necessary if and only if it is the case in all possible worlds. This too is uncontroversial. But then, with modal axiom S5 (which claims that if something is possibly necessary, it is necessary), we can infer that,

(6) If an omniscient being is possibly necessary, an omniscient being is necessary.

And (2) - (6) entail that,

(7) An omniscient being is necessary.\(^6\)

So, the necessary existence of an omniscient being follows from: (i) the definitions of “omniscience”, “possibility” and “necessity”, (ii) general conditional proof (a widely accepted inference pattern in logic and mathematics), and (iii) modal axiom S5.

\(^6\)Of course, one can obtain (7) from (2) - (6) by using modus ponens several times.
One might object: why can’t the atheist simply claim that there is at least one logically possible world that does not contain an omniscient being? Indeed, they already claim that the actual world is one such world. Then, the atheist can simply assert that there is at least one logically possible world that cannot contain an omniscient being, and so it is not the case that an omniscient being can exist in all logically possible worlds, and premise (2) is false. The problem with this objection is that it is trivially true that there could be an omniscient being in any random logically possible world: a being that knows as much as it is logically possible for a single being to know is obviously logically possible. And note that we can reformulate the argument in such a way that the ineffectiveness of this objection becomes clear: first, assume that there is a logically possible world that does not contain an omniscient being. That is, assume the very claim that the atheist asserts when making this objection. But also note that it is logically possible for there to be an omniscient being in any random world; this is trivial. But then it is logically possible for there to be an omniscient being in all worlds; and so there is a logically possible world where an omniscient being is in all worlds; and so there is a logically possible world where an omniscient being is necessary; and so an omniscient being is possibly necessary; and so an omniscient being is necessary. But this is a contradiction, so our assumption must be false. In other words, if we assume that there is a logically possible world without an omniscient being, an absurdity results, so it follows that there is no logically possible world without an omniscient being. This objection fails. But what about other possible objections?

4 Possible Objections

I conclude by considering some further possible objections. But first, note that many of the stock objections to ontological arguments do not seem relevant to the arguments given above. These arguments avoid some contentious claims of previous ontological arguments. They do not claim that existence is a predicate, and so they avoid Kant’s [8] famous objection to ontological arguments. They do not claim that existence is a perfection (like, e.g., Descartes’s [4] version of the argument), so any potential problems with that claim are avoided. They do not appeal to the claim that conceivability entails possibility.
Omniscience

One might have various worries about the concept of “omniscience”. Perhaps the concept entails a contradiction, for example? But again, the definition of “omniscience” used here is a modest one: it merely claims that an omniscient being would be one that knows the maximum amount that it is logically possible for a single being to know. Again, this definition makes the logical possibility of an omniscient being trivial. Contradictions are logically impossible, so given that the definition of “omniscience” makes such a being trivially possible, the definition of “omniscience” used here must not entail a contradiction. But one might worry that this definition of “omniscience” is perhaps too modest? How can we be sure that the omniscient being discussed throughout knows everything that we might want or expect an omniscient being to know? But note that this being will know everything that it is logically possible for a single being to know. So, what knowledge could this being possibly lack? It will not know contradictions because it is impossible to know a contradiction; but contradictions cannot be true, and so one cannot know them anyway. And even if there are things that this being cannot know, these will be things that are logically impossible for it to know, and so not knowing these things is no limit on the being’s omniscience, just like not being able to make two and two equal five is no limit on a being’s omnipotence. In short, the being will know everything that it is logically possible for a single being to know, so it seems it will know everything that we can reasonably expect an omniscient being to know.

The Gap Problem and the More than One God Objection

One common problem with some arguments in natural theology concerns the “gap” between the being that a given argument posits and the traditional conception of God. That is, often, the being that a theistic argument posits lacks many of the characteristics often ascribed to God. The argument from design is open to this criticism, as Hume [7] pointed out. Cosmological arguments also face this problem (see Pruss [10]). Ontological arguments generally fare better than these other arguments with this particular criticism at least, since ontological arguments generally work with a robust conception of God; God is held to be a supreme being, or is the greatest conceivable being, or is a perfect being and so has all perfections, and so many of the properties theists typically ascribe to God and
so on. However, the ontological arguments given here face a fairly severe version of the gap problem: all we know about this being is that it is omnipotent (and, in the case of the second argument, necessary) ... but this is consistent with various conceptions of God. Perhaps this omniscient being is not the creator of the universe, or does not take an interest in human affairs, etc? I grant this objection. All the arguments show – assuming they are successful – is that an omniscient being exists (in the case of the second argument) or that it is at least rational to believe that one does (in the case of the first argument). Even though this is a modest conclusion, it is still interesting. Likewise, the conclusions of the arguments are consistent with there being multiple omniscient beings. This is true; though it would be odd to make the leap from believing in zero omniscient beings to believing in multiple omniscient beings, if one needn’t do so.

Bibliography


A Modal Theistic Argument

JASON L. MEGILL and AMY REAGOR

1 Introduction

Consider the following modal claim, where the predicate “O” stands for the property of omnipotence:

CLAIM : $\Diamond \Box \exists x O(x)$.

That is, “it is possibly necessary that there is an omnipotent being.” CLAIM is interesting for at least two reasons. First, given the modal axiom S5 (i.e., $\Diamond \Box \phi \rightarrow \Box \phi$), CLAIM entails that it is necessarily the case that there is an omnipotent being.1 Second, CLAIM can be supported with a fairly plausible argument.2

1This isn’t new or surprising. S5 has been used in modal theistic proofs since at least Leibniz [11]. See also Plantinga [17] and Megill and Mitchell [15] for more recent uses of S5 in theistic arguments. Also see Maydole [13] (Maydole doesn’t use the S5 axiom per se, though the system he uses (2QS5) contains it). Some non-specialists misread S5 as claiming that if something is possible, then it is actual, but S5 does not claim or even entail this (e.g., the Barcan formula would, but the Barcan formula is not a theorem of the system S5). S5 follows straightforwardly from the definition of “possibility” and “necessity” used in possible worlds semantics: if something is possible, then there is a possible world in which it is the case, and if something is necessary, then it is the case in all possible worlds, so if there is a possible world that contains a necessary being, the being will be in all possible worlds, i.e., it will be necessary. In short, if something is possibly necessary, it is necessary. And of course, if an omnipotent being exists in all logically possible worlds, one exists in the actual world.

2Note that the argument will only attempt to show that an omnipotent being exists and not that an all-powerful, all-knowing and all-good being exists. The claim that
2 Omnipotence

“Omnipotence” is defined here in the following manner: a being is omnipotent if it can make any logically possible, contingent state of affairs obtain. Some have claimed that the concept of “omnipotence” is problematic, e.g., it entails a contradiction. But we think such challenges can be overcome; and we think it can be shown that an omnipotent being is logically possible, as we argue in the remainder of this section.

One commonplace example of a problem with omnipotence, the paradox of the stone, is irrelevant in this context. The definition of omnipotence used here is not afflicted with that paradox. The paradox of the stone asks, “Can God create a stone so heavy that God cannot lift it?” If God cannot create the stone, then there is something that God cannot do, and if God can create the stone, then God cannot lift the stone, so there is something that God cannot do; either way, there is something that God cannot do and omnipotence is impossible. This paradox calls into question the idea that omnipotence is the ability to do anything, but it is not applicable to the idea that omnipotence is the ability to make any logically possible, contingent state of affairs obtain. Either it is logically possible for God an omnipotent being exists still has theological interest; we say more about this issue below.

3See, e.g., Rosenkrantz and Hoffman [19] and Wierenga [23]. This definition of “omnipotence” aims to eliminate two possibilities. First, it denies that an omnipotent being can make logically impossible states of affairs obtain. For example, God could not make two and two equal five. Aquinas [1], among others, saw the need to qualify omnipotence in this way. However, note that if one agrees with, say Descartes [3], that God could do even the logically impossible, a version of the argument offered below will still succeed (for reasons that should be clear as we continue). Second, it denies that an omnipotent being can make logically necessary states of affairs obtain, for nothing can make a logically necessary state of affairs obtain. For example, God could not make two and two equal four either. See, e.g., Hoffman and Rosenkrantz ([6], section 1):

“Nor is it possible for an agent to bring about a necessary state of affairs (e.g., that all cubes are shaped). It is possible for an agent, a, to bring about a necessary state of affairs, s, only if possibly, (1) a brings about s, and (2) if a had not acted, then s would have failed to obtain. Because a necessary state of affairs obtains whether or not anyone acts, (2) is false. As a consequence, it is impossible for an agent to bring about either a necessary or an impossible state of affairs.”
to create a stone that God cannot lift or it is not. If it is not logically possible for God to create such a stone, then this is of no consequence: God can only make logically possible states of affairs obtain, so the fact that God cannot make a logically impossible state of affairs obtain is not problematic. And if it is logically possible for God to create a stone that God cannot lift, then the state of affairs in which God lifts the stone must not be logically possible, for if it is was, then God would be able to lift the stone. But then the fact that God cannot lift the stone is of no consequence: God can only make logically possible states of affairs obtain, so the fact that God cannot make a logically impossible state of affairs obtain is not problematic. Either way, there is no problem.

One might claim that it is logically possible for a person to run a marathon, but God is perhaps incorporeal, in which case God cannot run a marathon (nor do anything that requires physical embodiment); so “omnipotence” cannot mean the ability to do anything that is logically possible. But this is not the definition of omnipotence used here: “omnipotence” was defined as the ability to make any logically possible, contingent state of affairs obtain, and not as the ability to do anything that is logically possible. Indeed, either it is logically possible for God to run a marathon or it is not. If it is, then God can make that state of affairs obtain and there is no problem. If it is not, then there is still no problem for the definition of omnipotence used here since it merely claims that God can make any logically possible, contingent state of affairs obtain. That is, if it is logically impossible for God to run a marathon, then God cannot make it the case that God can run a marathon, but this is no more problematic than the fact that God cannot make two and two equal five, since both cases are logically impossible. It is possible for people to run marathons, but if it is impossible for God to run one, then God’s not being able to run one is no limit on God’s omnipotence. Or consider an example discussed by Geach [5]: people can break their word, but presumably God cannot. But again, if it is logically impossible for God to break God’s word, then the fact that God cannot break God’s word is no limit on God’s omnipotence, and such cases are not problematic for the definition of “omnipotence” we use.

Here is another potential problem with the concept of “omnipotence”. If theism is true, then an omnipotent being exists. Call this being “OB”. Given that OB exists (in, say, the actual world), it is a logically possible state of affairs that OB exists. And OB is omnipotent, so by definition, OB can make any logically possible state of affairs obtain, including the
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fact that OB exists. So OB should be able to make OB exist. But since nothing can cause itself (for something to cause itself, it would have to exist before it existed), this is incoherent. But recall that the definition of “omnipotence” used here claimed that an omnipotent being could make any logically possible, contingent state of affairs obtain. Also, the argument offered below claims that it is possibly necessary, and so necessary, that an OB exists. Given that, there is no reason to suppose that an OB could make itself exist; its existence is necessary, and it can only bring about contingent states of affairs.

Yet another potential problem with omnipotence concerns (a) states of affairs not brought about an omnipotent being and (b) uncaused states of affairs, and so states of affairs not brought about by anything, and so not brought about by an omnipotent being. Consider a state of affairs S, whatever it might be, that is not brought about by an omnipotent being, whether this is because S is uncaused or because S is caused by something else aside from an omnipotent being. It seems that it is possible to have a state of affairs S that is not brought about by an omnipotent being. But it appears that it is impossible for an omnipotent being to bring about S, because by hypothesis, S is not brought about by an omnipotent being. So there is at least one logically possible state of affairs that an omnipotent being cannot bring about, and omnipotence is impossible. It’s difficult to assess this objection until we are told precisely what the state of affairs S is. That is, the defender of the coherence of omnipotence might fairly ask, “What is the specific content of this state of affairs S?” If it is a state of affairs that is in fact impossible, then there is no reason to think an omnipotent being should be able to bring it about. And if it is a state of affairs that is contingent, then there is no reason to think that an

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\[4\] It appears that something can cause itself to continue to exist, once it exists. But when we claim that “nothing can cause itself”, we mean nothing can cause itself to come into existence in the first place. Something cannot be its own efficient cause.

\[5\] One might wonder why we should we think that an omnipotent being would be a necessary being in this sense. Many today (and over the last few centuries) think (and have thought) that if God exists at all, God would be necessary in this sense; i.e., God would exist in all possible worlds. But God’s existence hasn’t always been seen as being necessary throughout the tradition, and there are even some cases in which God was thought to be necessary, but in a different sense of “necessity” (e.g., Aquinas). Our response is that by the end of the argument, if the argument is successful, the being will have been shown to be necessary in this “modern” sense.

\[6\] Swinburne ([21], p. 156) discusses this issue.
omnipotent being could not bring it about. There would be no problem either way. And if we are to understand the objection as simply saying that it is possible that there is a state of affairs that is not caused by an omnipotent being, then this is no problem whatsoever for omnipotence. By definition, an omnipotent being can bring about any logically possible state of affairs. But this is consistent with it not bringing about a given state of affairs $S$ in some logically possible world. It might be that an omnipotent being could bring about $S$ (and so could remain omnipotent), but there is a possible world where the being does not in fact bring $S$ about (so $S$ can be logically possible).

However, there is an additional potential problem with omnipotence, namely, what Plantinga ([17], p. 180) calls “Leibniz’s Lapse.” Roughly, the problem is the following: consider a state of affairs $T$. Suppose that God actualizes $T$. In $T$, someone named “Curley” is offered a bribe. Curley has free will. Curley is free to take the bribe and free to reject it. If Curley accepts the bribe, then God has actualized a world that consists of $T$ plus Curley taking the bribe; call this world “$W$”. If Curley rejects the bribe, then God has actualized a possible world that consists of $T$ plus Curley rejecting the bribe; call this world “$W^*$”. God does not decide the matter for Curley; again, Curley has free will, so Curley does. But if Curley accepts the bribe, God could not have actualized $W^*$. And if Curley rejects the bribe, God could not have actualized $W$. Again, all God can do is actualize $T$, and Curley decides what world is actualized from there. But no matter what Curley does, there is at least one logically possible, contingent state of affairs that God could not actualize. It follows that an omnipotent being could not actualize all logically possible, contingent states of affairs. Therefore, the definition of “omnipotence” used here is incoherent.

To respond, again, by definition, an omnipotent being can make any logically possible, contingent state of affairs obtain. So an omnipotent being can make $T$ obtain. But then Curley faces a choice: accept the bribe or reject it. But it is possible for Curley to do either one. Again, Curley has free will, so he possibly accepts the bribe and he possibly rejects it. If Curley accepts the bribe, then world $W$ results. But note that if so, an omnipotent being can make $W$ obtain. The being can make $T$ obtain, and then – albeit with Curley’s help – $W$ obtains. Likewise, if Curley rejects the bribe, then world $W^*$ results. But note that if so, then an omnipotent being can make $W^*$ obtain. The being can make $T$ obtain, and then – albeit with Curley’s help – $W^*$ obtains. The point is that
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an omnipotent being can make either $W$ or $W^*$ obtain, depending upon what Curley does, of course. But it is not as if there is a logically possible, contingent state of affairs that an omnipotent being cannot in principle make obtain. One might say that it seems odd that an omnipotent being would need Curley’s help to do anything, but if it is not logically possible for an omnipotent being to actualize a given world without such help, then this is no more problematic than the fact that the being cannot make 0 equal 1.

Furthermore, it is not even clear that an omnipotent being would need Curley’s help to make a given world obtain. Consider an omnipotent being surveying the various possible worlds, deliberating on which world to actualize. $W$ and $W^*$ will be options (by hypothesis, they are both logically possible worlds): an omnipotent being could decide to actualize $W$ (in which Curley freely accepts the bribe) and an omnipotent being could decide to actualize $W^*$ (in which Curley freely rejects the bribe). That is, if one assumes that an omnipotent being will know beforehand what Curley will (freely) choose to do, and so will know beforehand that it is actualizing either $W$ or $W^*$, then it can actualize either one without Curley’s help. In sum, Plantinga’s argument purports to show that there is at least one logically possible world – and so one logically possible state of affairs – that God cannot actualize, because Curley’s decision precludes it. But we claim that this is not the case: in one scenario, God can actualize either world with help from Curley, and in another scenario, God can actualize either world without help from Curley; but ultimately, God can actualize either world, and so there is not at least one possible state of affairs that an omnipotent being cannot actualize.

Thus far, we have merely been trying to defend the concept of omnipotence from various charges of incoherence, inconsistency or impossibility. But one can offer positive arguments for the logical possibility of an omnipotent being. For example, one might argue: (i) if an omnipotent being is not impossible, then an omnipotent being is possible; (ii) an omnipotent being is not impossible; therefore, (iii) an omnipotent being is possible. The argument is obviously valid; it’s an instance of modus ponens. And premise (i) is obviously true; assume the antecedent and then derive the consequent with double negation elimination. So the question is, is (ii) true? But we have already argued that (ii) is true. We have just argued that an omnipotent being is not impossible, at least insofar as the various arguments that purport to demonstrate the impossibility of an omnipotent being can be overcome. Likewise, one can argue that: (i) if a concept is
consistent, it is possibly instantiated, so if the concept of “omnipotence” is consistent, it is possible that there is an omnipotent being; (ii) the concept of “omnipotence” is consistent; therefore, (iii) it is possible that there is an omnipotent being. Again, this is an instance of modus ponens. Premise (i) is plausible given the commonplace view that consistency entails possibility; see, e.g., Szabo-Gendler and Hawthorne [22], “On a standard sort of characterization, \( P \) is logically possible just in case no contradiction can be proved from \( P \) using the standard rules of deductive inference ...” So again, the question is, is (ii) true? But again, we have already argued that (ii) is true, at least insofar as the various arguments that purport to demonstrate the inconsistency of “omnipotence” can be overcome.

Here is a different argument for the logical possibility of an omnipotent being. Note that the concept of “omnipotence” cannot be inconsistent. The concept of omnipotence claims that “an omnipotent being can make any logically possible, contingent state of affairs obtain”. This definition is really a cluster of three separate claims: (i) an omnipotent being can make merely logically possible states of affairs obtain, but (ii) an omnipotent being cannot make necessary states of affairs obtain nor can it (c) make logically impossible states of affairs obtain (see above). The only way to generate a contradiction from (a) – (c), and so from the definition of “omnipotence”, is by deriving the claim that God can and cannot make at least one given state of affairs obtain.\(^7\) To do this, we would need to find a state of affairs that was an example of (a) but also an example of either (b) or (c). If we could find such a state of affairs, then we would have a state of affairs that an omnipotent being could and could not make obtain ... we would have our contradiction. But there cannot be such a state of affairs:

\[^7\text{And note that that is precisely what many of the various arguments against omnipotence try to do. For example, they try to show that an omnipotent being could and could not make itself obtain; an omnipotent being could and could not run a marathon; an omnipotent being could and could not break its word; an omnipotent being could and could not make a world where Curley accepts the bribe and so on.}\]

\[^8\text{And this is precisely the point we are trying make in many of our responses to the potential problems with omnipotence discussed above. With these various alleged}\]

In short, the concept of omnipotence cannot be
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internally inconsistent. But then, given that consistency entails logical possibility, an omnipotent being must be logically possible.

In sum, we have defended concept of omnipotence from various objections and have also offered positive arguments for the possibility of an omnipotent being.

3 The Argument

We now argue that CLAIM is true. The argument is nine steps. First,

(1) The definition of an “omnipotent being” entails that an omnipotent being could make any logically possible, contingent state of affairs obtain.

(1) does not assume that an omnipotent being exists, so it does not beg the question. (1) merely claims that if there is an omnipotent being, then it could make any logically possible, contingent state of affairs obtain.

difficulties with omnipotence, a pattern emerges: we are told that there is something that an omnipotent being both can and cannot do. But upon further inspection, it looks like the objection to omnipotence makes the mistake of trying to place an activity or a state of affairs in more than one “category”; for example, a given state of affairs is assumed to be contingent and necessary, or contingent and impossible. And when one recognizes this mistake, the difficulty with omnipotence disappears. For instance, we are told that an omnipotent being can both make itself obtain and cannot make itself obtain. But when we recognize that an omnipotent being is necessary, and that it can only make contingent states of affairs obtain, the difficulty vanishes. This is a case of a state of affairs (the being’s existence) being mistakenly placed in the contingent category and the necessary category. Or we are told that an omnipotent being both can and cannot run a marathon: there is a contingent state of affairs that is impossible to bring about for an omnipotent being. But once we see that running a marathon is impossible for the being, the difficulty disappears. This time, a claim is mistakenly being placed in both the contingent category and the impossible category. Or we are told that there must be something an omnipotent being cannot do: either it cannot make a stone it cannot lift or it cannot lift the stone. But again, either it can make the stone or it cannot. If it cannot, then making the stone is in the impossible category, so there is no problem. And if it can make the stone – so making the stone is in the contingent category, then lifting the stone is in the impossible category, so again, there is no difficulty. It is only when we implicitly posit the impossible situation that, say, lifting the stone is in multiple categories (e.g., contingent and impossible) that the illusion of a difficulty with omnipotence arises.
Note that (1) is true even if there is not an omnipotent being, just as the definition of “unicorn” entails that “a unicorn will have a horn” is true even though there are no unicorns. (1) is true by definition.

The second step is,

(2) Any random logically possible world is a logically possible, contingent state of affairs.

(2) is innocuous. Consider a random logically possible world. This world can be thought of as a logically possible, contingent state of affairs; it might be an immensely complex state of affairs, but it can be thought of as a state of affairs.\footnote{Note that the argument will be consistent with any stance on the ontological status of possible worlds, from modal realism (see, e.g., Lewis [12]) to any form of “ersatzism”. We are describing this (random) world as a logically possible “state of affairs”, but this does not entail that it is not concrete (it does not entail that it is concrete either). In short, the argument is agnostic about the ontological status of possible worlds.} Furthermore, given (1) and (2), we can infer that,

(3) The definition of an “omnipotent being” entails that an omnipotent being could make any random logically possible world obtain.

If the definition of an “omnipotent being” entails that an omnipotent being could make all $P$’s obtain, and any random $Q$ is a $P$, then the definition of an “omnipotent being” entails that an omnipotent being can make any random $Q$ obtain. Just as we saw above with (1), (3) does not assume that an omnipotent being exists; it merely claims that if one does, then it could make any random logically possible world obtain.

Step (4) is the following claim:

(4) If (the definition of an “omnipotent being” entails that an omnipotent being could make it the case that any random logically possible world obtains) AND an omnipotent being is logically possible, then an omnipotent being could coexist with any random logically possible world.

This claim is very plausible. It is very plausible because it is a specific instance of a more general principle, and the general principle itself is very plausible. The principle is,
PRINCIPLE: If ((the definition of an “x” entails that an x could make it the case that some logically possible state (or states) of affairs S obtains) AND an x is logically possible), then an x could “coexist” with S (in, e.g., a logically possible world).

So, for example, if the definition of “carpenter” entails that a carpenter could make it the case that there is a cabinet, and the existence of a cabinet is a logically possible state of affairs, and the existence of a carpenter is logically possible as well, then a carpenter and a cabinet could coexist. If the definition of “arsonist” entails that an arsonist could make it the case that there is a fire, and the existence of a fire is a logically possible state of affairs, and the existence of an arsonist is logically possible as well, then an arsonist and a fire could coexist. PRINCIPLE essentially claims that if a being and a given state of affairs are both logically possible, so if there is a world W that contains the being and there is a world W* that contains the state of affairs, and if the being could bring about the state of affairs, then they could coexist. PRINCIPLE is obvious, if not outright trivial: to deny it, one would have to claim that a given being (is possible) and can bring about a given state of affairs (that is also possible), but it is not possible for the two to coexist (together in, say, a logically possible world). This is not coherent, for if they cannot coexist, then clearly the being cannot bring the state of affairs about, as we have supposed. But again, (4) is simply a specific instance of PRINCIPLE. PRINCIPLE is true, so (4) is true as well. We can then consider a particular omnipotent being; call it “a”.

Note that for a given x, there might be two (or more) states of affairs – S and S’ – that x could bring about, and S and S’ might not be compossible. For example, perhaps there is a logically possible carpenter that could make it the case that there are only brown cabinets or only red cabinets. What (4) entails in this case is that there is a possible world with the carpenter and only brown cabinets, and there is a different possible world with the carpenter and only red cabinets, but the principle does not entail that there is a single possible world with only brown cabinets and only red cabinets.

One might try positing the following counterexample to (4): suppose that x is a suicidal person and S is the state of affairs in which x is dead. x could bring S about, but x cannot coexist with S, because then x would be both alive and dead. This counterexample does not succeed, however, because x and S can coexist in, e.g., the same possible world, and this is the sense of “coexist” that the principle is concerned with.
If ((a could make it the case that any random logically possible world obtains) AND a is logically possible), then a could coexist with any random logically possible world.

Step (5) follows from (4) with universal elimination, though this will become clearer in the following section when we give a formal version of the argument.

Step (6) is the antecedent of (5).

(a could make it the case that any random logically possible world obtains) AND a is logically possible.

To establish (6), we must establish two claims: (A). we must show that a could make it the case that any random logically possible world obtains. But (A) easily follows from step (3) (with universal elimination), which we have already established; and (B). we must show that a is logically possible, i.e., we must show that an omnipotent being is logically possible. But we have already argued for this claim in section two.

So, with modus ponens and (5) and (6) we can infer that,

a could coexist with any random logically possible world.

But note that the following claim is true,

If a could coexist with any random logically possible world, then there is a possible world in which an omnipotent being coexists with all logically possible worlds.

Prima facie, this premise might not be obvious, but it can easily be shown to be true. Suppose (for conditional introduction) that an omnipotent being could coexist with any random logically possible world. But if something could be the case, then it is possibly the case. And if something is possibly the case, then given the meaning of “possibility” in possible worlds semantics, there is a possible world in which it is the case. So, if an omnipotent being could coexist with any random logically possible world, then there is a possible world in which an omnipotent being coexists with any random logically possible world. Furthermore, if an omnipotent being

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11To deny this, one would have to claim that some P could be the case, but it is not possible that P, and with transposition, that some P is possible, but P couldn’t be the case. This is nonsense. For example, if something could be the case, then it must be possible.
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can coexist with any random possible world, then it can coexist with all logically possible worlds.\footnote{Of course, the inference from “an omnipotent being can coexist with any random logically possible world” to “an omnipotent being can coexist with all logically possible worlds” is an instance of general conditional proof, a fundamental principle of quantified formal logic that is used all throughout mathematics, for example. If one can show that something holds of any random element in a domain, one has shown that it holds of all elements in the domain.} We have assumed the antecedent of (8) and derived the consequent; (8) is true.

We can now infer, with modus ponens on (7) and (8), that,

(9) There is a possible world in which an omnipotent being coexists with all logically possible worlds. But if something “coexists with all logically possible worlds”, it is necessary, for all that “necessary” means (in possible world semantics) is to exist in all possible worlds. That is, an omnipotent being is possibly necessary.

And (9) is simply CLAIM, so CLAIM is true. Given S5, an omnipotent being necessarily exists.

Call the argument the “modal theistic argument”, or the “MTA”. Even though the MTA is different from traditional ontological arguments, be they modal or not, the MTA is arguably a version of the modal ontological argument. Just as the traditional ontological argument held that we can infer that God exists from the idea of “God” (a being that is allegedly perfect etc), the MTA claims that we can infer that an omnipotent being exists from the idea (or from the definition of) “omnipotence”.

4 A Formal Version of the Argument

In this section, we offer a formal version of the argument. Hopefully, this will (i) clarify exactly what the argument is and (ii) make it apparent that the argument is at least valid.

Recall that premise (1) states, “the definition of an “omnipotent being” entails that an omnipotent being could make any logically possible, contingent state of affairs obtain”. Suppose that the predicate “O” stands for the property of omnipotence, the predicate “S” stands for “is a state of affairs”, and “B” stands for the relation “can make obtain”. If so, we can translate premise (1) in the following manner:

\begin{align*}
\forall x \forall y (O(x) \land \Diamond S(y) & \rightarrow B(x, y)).
\end{align*}
That is, if a being $x$ is omnipotent, and $y$ is a logically possible state of affairs, then the being can make that state of affairs obtain. (1) is simply the definition of omnipotence, so it is true by definition. Furthermore, recall that premise (2) is “any random logically possible world is a logically possible, contingent states of affairs”. If we suppose that the predicate “$W$” stands for “any random world”, then we can translate (2) as,

\[(2) \forall y (\Diamond W(y) \rightarrow \Diamond S(y)).\]

Premise (2) is trivial. But given (1) and (2), we can infer (3),

\[(3) \forall x \forall y (O(x) \land \Diamond W(y) \rightarrow B(x,y)).\]

That is, an omnipotent being can make any random logically possible world obtain.\footnote{Claim (3) is in fact a logical consequence of (1) and (2). Here is a proof (in a standard natural deduction, Fitch-style logic):

\begin{enumerate}
\item $O(a) \land \Diamond W(b)$ (this is an assumption for universal introduction).
\item $O(a) \land \Diamond S(b) \rightarrow B(a,b)$ (this is universal elimination on premise (1) of the MTA).
\item $\Diamond W(b) \rightarrow \Diamond S(b)$ (universal elimination on premise (2) of the MTA).
\item $\Diamond W(b)$ (conjunction elimination on (1) above).
\item $\Diamond S(b)$ (conditional elimination on (3) and (4)).
\item $O(a)$ (conjunction elimination on (1) above).
\item $O(a) \land \Diamond S(b)$ (conjunction introduction on (5) and (6)).
\item $B(a,b)$ (conditional elimination on (2) and (7)).
\item $\forall x \forall y (O(x) \land W(y) \land B(x,y))$ (universal introduction on lines (1) through (8)).
Of course, (9) is step (3) of the MTA.}

Step (4) claims that, “If ((the definition of an “omnipotent being” entails that an omnipotent being could make it the case that any random logically possible world obtains) AND an omnipotent being is logically possible), then an omnipotent being could coexist with any random logically possible world”. We translate (4) as the following:

\[(4) \forall x \forall y ((O(x) \land \Diamond W(y) \rightarrow B(x,y)) \land \Diamond O(x) \rightarrow C(x,y)).\]

(4) is very plausible because it is a particular instance (that involves the concept of “omnipotence”) of a very plausible principle. To deny this principle, one would have to claim that a given (possible) being could bring
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about a given (possible) state of affairs, yet the being could not coexist with the state of affairs. This is not coherent. Step (5) easily follows from (4) with universal elimination.

\[(5) \ (O(a) \land \Diamond W(b) \rightarrow B(a, b)) \land \Diamond O(a) \rightarrow C(a, b).\]

Claim (6) is,

\[(6) \ (O(a) \land \Diamond W(b) \rightarrow B(a, b)) \land \Diamond O(a).\]

Of course, (5) is a conditional and (6) is the antecedent of that conditional. (6) is a conjunction with two conjuncts, one of which is a conditional itself. The first conjunct claims that if \(a\) is omnipotent and \(b\) is a random logically possible world, then \(a\) can make \(b\) obtain. This claim is entailed by step (3) of the MTA (with universal elimination); given that (3) is true, it is true as well. The second conjunct claims that an omnipotent being \(a\) is logically possible; we argued for this claim above.

Step (7), which follows from (5) and (6) with modus ponens, is,

\[(7) \ C(a, b).\]

Step (7), which is true (it follows from the meanings of “could” and “possibly”), is,

\[(8) \ C(a, b) \rightarrow \Diamond \Box \exists xO(x).\]

Finally,

\[(9) \ \Diamond \Box \exists xO(x).\]

This follows from (7) and (8) with modus ponens. (9) is CLAIM. With S5 and (9), we can conclude that it is necessarily the case that there is an omnipotent being.

5 Possible Objections

We now consider some possible objections. Some of what follows also allows us to sharpen and further clarify the MTA.

OBJECTION ONE: Consider what we can call a “wimp world”. This is a world that does not contain an omnipotent being.\(^{14}\) The atheist thinks that such a world is logically possible. God is supposedly a necessary

\(^{14}\) Plantinga [17] considers such a world when formulating his version of the modal ontological argument.
being if God exists at all, so there must be at least one wimp world if this necessary being does not exist. So why can’t the atheist simply claim that a wimp world is logically possible to defeat the MTA? For example, if there is at least one logically possible world that an omnipotent being cannot make obtain, namely, a wimp world, then an omnipotent being will be unable to make just any random logically possible world obtain; so step (3) is false. The MTA cannot simply deny that a wimp world is logically possible, for that would beg the question (given that if such a world is not possible, theism is true). So, in short, what is to prevent the atheist from simply claiming that a wimp world is logically possible (indeed, she already claims as much), and therefore an omnipotent being cannot make all random logically possible world obtain etc., and so the MTA fails?

Response to Objection One: It is true that simply denying that a wimp world is logically possible would beg the question against the atheist, for such a denial would entail that it is necessarily the case that there is an omnipotent being. But it is also true that an assertion that a wimp world is logically possible would beg the question against the theist. To avoid begging the question in either direction, when discussing the possibility or impossibility of God, we should consider what is possible or impossible without assuming either that a wimp world is possible or impossible. So, e.g., we should try to ascertain whether an omnipotent being could exist in any random logically possible world if we simply think about this world as it is aside from the presence or absence of an omnipotent being. This is precisely what the MTA does. To elaborate, consider a logically possible world $W$. Say that $W$ is the actual world. There are a lot of buildings, people, and stars and so on in the actual world. But we want to know if there is also an omnipotent being in the actual world (and any other world, for that matter). To avoid begging the question, we should think of $W$ (and any other world), at least initially, as it is apart from whether or not it contains an omnipotent being. We should simply consider these buildings, people, stars etc. on their own, independently of the presence or absence of an omnipotent being, and then try to determine if an omnipotent being exists in $w$ (or any and all other worlds). If we do this, we will not beg the question in either direction. But when we do this, the MTA shows that it is necessarily the case that there is an omnipotent being.

To be perfectly clear, in the MTA, when we claim, for example, that an omnipotent being could make any random logically possibly world obtain (as we do in step (3)), the logically possible worlds are thought of as they are apart from or aside from the presence or absence of an omnipotent
being. The actual world, for example, is *initially* thought of as the atheist thinks of it, as simply a bunch of entities (buildings, people, stars etc.) that does not include an omnipotent being. The point of the MTA is that when we think of all worlds in this fashion, when we consider all worlds apart from the presence or absence of an omnipotent being, we eventually see that an omnipotent being *could* coexist with all logically possible worlds, i.e., it is possibly necessary, and so necessary, that there is an omnipotent being.

Again, the claim is that when we are trying to rationally determine whether or not there is an omnipotent being, we should not immediately assume that there is or is not a wimp world, for that would immediately and unreflectively settle the issue. Rather, we should consider worlds as they are independently of the presence or absence of an omnipotent being; and when we do so, the MTA suggests that there is an omnipotent being. Of course, many people already adopt the viewpoint that we claim that we should adopt when we begin to examine the question: agnostics. The agnostic does not claim that a wimp world is possible, for that would entail that a necessary divine being is impossible, and this is simply atheism. And the agnostic does not claim that a wimp world is impossible, for that would entail that a necessary divine being exists, and this is simply theism. What this suggests is that even if atheists are not persuaded by the MTA (because, for example, they refuse to suspend their judgment that a wimp world is possible), agnostics – at least – should be. This appears to make the MTA worthwhile even if it fails to convince *any* atheists.

**OBJECTION TWO:** The MTA claims that the property of omnipotence entails (necessary) existence. But consider the following putative being: an omnipotent round square. If omnipotence entails existence, this being must exist. But this being obviously does not exist; a round square is logically impossible. Therefore, the MTA must be mistaken.

**RESPONSE TO OBJECTION TWO:** Casting doubt on ontological arguments by showing that they can be used to prove the existence of absurd entities – such as the greatest conceivable island, in Gaunilo’s famous parody of Anselm – is a strategy almost as old as the ontological argument itself. The strategy is still used today; for instance, Gödel’s version of the argument can be parodied (see Oppy [16]). However, the MTA cannot be used to show that an omnipotent round square exists. Note the structure of the MTA: for the MTA to establish the necessary existence of a being, the being itself must be logically possible. If the being is not logically
possible, then step (6) will be false. But an omnipotent round square is not logically possible. So the MTA cannot establish that such a being necessarily exists.

OBJECTION THREE: You have just ruled out the possibility that the MTA entails that self-contradictory entities like omnipotent round squares exist. But perhaps one can construct a parody of the argument to show that some other absurd entity exists, e.g., a necessary red thing?

RESPONSE TO OBJECTION THREE: The MTA cannot be used to show that a necessary red thing exists either. Note again the structure of the MTA: for the MTA to establish the necessary existence of a being, the being must be capable of bringing about any random logically possible world. If the being cannot bring about any random logically possible world, then for example, (3) will be false. There is no reason to think that a red thing can bring about any random logically possible world. There is a reason, however, to think that an omnipotent being could bring about any random logically possible world; it could do so because of the definition of “omnipotence”. Whatever faults the MTA has, it cannot be easily parodied.

OBJECTION FOUR: Existence is not a predicate or a property etc.

RESPONSE TO OBJECTION FOUR: This famous objection to traditional ontological arguments is generally attributed to Kant [10], although Gassendi arguably makes the same or at least a similar point in the 5th set of objections to Descartes’s Meditations. But at no place in the MTA do we state or implicitly assume that existence is a predicate; indeed, we use an existential quantifier; this traditional objection is simply irrelevant to the MTA. In any event, this objection is too often presented as a conclusive refutation of traditional ontological arguments, when it is not conclusive. It is far from clear that existence is not a property; there are some reasons for thinking that it is (see McGinn [14]). Indeed, it is common to posit an existence predicate in quantified modal logic; see, e.g., Hughes and Cresswell ([8], p. 292). But again, this objection is irrelevant in any case; we mention it only because it is so ubiquitous. Incidentally, the MTA never claims that existence is a perfection either, nor does it ever rely on what is

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15See Descartes [3] for the exchange between Descartes and Gassendi. Forgie [4] has argued that Kant and Gassendi were actually making different claims, which is why we say that they are “arguably” making the same or a similar claim.
conceivable to determine what is possible. So other potentially problematic claims of (at least some) traditional ontological arguments are avoided as well.

**OBJECTION FIVE:** An objection to ontological arguments is the following: ontological arguments claim that something (a perfect being, a supreme being etc.) exists by definition; the definition of “perfect”, for instance, entails that such a being must exist. So, for example, echoing Descartes, an argument might claim that (i) a perfect being has all perfections; (ii) existence is a perfection; therefore, (iii) a perfect being exists. But what the argument should say is this: (a) if a perfect being exists, then a perfect being has all perfections, (b) existence is a perfection, so (c) if a perfect being exists, a perfect being exists. Claim (i) simply gives the definition of a perfect being; and so only gives the necessary conditions that a being must meet to count as an example of a perfect being. This implies that (i) should really be thought of as a conditional, i.e., (a). But then we must turn the conclusion (i.e., (iii)) into a conditional as well (i.e., (c)). An atheist can grant that (a) is true; they can simply claim that the antecedent is false. And an atheist can even grant that the conclusion (c) is true; (c) poses no threat to atheism. And the theist cannot simply assert the antecedent of (c) for that would beg the question. The MTA is an ontological argument that claims that the definition of “omnipotence” entails that an omnipotent being exists; it stands to reason that the MTA faces a similar problem. Why can’t the atheist simply turn the conclusion of the MTA into a conditional?

**RESPONSE TO OBJECTION FIVE:** The MTA does not fall prey to this difficulty; in fact, it was largely formulated in the manner that it was to avoid this objection. For example, recall that premise (1) was, “the definition of an “omnipotent being” entails that an omnipotent being could make any logically possible, contingent state of affairs obtain”. We claimed that this premise was true by definition and that it could be reformulated as a conditional; specifically, if there is an omnipotent being, then it could make any logically possible, contingent state of affairs obtain. This is precisely the conditional that objection five would have us turn (1) into; but it is already in that form (or at least can be easily put into it). The same is true of (3): one can reformulate (3) as a conditional (of the relevant sort); i.e., if there is an omnipotent being, then it could make any random logically

\[16\] Indeed, given that (c) is a tautology, the atheist should endorse it; but again, (c) poses no threat to atheism.
possible world obtain. Reformulating these premises as conditionals does not affect the argument; e.g., it does not force us to turn the conclusion into a conditional; these premises already are (or can be) in the form of a conditional of the relevant type. Perhaps the atheist can try to turn (1), i.e., “the definition of an “omnipotent being” entails that an omnipotent being could make any logically possible, contingent state of affairs obtain”, into this conditional, “if there is an omnipotent being, then the definition of an “omnipotent being” entails that an omnipotent being could make any logically possible state of affairs obtain”. But this is to no avail, since the consequent of the new conditional is true whether the antecedent is true or not; we discussed above how (1) will be true whether an omnipotent being exists or not. So given that the consequent of the conditional is true regardless, and the consequent of the conditional simply is (1), and it is (1) that the MTA needs, turning (1) into a conditional does nothing to harm the MTA. One could say the same about (3).

OBJECTION SIX: All we know about this being is that it is omnipotent. But for all we know, the being lacks the other divine attributes, e.g., omniscience and omnibenevolence. Perhaps the being dislikes us, and this explains the existence of evil and the sorry state we find ourselves in. This being might not be worthy of worship etc.

RESPONSE TO OBJECTION SIX: It is a common and venerable strategy to attack theistic arguments by pointing to the “gap” between the being that a theistic argument establishes and the traditional conception of God. For example, Hume [9] used this strategy against the argument from design (perhaps the designer is an infant deity that was ashamed of its performance etc.), and cosmological arguments are also vulnerable to such an attack (see Pruss [18]). However, there are several possible responses. First, note that this objection concedes that the being in question exists, but points to the gap between what we know about this being (at least given the MTA) and the traditional conception of God; but this is already a major victory for theism. Clearly, the existence of an omnipotent being would be theologically interesting. Second, for all we know, the traditional conception of God is mistaken. The argument never set out to show that the traditional conception is true, but only that there is a being that has at least one of the divine attributes, omnipotence. This objection concedes that the goal has been met. Third, it appears that we could at least consistently ascribe some more of the traditional divine attributes to this necessary omnipotent being. For instance, there is no reason to think that
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an omnipotent being could not also be omniscient, or the creator of the universe. If one thinks there is an adequate solution to the problem of evil, and that omnibenevolence is consistent with omnipotence and other divine attributes, one can perhaps ascribe the property of omnibenevolence to this entity as well? In short, it might be possible to extend the argument to show that this being has additional divine attributes. Fourth, there might be various logical relationships between some of the divine attributes so that, e.g., if an agent has one of the divine attributes, the agent also must have a different divine attribute; if it can be shown that omnipotence entails other properties, then we can infer that the being has those properties as well. For instance, one might think that omnipotence entails omniscience. If an agent knowing that $P$ can be thought of as a logically possible state of affairs, and an omnipotent agent can bring about any logically possible state of affairs, then an omnipotent agent can make itself omniscient, at least. We confess, however, that it doesn’t appear that omnipotence or omniscience entails omnibenevolence; so this strategy probably has its limits.\textsuperscript{17}

**OBJECTION SEVEN:** Perhaps there are multiple necessary omnipotent beings?

**Response to Objection Seven:** This objection, like the sixth, follows a strategy used by Hume [9]; e.g., against the argument from design, Hume suggests that perhaps there was a team of designers etc. But this objection also concedes that at least one omnipotent being exists in the actual world; establishing this was the principle aim of the MTA, so this “objection” is not even really an objection. Second, it might be impossible to have more than one omnipotent being in a given possible world. Hoffman and Rosenkrantz ([7], section one) write,

\begin{quote}
\textit{“... could there be two coexistent omnipotent agents, Dick and Jane? If this were even possible, then possibly, at some time, t, Dick, while retaining his omnipotence, attempts to move a feather, and at t, Jane, while retaining her omnipotence, attempts to keep that feather motionless. Intuitively, in this case, neither Dick nor Jane would affect the feather as...”}
\end{quote}

\textsuperscript{17}Note that if the doctrine of divine simplicity is true, then all of the divine attributes are identical to one another in some sense, so if a being has one divine attribute, it will have all of them. So the modal theistic argument along with the doctrine of divine simplicity will entail that God – as traditionally conceived – exists. However, we won’t press this point because we are somewhat suspicious of the doctrine of divine simplicity.

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to its motion or rest. Thus, in this case, at t, Dick would be powerless to move the feather, and at t, Jane would be powerless to keep the feather motionless! But it is absurd to suppose that an omnipotent agent could lack the power to move a feather or the power to keep it motionless. Therefore, neither Dick nor Jane is omnipotent.”

Third, this objection is fairly strange; it would be odd for an atheist to move from atheism to polytheism simply to overcome the MTA. To sharpen this response, atheists are often quite fond of Occam’s razor, and indeed, occasionally use it to try to defeat theism (see Dawkins [2], for example). This atheistic argument is sometimes called the “argument from parsimony”. But in this case, Occam’s razor would have us reject the objection: the MTA only shows that at least one necessary omnipotent being exists, so positing more than one such entity would be needlessly multiplying entities, and we agree with the atheists that we don’t want to do that.18

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18We’d like to thank two anonymous referees for extremely helpful comments on an earlier draft.
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A Debate on God: Anselm, Aquinas and Scotus

Marcin Tkaczyk

In this paper I deliver a case study of main scholastic attempts to prove God’s existence – those from Anselm of Canterbury, who was the first to prove God’s existence a priori, Thomas Aquinas, the prominent antagonist of such proofs, and John Duns Scotus, who was a moderate antagonist against Anselm, although is commonly considered a supporter of ontological proofs even in esteemed reference books. The texts I focus on are classical, but they are analyzed systematically, from a contemporary point of view, rather than historically. I do not pretend to reconstruct every single detail of the medieval theories, but I ask, what can a contemporary philosopher learn from them.

The working hypotheses I put forward and try to confirm are following. There is no very sharp difference between ontological proofs and those non-ontological. The proofs provided by leading philosophers are usually formally correct, and yet they must not be judged as sound or as convincing. The existence of God is usually smuggled in a way into the premises. This seems to be inevitable, if the concept of proof is to be understood strictly.

The Reader should remember that, what I offer, is a case study rather than fully systematic exposition. I attempt to examine the prominent proofs with respect to their premises rather than their validity. And I focus on modal concepts involved in those premises. However, as it is a case study, the debate remains open.
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1 Who is God?

When a philosopher proves God’s existence (as well as non-existence), they refer to an object people think and speak of before they philosophize. The issue is not a proof of existence of an arbitrary object, called God subsequently by a convention. It is rather the issue of existence or non-existence of a definite object of which some people think it exists and other do not. What stands generally, applies especially to Jewish, Christian and Muslim thinkers. They face the challenge to justify existence of the God of Revelation. Hence, to demonstrate an object \( x \) to exist is half way home only. The other half is to identify the \( x \) with the One, and it is not a bit easier than the former. For a Christian thinker the main principle in the field is the opening article of the Nicene Creed: “we believe in one God, the Father, the Almighty, maker of all that is, seen and unseen” (πιστεύομεν εἰς ἕνα Θεόν, Πατέρα, Παντοκράτορα, πάντων ὅρατῶν καὶ ἑκάτερων ποιητήν). Half century later the words “heaven and earth” were added by the First Council of Constantinople, hinting most clearly to the first sentence of the Holy Scripture: “in the beginning God created the heaven and the earth” (ἐν ἀρχῇ ἐποιήσεν ὁ θεὸς τὸν ὄφραν καὶ τὴν γῆν) (Ge 1:1). As a result the credendum took the final shape: “we believe in one God, the Father, the Almighty, maker of heaven and earth, and of all that is, seen and unseen” (πιστεύομεν εἰς ἕνα Θεόν, Πατέρα, Παντοκράτορα, ποιητὴν ὄφρανος καὶ γῆς, ὅρατον τε πάντων καὶ ἀφανῶν). Hence, as early as fourth century, the basic Christian dogma claims that (i) the mysterious figure Jesus of Nazareth called Father (the Father), (ii) the supreme reason which governs the universe according to the Greek philosophy (the Almighty) and (iii) the God of the Jewish Old Testament (the maker of heaven and earth), are identical (one God).

In this paper I focus on some complicated interplay of two terms in main medieval accounts of God: “necessary being” and “contingent being”. Those term play substantial rôles in proving God’s existence with respect to Nicene description of God as the maker of all that is as well as the supreme being.

2 Modal concepts, modal logic.

Theory of modality has been designated as crux logorum not accidentally. Modal concepts, like necessity, contingency, possibility and impossibility, definitely play an important rôle in almost all kinds of knowledge. Still the
theory of modality remains most obscure, and not a bit less controversial than proving God’s existence. So, in my view, applications of modal logic to philosophical issues provide philosophical claims with no justification, and serve rather understanding of modal logic than the claims. Nevertheless, there is some, very small, piece of non-controversial modal logic, which may be of some help in philosophy, mostly in search for formal structure of some complicated expressions. The piece of modal logic is probably close to what was called by Jan Łukasiewicz basic modal logic [13]. The basic modal logic was also a common view of ancient and medieval scholars. Here is some useful revision.

In the basic modal logic classical connectives may appear: negation “¬”, conjunction “∧”, disjunction “∨”, implication “→”, equivalence “↔”, and modal connectives of necessity “□” and possibility “◇”. The connective of negation and modal connectives are one-place, whereas other connectives are two-place. Formation rules are typical and in absence of parentheses the order of connectives is like that: ◇, □, ¬, ∧, ∨, →, ↔. In metalanguage variables: “A”, “B” and “C” are used to speak of any object language formulas, as well as symbols: “|=”, “⊧” and “∼” for consequence, denied consequence and logical equivalence. Sets of formulas are logically equivalent if and only if they have the same consequences.

Principles of the basic modal logic are well grounded in the vernacular use of modal expressions. Let A be an arbitrary formula. Aristotelian way of mutual definability of modal connectives is considered obvious:

\[ \Box A \sim \neg \Diamond \neg A, \]  
\[ \Diamond A \sim \neg \Box \neg A. \]  

Interrelations between modalities are also considered obvious to the effect that necessity is something more (something deductively stronger) than mere occurrence, whereas possibility is something less (something deductively weaker) than mere occurrence:

\[ \Box A \vdash \Diamond A, \]  
\[ \Box A \vdash A, \]  
\[ A \vdash \Diamond A, \]

Those principles are counterparts of scholastic dicta: “‘it is necessary that’ entails ‘it is possible that’” (a necesse ad posse valet consequentia formalis), “‘it is necessary that’ entails ‘it is the case that’” (a necesse ad esse valet consequentia formalis) and “‘it is the case that’ entails ‘it is possible that’” (ab esse ad posse valet consequentia formalis) respectively.
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And here are opposite principles:

\[ \Diamond A \nvdash \Box A, \quad (2.6) \]
\[ A \nvdash \Box A, \quad (2.7) \]
\[ \Diamond A \nvdash A, \quad (2.8) \]

for some formula \( A \). Those principles are counterparts of dicta: “‘it is possible that’ does not entail ‘it is necessary that’” (\textit{a posse ad necesse non valet consequentia formalis}), “‘it is the case that’ does not entail ‘it is necessary that’” (\textit{ab esse ad necesse non valet consequentia formalis}) and “‘it is possible that’ does not entail ‘it is the case that’” (\textit{a posse ad esse non valet consequentia formalis}) respectively.

Further commonly accepted principle of modal logic is the Aristotelian rule of inheritance of modalities. If a consequence occurs:

\[ A_1, A_2, \ldots, A_n \vdash B, \]

then so do the consequences:

\[ \Box A_1, \Box A_2, \ldots, \Box A_n \vdash \Box B, \]
\[ \Diamond A_1, \Box A_2, \ldots, \Box A_n \vdash \Diamond B, \]
\[ \Diamond (A_1 \land A_2 \land \ldots \land A_n) \vdash \Diamond B. \]

Hence, in the case of possibility only some restricted inheritances occur. Those rules are derivable in weak regular modal logics, like C2 and K, and will be at use in this paper.

3 Aristotelian Modalities.

First chapters of Aristotle’s \textit{Prior Analytics} contain purely assertoric, i.e. non-modal, syllogistic, presumably the first formal theory ever. In accordance with Platon’s view of simple statements, Aristotle thinks of formulas as predications one thing of one thing (\pi\nu\omicron\omicron\zeta \dot{\alpha}\pi\omicron \pi\nu\omicron\omicron\zeta). One thing, called predicate, belongs or not to the other, called subject:

\[ a \text{ belongs to } b, \]
\[ a \text{ does not belong to } b, \]

where a subject is usually supplemented with a quantificational term: “all” or “some” ([3], p. 121 - 123). Such an account of simple units of information was to become a common knowledge, and even today, despite of the development of mathematical logic, is not to be ignored.
Assertoric syllogistic, known as Aristotle’s second logic, as opposed to his early, mostly rethoric or semantical works constituting the first one, turned out soon insufficient. So Aristotle’s third logic came into being, that is modal logic that suits Aristotelian essentialism. Basic description of modal syllogistic may be found in chapters 8–24 of the first book of *Prior Analytics*. To expand syllogistic on changing entities, mostly living organisms, Aristotle accepts three basic relations between terms instead one:

\[
\begin{align*}
  &a \text{ (merely) belongs to } b, \\
  &a \text{ necessarily belongs to } b, \\
  &a \text{ possibly belongs to } b,
\end{align*}
\]

forming assertoric, apodeictic or problematic formulas respectively (*Prior Analytics* 1.8). Each one of them may be negated as well as supplemented with quantificational term. Some further ideas on modal logic may be found in Aristotle’s *Metaphysics*.

Although assertoric syllogistic became a pillar of western culture and universal academic decorum (*pueris dabatur*), Aristotle’s modal logic has always been found highly controversial and some of its paradoxes still remain not solved. Chronic ambiguity of modal expressions certainly belongs to prominent causes of, all by all, failure of modal logic.

According to Robert Patterson, from the very beginning Aristotle makes use unawares of two different concepts of necessity. The necessary belonging may be identified either with a’s belonging to the nature of b, or with its belonging to the concept of b. In the first case the modality is natural or essential, whereas in the other the modality is conceptual ([15], p. 11 - 14). Consider a simple example. The sentence:

mammal necessarily belongs to every tiger

is true under both concepts of necessity. Now imagine a cage containing tigers exclusively. The sentence:

mammal necessarily belongs to every animal in the cage

is true only under the concept of natural necessity. For there is no contradiction in a supposition that in the cage there is a non-mammal animal, say a snake. But, if it is – even accidentally - the case that all animals in the cage are tigers, being a mammal is part of the nature of every single animal in the cage in question.

In my view, analogical distinction should be drawn with respect to Aristotelian possibility. Conceptual possibility would be closely connected with
consistency, whereas natural or essential possibility would be related to natural inclinations of an object, regardless its description. For example among the sentences:

flying possibly belongs to some birds,

flying possibly belongs to some animals on the roof

first employs the conceptual possibility, and the other the natural one. To affirm or deny the latter sentence one needs to check, what animals there are on the roof. But to know that the denial of the first sentence is false, it is enough to understand the sentence.

Unaware use of conceptual and natural modalities – as it should shortly get clear – may cause serious misunderstandings in ontology, particularly in theodicy.

4 Rise of existential modalities.

Neither in assertoric syllogistic nor in modal one any use is made by Aristotle of the term “being” or “existence”. And there is nothing surprising in it, for, according to Aristotle, there is actually no concept of being whatsoever or, more precisely, there are as many different concepts of being as categories. So being is no species. That is the fundamental claim of Aristotelian theory of being qua being: there is no being qua being. So, statements like:

existence belongs to $a$,
existence does not belong to $a$

do not appear within the strictly Aristotelian horizon. As it is stated in Posterior Analytics, Aristotle would rather accepted such a formal representation of existence as

$$a \text{ exists } \leftrightarrow \exists b (b \text{ belongs to } a).$$

So $a$ exists if and only if there is some predicate which can be attributed to $a$. But existence is not anything which might be attributed itself in the sense. The more so neither necessary nor possible existence are. The conceptual framework of peripatetic philosophy was expanded on existence by two Persian, Muslim thinkers: Al Farabi ($9^{th} - 10^{th}$ centuries) and Avicenna (properly Hakim Ibn Sina, $10^{th} - 11^{th}$ centuries).

Both Al Farabi and Avicenna thought – perhaps erroneously – Aristotle had been aware of the peculiarity of concepts of existence. They based their
conclusions on distinctions between first and second substance, introduced in *Categories*, and mostly on one excerpt of *Posterior Analytics* (92 b10), Aristotle’s main work on philosophy of science. In the latter text Aristotle differentiates two kinds of scientific problems: a question, what something is, and a question, if it is (τι ἐστι δημοσίᾳ το ἐνα δημοσίᾳ) ([11], pp. 131 - 132).

It was presumably Al Farabi who founded the difference between essence and existence. And he drew the conclusion that a combination of essence and existence, that forms a being, requires explanation. According to Al Farabi the only satisfactory explanation of it is a being which explains its existence itself. Its existence should occur in combination with its essence by means of the essence only. Such a being is designated as necessary, as opposed to other, contingent beings, and is identified with God, creator of the world, almost automatically. So Al Farabi founded a proof of God’s existence from the contingency: all objects we know in the world are contingent, so there certainly is another, necessary object, which decided about existence of contingent objects — for any infinite sequence of contingent causes should be excluded ([11], pp. 132 - 133).

A century after Al Farabi’s death his general ideas were to be elaborated and developed by Avicenna, who introduced existence into the conceptual framework of Aristotelian theory of modality. He allows to speak of existence as belonging or not to an object:

existence (merely) belongs to \( a \),  
existence necessarily belongs to \( a \),  
necessity possibly belongs to \( a \).

An object \( a \) exists if and only if existence belongs to it. The object is a necessary being if and only if existence belongs to it necessarily, i.e. by means of its own nature, its own essence. Otherwise existence belongs to \( a \) “as if it was an accident” – says Avicenna. Such a being is contingent, it exists contingently. Contingent objects possess – if any – some borrowed existence, reflected light [8].

Here is the source for Al-Farabi-like proof of God’s existence. For Avicenna was presumably the first thinker who quite consequently regarded God as the unique necessary being. According to Avicenna God exists necessarily if and only if his existence is possible:

\[ \Diamond A \sim A \sim \Box A, \]

for \( A \) meaning that God exists. And nothing but God has the curious feature ([17], p. 198). Such an account of God is based on Al Farabi’s
proof of God’s existence. God is simply equipped with such features that he can serve as an explanation of existence of the contingent objects.

Thomas Aquinas, who was to adapt Al Farabi’s proof in the thirteenth century, called it the proof from necessity and possibility. He obviously identified Al Farabi’s necessary being with the philosophical Almighty as well as the Jewish maker of the heaven and the earth, and of all that is, seen and unseen, of the Nicene Creed.

Introduction of problematic existential statements is not as automatic as apodeictic or assertoric ones. As long as conceptual modalities are referred to it seems quite acceptable to adapt the interrelation (2.2) for a definition. An object would be then possible if and only if its existence is not excluded by its concept. The proverbial square circle would be an example of an impossible object. Notwithstanding, if natural modalities are to be referred to, the problem becomes much more difficult. Generally speaking, b possibly belongs to a if and only if b is not prohibited by a’s nature. However, if a is allowed to stand for a non-existent object and b is allowed to be existence, the idea gets complicated. If a did not exist, it would be able neither to entail nor to exclude anything. So either everything or nothing should be considered possible for a non-existent object. Notice, that no such difficulty arises within the confines of purely Aristotelian modalities. Since, by definition, only existent objects are there the issue, and hence, there is always a nature to entail or prohibit some predication.

Avicenna’s solution is a version of modal realism. The Persian thinker claims natures or essences may exist whether or not they are exemplified by existing objects. As Goichon claims, possibility of existence of an object a is identified as existence of a’s essence ([11], p. 136). As far as I am aware it is the first appearance of the possible worlds ontology ever. It is worth noting, although possibility is here reduced to existence of object of a sort, necessity is not strictly identified with general quantification over possibilities.

By Avicenna’s account essences exist in the mind of God, the unique necessary being, who is in a position do present some of them with existence. An anticipation of Leibnizian theodicy is obvious. In connection with those hypotheses Avicenna accounts for universals: they are independent things (ante res) as God’s thoughts, in things (in rebus) as their essences or natures and come into being after things (post res) as abstract concepts situated in human minds.

Persian ontology, as proposed by Al Farabi and Avicenna, may be found
tempting. Nevertheless, when God’s existence is questioned and being proved, the idea of divine mind as a container for mere possibilities must be regarded as useless. Still, a version of modal realism may be regarded as a necessary presumption of some theological investigations. And one should not overlook the fact that – like in the case of Anselm – Avicennian modalities are related to mind, even if the mind is divine. So possibility is a kind of conceivability.

It should be emphasized that terms like “necessary being” and “contingent being” have not been invented by the Persians. The Greek philosophy, as well as early Christian thinkers, use the terms. However, the meaning changes. Before Al Farabi a necessary being means something like mathematical objects — invariable, permanent, eternal. On the other hand contingent beings rise into existence, change and finally perish. The general idea has been harvested from Parmenides and Heraclitus and developed by Plato. Aristotle summarizes it in his Ethica Nicomachea, identifying the necessity as lack of any origin, change or perish: (τὰ γὰρ ἐξ ἀνάγκης ὀντὰ ἀπλῶς πάντα ἄδια, τὰ δ’ ἄδια ἀγένητα καὶ ἄφθαρτα) ([2], 1139 b). However, it is Al Farabi, and then Avicenna, who develop the idea of borrowing and granting existence. They introduce the concept of contingent being as a being which requires some external interference to exist, as well as the concept of necessary being as a being which requires no such interference.

5 Prosligion.

The history of ontological proofs begins in 10th century with Anselm of Canterbury, the abbot of Le Bec and future primate of England. In 1077 and 1078, at his fellow monks’ request he wrote a book called Prosligion: Discourse on the Existence of God to the effect that God’s existence, the monks believed in firmly, might be recognized and proved with no appeal to faith.

The number of analyses dedicated to Ratio Anselmi is enormous, and so is the number of different – sometimes vitally different – interpretations of Anselm’s argument. What Anselm claims in Prosligion is so difficult that among scholars there is no agreement even regarding what kind of text one has to do with. M. Cappuyns, in his excellent summary [4] of all studies concerning Ratio Anselmi until 1934, distinguishes four general kinds of interpretations or attitudes towards Anselm’s text: logical, psychological, cosmological and theological. Cappuyns’ summary seems to remain standing. Only in the first interpretation Ratio Anselmi is regarded as an
argument (either sound or not) from some premises to a conclusion. In the psychological interpretation Anselm simply affirms the fact of God’s presence in mind, in the cosmological interpretation one has to do with a supplement to *Monologion*, an earlier work from Anselm, and in the theological interpretation one has to do with an affirmation of the act of faith. It should be claimed that in this paper it is just assumed that *Ratio Anselmi* is an argument, an attempt to prove God’s existence. Even if one is fully convinced that Anselm’s text is an argument, there is still a lot of questions with respect to its soundness as well as the proper meaning.

In the chapter 2 Anselm concludes from conceivability of a being than which nothing greater can be conceived to the real existence of the being (“existit ergo procul dubio aliquid quo maius cogitari non valet, et in intellectu et in re”) ([1], ch. 2). Most readers regard the expression “a being than which nothing greater can be conceived” as a definition of God. According to Cappuyns it is just the case and the definition is most obvious and most common (admise par tout le monde et, en réalité, tautologique) ([4], p. 324). The chief objection voiced commonly to that argument concerns the way Anselm makes use of the concept of God. It is claimed to be illegitimate to conclude from a definition or a concept of an object to existence of the object (l’objection calassique contre l’argument de S. Anselme est qu’il fait sortir l’existence de la pensée) ([10], p. 6). A simplified reading of the chapter 2 gave rise to some versions of ontological proof most naive and even considered discreditable for the idea itself. In such account the ontological proof would proceed approximately like that: all perfections should be, by definition, attributed to God, existence is a perfection, therefore God exists. Such claims appear e.g. in Bonaventure and Kant, and deserve no debate. On the other hand many scholars accounted for Anselm’s proof as a piece of reasoning much more sophisticated.

The subsequent chapters of *Proslogion* must not be overlooked. In the chapter 3 it is claimed that God cannot even be conceived not to exist (“quod utique sic vere est, ut nec cogitari possit non esse”) ([1], ch. 3). In the chapter 5 God is regarded as the highest of all beings, and it is claimed that God alone exists through himself and creates all other things from nothing (sed quis es, nisi id quod summum omnium solum existens per seipsum, omnia alia fecit de nihilo?) ([1], ch. 5). In the chapter 13 God is regarded as uncircumscribed and eternal (*tu ergo, domine, singulariter es incircumscriptus et aeternus*) ([1], ch. 13), and this topic is returned to in the chapters 18, 19 and 20. Those are summarized in the chapter 22 to the effect that God alone is fully self-identical (*tu solus ergo, Domine,*
es quod es, et tu es qui es [...] et tu es qui proprie et simpliciter es; quia nec habes fuisse aut futurum esse, sed tantum praesens esse, nec potes cogitari aliquando non esse) ([1], ch. 22). Those claims seem to be vital for modal characterization of Anselmian God. And, by the way, in the chapter 15 some piece of negative theology is delivered to the effect that God is greater than can be conceived (non solum es quo maius cogitari nequit, sed es quiddam maius quam cogitari possit) ([1], ch. 15)

6 General framework of Anselm’s proof.

What exactly is Anselm’s reasoning and what it exactly claims, remains a matter of a never ending discussion. There is no agreement either on interdependences between chapters of *Proslogion* or on the train of Anselm’s thought. It has been claimed that the chapters constitute the proof inclusively, and that the proof is included only in the chapter 2. According to Charles Hartshorne [12] the chapters 2 and 3 include two separate proofs, the former a non-sound and the latter a sound one. Whatever the particular interpretative ascertainments might once turn out, the framework of modal concepts involved in the proof remains unchanged and constitute the general template of classical ontological argument, or at least its modal account:

\[
\text{God’s existence is possible,} \quad \\text{God’s existence cannot be contingent,} \\
\therefore \text{God exists.} \quad (6.1)
\]

The framework was more or less present in most accounts of an ontological proof, but has been firmly established in the profound study from N. Malcolm [14]. Depending on particular interpretation of *Proslogion* the definite meaning of modal expressions as well as the way to justify the premises may change.

In the chapter 2 of *Proslogion* Anselm argues to the effect that God exists. No clues on either necessity or contingency of his existence are given straightforwardly. God is designated as a being than which nothing greater can be conceived (*id quo maius cogitari non potest*) ([1], ch. 2). Modalities are here introduced by means of a concept of *conceivability*, which might be a counterpart for possibility, and *inconceivability*.

The first premise of the argument (6.1) is grounded on two Anselm’s claims. First, God is conceivable. Some people believe in God, others do not. And they can disagree and even argue to one another. It follows
they understand one another, at least from time to time. So, as it was just stated, God is conceivable. Anselm invokes the Biblical fool, who “hath said in his heart: ‘there is no God’ ” (dixit insipiens in corde suo: non est deus, cf. The Book of Psalms 14:1 and 53:1, in the Biblia Sacra Vulgata, the Latin version Anselm used, 13:1 and 52:1 respectively). If the fool thinks there be no God, he must conceive of God (convincitur ergo insipiens esse vel in intellectu aliquid quo nihil maius cogitari potest) ([1], ch. 2). According to the other claim God’s existence is conceivable, provided God is generally conceivable. (si enim vel in solo intellectu est, potest cogitari esse et in re) ([1], ch. 2). Anselm seems to claim: if x exists in, say, the universe of possibilities, then x’s real existence is possible. In my view, that profound and rather controversial claim deserves much more careful study than it has been given thus far. It is presumably a thought analogical to metalogical claim x be provable in T if and only if x’s provability is provable in T. Nevertheless, in this paper I disregard that problem. The point is here, the two claims: God’s conceivability and the conceivability of his existence as a consequence of the former aspire to ground together the first premise of the argument (6 .1): God’s existence is at least possible.

The second premise of the argument (6 .1), in Anselm’s account, is grounded generally in the following view: if God existed contingently, he would not be God, he would not be himself. In the claim a hierarchy is assumed among all possible objects: some of them are greater than others. The idea of that hierarchy has been involved in the first premise already, actually in the concept of God itself. Now the idea becomes substantial. As Anselm claims:

an existent being is greater than a merely conceivable one, (6 .2)
a necessary being is greater than a contingent one. (6 .3)

The claim (6 .2) appears in the chapter 2 of Proslogion (si enim vel in solo intellectu est, potest cogitari esse et in re, quod maius est) ([1], ch. 2). The claim (6 .3) is central for the chapter 3. A necessary being is called a being which is conceivable generally, but its non-existence is inconceivable (nam potest cogitari esse aliquid, quod non possit cogitari non esse) ([1], ch. 3). Such a being is considered greater than a contingent one (quod maius est quam quod non esse cogitari potest) ([1], ch. 3). It seems to follow, God could not exist contingently.

It seems there be two versions of the second premise of the ontological
proof: the claim (6.2), to the effect that God’s existence cannot be a mere possibility, and the claim (6.3), to the effect that God’s existence cannot be contingent. So, there would be two versions of the ontological proof or even, as Hartshorne claims, two (absolutely) independent ontological proofs. If the claim (6.3) is being taken into account, the argument (6.1) is immediately achieved. If it is the claim (6.2) to be accounted, the ontological proof should be summarized as follows:

\[
\text{God’s existence is possible,} \\
\text{God’s existence cannot be merely possible,} \\
\therefore \text{God exists,} \\
\text{(9*)}
\]

which is a definitely valid argument. Nevertheless over a span of ages the philosophical criticism was leveled at the claim (6.2), while the claim (6.3) remains almost untouched. Because the second premise of the argument (9*) was commonly considered absolutely unacceptable. Hence, most philosophers, who are interested in ontological proofs, take the argument (6.1) into consideration, involving the chapter 3 of *Proslogion*. It is a fact most peculiar, for the claim (6.2) is clearly entailed by the claim (6.3), provided modal terms are not ambiguous. I presume, when reading the sentence (6.3) philosophers do not feel the ontological commitment as sharply as in the sentence (6.2). Still, it is only the philosophers’ feeling.

It should be also noted, as I have already mentioned, Anselm is perfectly aware of the idea of a necessary being as a being existing by its own power. For in the chapter 5 Anselm calls God such a being — the unique being which exists by himself (*solum existens per seipsum*) ([1], ch. 5). And in the subsequent chapters 13, 18, 20 and 22 he calls God eternal, self-identical etc., hence, necessary in the classical Greek sense. Furthermore, in an earlier work of mine I argued to the effect that there is another premise in Anselm’s proof, a hidden one, to establish the required interdependence between the conceivability qua existent and the real existence ([21], pp. 297 - 298). I consider it absolutely vital for evaluation of Anselm’s reasoning as valid or not. Nevertheless, the point plays no rôle in the case of modal concepts here involved, and so may be disregarded.

## 7 Anselmian modalities.

If the ontological proof was identified as the argument (9*), it would be obviously valid. Since it seems to be just a case of Modus Ponens:

\[
\Diamond A, \Diamond A \rightarrow A \vdash A,
\]
A Debate on God: Anselm, Aquinas and Scotus

where $A$ means that God exists. Furthermore, validity of the argument (9*) might be established with no appeal to modal logic whatsoever. Let $A$ mean that God’s existence is possible and let $B$ mean that God exists, so, mere possibility of God’s existence would be meant by the conjunction $A \land \neg B$.

In that account the argument (9*) would be represented by the schema:

$$A, \neg(A \land \neg B) \vdash B, \quad (7.1)$$

which is a valid schema of the classical propositional calculus. But, as the claim (6.2) has been held discredited for centuries, philosophical debate – almost since the very beginning – has been focused on the argument (6.1).

There are two basic readings of the argument (6.1), differing with respect to a schema and deductive strength of the second premise. Let again $A$ mean that God exists. The first premise of (6.1) has an obvious counterpart:

$$3A, \quad (7.2)$$
as above. The other premise usually is to be read as follows:

$$\neg \Box(A \land \Box \neg A), \quad (7.3)$$

which is logically equivalent to the formula

$$\Box(A \rightarrow \Box A). \quad (7.4)$$

This is, among others, the view of Hartshorne ([12], p. 51). As it was indicated by C. L. Purtill, the entailment

$$\Box A, \Box(A \rightarrow \Box A) \vdash A \quad (7.5)$$

holds in a modal logic which contain – above the uncontroversial rule (2.4) – the interdependence:

$$\Box(A \rightarrow \Box A) \vdash \Box(\Box A \rightarrow A), \quad (7.6)$$

which is by no means obvious. Hartshorne reconstructed Anselm’s reasoning in the very strong system S5, so the rule (7.6) was secured. However, the system S5 is not free of doubt, hence an embarrassing game begun to search for a logic ready to prove God’s existence. It is a well known fact that in the system T the necessary being has not come to being yet, but in the system S5 it fortunately begins to exist. In such circumstances another reading of the second premise of (6.1) might seem to be a considerable
solution. For as early as in Malcolm’s classical work ([14], pp. 148 - 149) it was claimed Anselm had argued to the effect that God’s existence is either necessary or impossible:

\[ \neg \Diamond (\Diamond A \land \Diamond \neg A), \quad (7.7) \]

which is logically equivalent to the formula

\[ \Box (\Diamond A \to \Box A), \quad (7.8) \]

a stronger one than (7.4). The entailment

\[ \Diamond A, \Box (\Diamond A \to \Box A) \vdash A \quad (7.9) \]

holds in quite weak, uncontroversial calculi, hence validity of the ontological proof would be undoubtful under the reading (7.7) of the second premise of (6.1).

The choice between readings (7.4) and (7.7) is by no means a matter of philosophical taste. It is not allowed just to prefer the reading (7.7) to (7.4), and solve the problem of validity of the argument (6.1) that way. For the readings in question differ philosophically from each other and the difference is substantial. The weaker formula (7.4) means that God exists necessarily, provided God exists. That seems possible to justify from many points of view. Even such critics of Anselm like Gaunilo and Aquinas would be ready to accept the statement (7.4). Arguing against Anselm, Aquinas wrote that to state that something greater can be thought than anything given in reality is a difficulty only to a person who admits that there is in reality something than which nothing greater can be conceived (non enim inconveniens est, quolibet dato vel in re, vel in intellectu, aliquid majus cogitari posse, nisi ei qui concedit esse aliquid, quo majus cogitari non possit in rerum natura) ([19], 1.11). But the statement (7.4) seems too weak to guarantee validity of the ontological proof, at least within the confines of an uncontroversial modal logic. The formula (7.7) means that God exists necessarily, provided God’s existence is possible. It follows that, if God does not exist, God’s existence is impossible. This, however, clearly requires an ontological base for the existential modality, a base independent from God himself. Otherwise, if God did not exist, there would be nothing to prohibit God’s existence. In such circumstances the argument (6.1) seems sentenced to conceptual modalities rather than natural. The term “conceivable” must be taken serious: according to Anselm, either God’s existence or God’s non-existence must be inconceivable.

Some use of conceptual modalities is attributed to Anselm straightforwardly by Malcolm, who speaks even of *logical* necessity and impossibility.
According to Malcolm, Anselm proved that God’s existence is either logically necessary or logically impossible ([14], p. 149). In such a view the reading (7.7) seems legitimate, and the argument (6.1) is valid, but the ontological proof gets into another trouble.

Malcolm is certainly wrong — and so is a huge number of other scholars — when he claims that, in the light of the claim (7.7), the unique way to disprove God’s existence is to prove God’s existence be logically impossible ([14], p. 149). Of course, under uncontroversial modal rules, like (2.4) and (2.5), the claim (7.7) entails interdependences:

$$\diamond A \sim A \sim \square A.$$  \hfill (7.10)

Those mean that the possibility of God’s existence, God’s existence itself and God’s necessary existence are all the same. So, with respect to conceptual modalities God does not exist if and only if God’s existence entails a contradiction. However, in this very point many versions of the ontological proof, those old as well as those new, get caught in a trap. For the logical equivalence is mutual. Hence, provided the claim (7.7) holds, to prove that there is no God, it is enough to prove that God’s non-existence is logically possible, that it is not contradictory. And it certainly is so. The rules:

$$\square (\diamond A \rightarrow \square A), \diamond A \vdash A,$$
$$\square (\diamond A \rightarrow \square A), \diamond \neg A \vdash \neg A$$

are equally valid. The conceivability of God’s non-existence proves God’s non-existence equally well as conceivability of God’s existence proves God’s existence. And, for both, God’s existence and non-existence, are equally conceivable, the rules (7.4) and (7.7), as involving some conceptual modalities, are simply unacceptable.

An immediate strategy of a defender of the argument (6.1) is simply harvested from Anselm himself. The Biblical fool is invoked, who “hath said in his heart: ‘there is no God’”. And to deny the existence of God one must use the concept of God, so the concept must be consistent. However, such a defense is useless, for the concept of uncreated universe must be equally consistent, if one is to deny the claim of atheism. Hence, an attempt to base the ontological proof on any conceptual account of modality is rather hopeless. Atheism is at least consistent.
8 Aquinas’ account.

The objection to Anselm’s proof was risen immediately, when Anselm had just issued *Proslogion*, by a mysterious monk called Gaunilo of Marmoutiers, the author of *Liber pro insipiente*. Gaunilo points out to a concept of a perfect desert island, lost somewhere in oceans and absolutely unknown (*aiunt quidam alicubi oceani esse insulam, quam [...] fabulantur multo amplius quam de fortunatis insulis fertur, divitiarum deliciarumque omnium inaestimabili ubertare pollere, nulloque possessor aut habitatore uniuersis allis quas incolunt homines terris possidendorum redundantia usquequaque praestare*). If Anselm’s reasoning was correct, no one, who understood the concept, might doubt such an island to exist in reality. For an existent island is greater (i.e. better) than a non-existent one, and so the non-existent one is not perfect (*non potest ultra dubitare insulam illam terris omnibus praestantiorem uere esse alicubi in re [...] quia nisi fuerit, quaecumque alia in re est terra, praestantior illa erit, ac sic ipsa iam a te praestantior intellecta praestantior non erit*) ([9], ch. 6).

The objection took classical shape in *Contra Gentiles* by Thomas Aquinas. He claimed clearly there be no legitimate conclusion from a concept to existence in reality, although there may be a legitimate conclusion from a concept to existence in understanding (i.e. conceivability or conceivability qua existent). So, Anselm’s reasoning must not be correct (*eodem enim necesse est poni rem et nominis rationem; ex hoc autem quod mente concipitur quod profertur hoc nomine ‘deus’ non aequitur Deum esse, nisi in intellectu [...] et ex hoc non sequitur quod sit aliquid in rerum natura, quo non majus cogitari non possit*) ([19], 1.11).

The objection of Gaunilo and Aquinas was to be repeated over and over again during centuries, but no one put it better than they. And Aquinas’s is an icon of opposition to Anselm’s proof even more that the mysterious Gaunilo. In my view it is most interesting that, as I have already mentioned, such objections were practically always leveled at the claim (6.2) only, never at the claim (6.3).

Having rejected Anselm’s theory Aquinas claims, nevertheless, God’s existence to be provable. Aquinas hardly invented any proof. His famous five ways consists actually of one chief idea, harvested from Aristotle, Al Farabi and Avicenna, and reduplicated in five slightly different ways: motion, causation, contingency, values and design. Each time a feature is attributed to some existing objects that requires explanation. And, because an infinite sequent of explanation is excluded, an ultimate, absolute
explanation is required. For example, in the *ex motu* proof Aquinas has adopted from Aristotle the principle that things, which change, require an unchanging source. The chief idea is most revealed in the third way, where a concept of necessary and contingent being is involved directly. It is quite fair to say, the general template of Aquinas’ proof is:

\[
\text{some existing objects are contingent,}
\]
\[
\text{if contingent objects exist, there must exist God,} \\
\therefore \text{God exists.}
\]

And this argument is usually considered much more convincing than that of Anselm because of the presence of the contingent objects in the first premise. Such a premise is widely regarded as empirical. So an argument (8.1) looks like a piece of empirical knowledge, whereas arguments (6.1) or (9*) are *ontological*, which means bad.

Let me ask the question, if Anselm and Aquinas differ from each other, when proving God’s existence, that profoundly. Notice, the argument (8.1) is an abbreviation for a longer argument:

\[
\text{some existing objects are contingent,} \\
\text{if contingent objects exist, there must exist a necessary one,} \\
\text{God is the unique necessary object,} \\
\therefore \text{God exists,}
\]

I find this observation most instructive. For it is the chief objection Gaunilo and Aquinas addresses to Anselm: his God exists by definition. But at the same time Aquinas makes use of the concept of God *exactly in the same way*. Actually, some reference to a concept of God is absolutely inevitable in any proof of God’s existence. One certainly cannot prove that there exists the highest natural number in every set of natural numbers without having defined what exactly the highest number is. Similarly one cannot prove God’s existence or non-existence without having established what exactly God is. Whether or not a particular concept of God is legitimate for a religion or a philosophical system is, of course, a totally distinct matter. But it simply must be emphasised that Aquinas makes use of a concept of God no less than Anselm, and that the concepts they use appear to be the same.

It is striking that the third premise of the argument (8.1*) is actually identical with the second premise of the argument (6.1), or at least of the argument (9*).
In the argument (8.1), or equally (8.1*) there is even something more striking. For the first premise, Aquinas’ proof is grounded on, describes existing objects as contingent. Whereas the second premise has been considered doubtful from time to time, mostly with respect to denial of possibility of infinite sequences, the first premise is usually taken for granted. In my view it hardly is so.

When Aquinas establishes the first premise of his argument (8.1), he speaks of contingent objects, which can exist as well as not exist (sunt possibilia esse, et non esse) ([20], 1.2.3). The reason to consider a being contingent is here their rise and perish (inveniantur generari, et corrumpi) ([20], 1.2.3). That claim is supposed to be empirical (invenimus enim in rebus) ([20], 1.2.3) and such account of contingency is possible to find in Aristotle. And yet, Aquinas clearly jumps to the conclusion that a necessary being must exist. For he claims contingent beings must borrow their existence, which is clearly a piece of Persian ontology from Al Farabi and Avicenna. But this is no empirical claim any more. Hence, the claim of contingency may mean either (a) that objects we know by acquittance rise, change and finally perish, unlike mathematical objects, or (b) that objects we know by acquittance have been brought into being by something other. The claim (a) is well grounded and truly empirical, but the claim (b) is ill grounded and a piece of Persian ontology. The latter claim is ill grounded, for it clearly assumes what pretends to be justified. When one claims the Moon is shining reflected light, one clearly presupposes there is the source of the light. Analogically, when one claims the universe exists by borrowed existence, one presumes there is something able to grant the existence.

I do not defend Anselm, but in my view Aquinas smuggles God in the concept of contingency no less than Anselm in the concept of necessity. And ambiguity of modal expressions seem to be involved here no less than in Anselm’s case. It is astonishing that Duns Scotus, as early as 13th century, had at least an inkling of this failure of Aquinas’ proof. And the failure was then to be practically overlooked by philosophers occupied with Aristotelian infinite sequences. Although Duns Scotus was not able to identify the failure precisely, as we shall see, he was perfectly aware that Aquinas’ first premise is most problematic.
9 Duns Scotus’ account.

It was presumably John Duns Scotus, who was first to realize mutual relationships between Anselm’s and Aquinas’ accounts of God’s existence. According to the Doctor Subtilis both, Anselm and Aquinas, reason correctly, but fail to provide adequate justification of their premises. A modern logician would say, both proofs of God’s existence, the ontological and the cosmological one, are valid, but not sound. And the error they both commit is the petitio principii. The error an argument is reproach with if and only if at least one premise fails to be justified adequately.

In Duns Scotus’ view neither Anselm nor Aquinas established God’s existence. On the other hand neither of them was completely wrong. And of some pieces of their analyses Duns Scotus constructed his own proof. Whether or not Duns Scotus’ attempt is successful, it is definitely a piece of logical virtuosity. And so it remains most instructive.

Although neither Anselm nor Aquinas proved God to exist – Duns Scotus reasons – they both succeeded in proving something. One should analyze carefully, what exactly each of them proved. It turns out that their arguments, joint together, provide a proof of God’s existence. Or, at least, that is what Duns Scotus thinks.

Anselm’s reasoning – reinforced with Avicenna’s ontology – is successful in proving that either God exists necessarily or it is impossible for God to exist. So, Duns Scotus accepts the second premise of the argument (6 .1), an finds the stronger version (6 .3) of it well grounded in Avicenna’s ontology. But Anselm’s first premise, the possibility of God’s existence has been found by Duns Scotus ill grounded. Hence, if one found an independent proof of mere possibility of God’s existence, the work would be done.

And Duns Scotus discovers a proper proof of mere possibility of God’s existence in . . . Aquinas’ Five Ways. Although the original Aquinas’ argument (8 .1) has not been found satisfying a weaker argument seems sound to Scotus:

\[
some \text{ existing objects are possibly contingent, if contingent objects exist, there must exist God,} \quad (9 .1) \]
\[
\therefore \text{ God possibly exists.} \quad (9 .2)
\]

The modal principle is invoked here:

\[
\Diamond A, \Box (A \rightarrow B) \vdash \Diamond B. \quad (9 .2)
\]
Hence, if existence of contingent objects entails existence of God, then possibility of contingent objects entails possibility of God. Duns Scotus finally puts reinforced Anselm’s argument (6.1) together with weakened Aquinas’ argument (9.1) to achieve a proper proof of God’s existence:

\[ \text{some existing objects are possibly contingent}, \]
\[ \text{if contingent objects exist, there must exist God,} \]
\[ \text{God could not exist contingently,} \]
\[ \therefore \text{God exists,} \]

based on the valid schema:
\[ \Diamond A, \Box(A \rightarrow B), \neg(\Diamond B \land \Diamond \neg B) \vdash B. \]

He just uses the weakened version of Aquinas’ argument to justify the first premise of Anselm’s original argument. In doing so Duns Scotus reveals true fluency in deductive craft. He acts exactly like 20th-century logicians working in principles of mathematics. Notice, that the last premise of the argument (9.3), i.e. Anselm’s second premise in the argument (6.1), is practically, as I have already mentioned, equivalent to the last premise of Aquinas’ argument (8.1*). And the other premises of the argument (9.3) are weakened Aquinas’ premises. So, the main claim of Duns Scotus seems to be the possibility to derive God’s existence from weaker premises than those of Aquinas. Duns Scotus takes just the full advantage of the concept of necessary being.

10 Duns Scotus on Anselm.

Duns Scotus’ main criticism, as regards Anselm’s proof, is turned to the first premise of the argument (6.1), or equally (9*): possibility of God’s existence. According to Anselm, God is conceivable, any conceivable object is conceivable qua existent and that is what Duns Scotus regards as the ground for the first premise. And conceivability of God is based on the use, a theist as well as an atheist must make of the concept of God. Duns Scotus finds such justification deeply insufficient, for Anselmian conceivability is actually a mere appearance of conceivability, an appearance of consistency. No concept of God, says Duns Scotus in *Ordinatio*, is simple and clear enough to be judged as consistent like that (*nullus autem conceptus quem habemus de Deo proprius sibi et non conveniens creature est simpliciter simplex, vel saltem nullus quem nos distincte precipimus esse proprium Deo et simpliciter simplex*) ([5], 1.3.1), cf. also his *Reportata*
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([6], 1.3.2). I think, Duns Scotus’s criticism hits the bull’s-eye. As I have said, the conceivability delivers as good ground for possibility of God’s existence, as for possibility of God’s non-existence.

If modalities were to be conceptual, if they refer to conceivability, the other premise of (6.1) should be found by Duns Scotus also unjustified: neither existence nor non-existence is contradictory. This is why Duns Scotus is searching for another account of modalities to improve Anselm’s reasoning. And, as opposed to Anselm, he is fully aware of ontological legacy of the Muslim thinkers. Several generations after Anselm Duns Scotus – who was actually a kind of expert in Avicenna – was able to abandon the idea of conceivability and inconceivability, and transfer the structure of Anselm’s argument into the field of higher developed ontology. Although the argument (6.1) is formally adapted, the meaning of modal concepts change. For Duns Scotus, like for Aquinas, a necessary being is a being which exists by means of its own nature alone. Such a feature of God has been mentioned in the chapter 5 of Proslogion, but no use of it has been made in Anselm’s proof.

Duns Scotus introduces a new, non-Aristotelian concept of causation: essential cause. An essence (nature) of an object is an essential cause of the object if and only if the object either exists or not by power of the essence alone ([18], pp. 12 - 14). In Avicenna’s account such an object is either necessary or impossible – exactly like Avicennian God. So, Duns Scotus reasons, there are two possibilities: either among actually existent objects there are a necessary one, or not. But, if no existent object is necessary, then a necessary object is impossible. For a nature causing existence cannot, by definition, be granted. So, if an existent being is necessary, it exists necessarily, but if no existent being is necessary, the existence of such being is impossible.

Duns Scotus identifies God as the unique necessary being, which is based on Avicenna’s theory, but also remains perfectly in concord with Anselm’s views. Having expanded Aristotle’s account for causality, Duns Scotus speaks of essential cause of existence. A being is necessary if and only if its essence or nature causes its existence. Otherwise the being is contingent. These statements are perfectly Avicennian. Hence, for a being to be necessary it is not required that its concept contradicts non-existence. No contradiction must be entailed by a statement of non-existence of such a being. So, no conceptual modality is involved. This is an advantage in comparison with the original Anselm’s proof. But, although in the strict proof of God’s existence Anselm clearly employed some conceptual modal-
ities, he admitted, when analyzed the concept of God, that God is the unique necessary being and grants existence to others. More accurately he claimed that God alone exists through himself and creates all other things from nothing (*sed quid es, nisi id quod summum omnium solum existens per seipsum, omnia alia fecit de nihilo?*) ([1], ch. 5).

Modal concepts, Duns Scotus refers to in this point, are clearly natural modalities (cf. page 117). That means, they have nothing to do with conceptual relations, conceivability or consistency. The claim: existence necessarily belongs to God does not mean an atheist must believe a contradiction. Like in the previous example: all animals in the cage are necessarily mammals, provided they are all tigers. And, nevertheless, there is no contradiction whatsoever in the claim: that animal in the cage is no mammal. Duns Scotus does not suggest God’s non-existence be unconceivable. The considerations stay on strictly ontological level.

As I have shown, there are two versions of the premise in question: (6 .2) and (6 .3). It is a big advantage of Duns Scotus’ account that he delivers ontological justification for the stronger version (6 .3). Hence no question arises as regards validity of the argument. Duns Scotus moves a claim to the effect all claims concerning possibility or necessity are necessary. First he formulates a general claim that descriptions of existent objects are contingent, whereas descriptions of those natures are necessary (*illae etiam de actu sunt contingentes […] istae de possibili sunt necessariae; illae ad ens existens, istae ad ens etiam quidditative sumptum possunt proprie pertinere*) ([7], 3.9). The claim is then applied particularly to the case of God. If a being does not exists necessarily, it is impossible for it to exist necessarily (*quod non est a se, non potest esse a se, quia tunc non-ens produceret aliquid ad esse, quod est impossibile; et adhuc: tunc illud causaret se, et ita non tenetur incausabile omnino*) ([7], 3.19). Those claims constitute the theorem characteristic of the modal logic S5:

$$\neg \Box A \rightarrow \neg \Diamond \Box A,$$

logically equivalent to the formulas:

$$\Diamond A \rightarrow \Box \Diamond A,$$

$$\Diamond \Box A \rightarrow \Box A,$$

which all deserve the name of Duns Scotus’ modal principles. Duns Scotus’ reasoning seems quite clear: if a necessary being had been conferred the
existence from any object, including the being itself, it would not have been necessary in the Avicennian sense, it would not have existed by its own nature. If among really existent beings there is no necessary one, it follows there is no nature causing existence. For, if there was one, it would cause the existence, and a necessary being would exist. Therefore, if there is no necessary being really existent, it is impossible for such a being to exist.

So, based on Aristotelian view of modality and Muslim theory of being, Duns Scotus reaches a conclusion strictly analogical to Anselm’s second premise in the argument (6.1): God could not exist contingently, i.e. either God necessarily exists or it is impossible for God to exist. However, the meaning of terms Duns Scotus is using is completely different to that of Anselm. Duns Scotus reasons approximately like that: either God’s nature exists or not. If it exists, so does God himself, for his nature causes existence. If the nature does not exists, God’s existence is impossible, for natures constitute the base for modalities.

11 Duns Scotus on Aquinas.

No doubt, Duns Scotus must have been a fierce polemicist. Having had rejected Anselm’s proof, he got even with Aquinas. According to Scotus, Aquinas’ Five Ways provide no evidence for God’s existence. Like in Anselm’s case, the error Duns Scotus attributed to Aquinas is petitio principii – lack of justification of premises. However, Aquinas was treated by Duns Scotus quite leniently.

Turning to the argument (8.1), Duns Scotus claims the first premise not to be justified adequately: some existing objects are contingent. Personally Duns Scotus is convinced all members of our universe to be contingent. In his personal view Aquinas’ premise is “evident enough” (satis manifestae). However, it is a contingent claim, not a necessary one (ille etiam de actu sunt contingentes, licet satis manifestae), and so unacceptable in scientific demonstration ([7], 3.6).

The passage just quoted refers to the classical account of science, openly expressed by Aristotle in his Nicomachean Ethics. According to Aristotle – who seems here to be a sincere Platonist – no variable thing may be an object of scientific knowledge. When a thing that can vary is beyond the range of our observation, we cannot be certain whether it exists or not. An object of scientific knowledge therefore exists of necessity, an so must be eternal, cannot either come into existence or perish (ἐξ ἀνάγκης ζησοῦν...
And, according to Duns Scotus, existence of contingent beings is nothing eternal, and so no object of science. All Aquinas’ Five Ways deserve analogical rejection. For they all include contingent premises, premises stating contingent facts. This is exactly why, unlike *Ratio Anselmi*, they are not considered *ontological*.

Instead of Aquinas’ claim, that some existing objects are contingent, Duns Scotus affirms only that existence of contingent beings is possible. Although it has not been justified enough strong that existing objects received their existence from other objects, it is clear that is could be like that. So there are some natures – in Avicenna’s sense – which could receive existence (*possibilis esse post non esse, ergo non a se, nec a nihilo; utroque modo ens foret a non ente, ergo ab alio effectibilis; tum, quia aliqua natura est mobilis vel mutabilis, quia possibilis carere aliqua perfectione possibili sibi inesse; ergo terminus motus potest incipere, et ita effici*) ([7], 3.5). From that claim Duns Scotus concludes to possibility of an object lending existence (*alia est natura in entibus effectiva*) ([7], 3.4). This conclusion is based, as Duns Scotus says, simply on the interrelation between cause and effect: possibility of an effect entails possibility of its cause (*consequentia patet per naturam correlativorum*) ([7], 3.5). This claim is called the conclusion 1.

Duns Scotus proceeds quite analogically with respect to teleological order ([7], 3.27, conclusion 7) and order of perfection among natures ([7], 3.35, conclusion 11). He thus develops practically the entire domain of Aquinas’ Five Ways. I focus on the basic proof solely.

Having established the initial point, Duns Scotus attempts to exclude infinite sequences of interdependent natures. First he focuses on the concept of order of nature ([7], 1 - 2). Then proceeds to search for an absolutely first nature ([7], 3.8 - 3.15). Finally he claims infinite sequences to be impossible. Those expositions are most obscure, and furthermore devoid of any examples. Even profound studies do not provide any undoubted answer on what exactly the order of natures would be. There are two main attitudes to that problem. Predominantly it is claimed that Duns Scotus just mirrors common scholastic idea, harvested from Aristotle by Al Farabi and accepted by Avicenna and Aquinas. All the novelty would be substitution of existent objects with mere natures. In this view Duns Scotus argues to the effect that no infinite sequence of possible interdependent objects is possible. That would be the view of J. T. Rotondo [18]. On the other hand J. F. Ross and T. Bates suggest that the order of natures might by the order of explanation [17]. In this account Duns
Scotus seems to claim that nothing is explained, unless there is a final explanation. Whether were Duns Scotus' original intentions, by means of exclusion of infinite sequences, he derives existence of the first nature able to act \((\text{infinitas essentialiter ordinatorum est impossibilis; et infinitas accidentaliter ordinatorum est impossibilis}) \) \(\text{sic etiam: negetur ordo essentialis, infinitas est impossibilis; igitur omnino est impossibilis infinitas in essentialiter ordinatis; sic etiam: negatur ordo essentialis, infinitas est impossibilis; igitur omnino est aliquod primum simpliciter effectivum}\) ([7], 3.12). This constitutes the base for his claim called the conclusion 2: there is a simply first active nature \((\text{aliquod eff ectivum est simpliciter primum})\) ([7], 3.7). And remember that nature means a possible being here.

Now, Duns Scotus has apparently made two steps thus far. Firstly he established possibility of contingent objects, and secondly he derived existence of a supreme being from existence of contingent objects. This is sufficient to derive possibility of the supreme being:

\[
\Diamond A, \Box(A \rightarrow B) \vdash \Diamond B
\]

(11.1)

by means of rather uncontroversial part of modal logic. And, being aware of that, Duns Scotus claims possibility of God's existence on this base \((\text{rationi pri mi effect ivi simpliciter}) \) \(\text{potest esse ex secunda [conclusione]}\) ([7], 3.19). Actually the claim does not follow from the conclusion 2, but rather from both the entailment of the conclusion 2 and form the conclusion 1 itself. Duns Scotus often seems not to be aware of some subtleties like the deduction theorem. So, the argument (9.1) appears to be sound, and hence, two first premises of the argument (9.3) appear to justify possibility of God's existence.

12 Duns Scotus' error.

Logical virtuosity would be a proper description of what has been done by Duns Scotus in the field of theodicy. And yet, one could hardly admit that he proved God’s existence. Duns Scotus relinquished the concept of conceivability and the whole idea of conceptual modalities as a base for the proof. God’s existence has no sign of tautology nor of contradiction any more. On the contrary, the Persian ontology of existential modalities has been adapted and the possibility of God's existence has been established empirically. And such a brilliant train of thought works... until the time comes to draw the conclusion.

When Duns Scotus is to draw the theistic conclusion, he acts discreetly
as if he used conceptual modalities all the time. Consider the argument (9.3) once again.

The first premise sounds: some existing objects are possibly contingent. That is clearly a claim concerning the nature of existing objects we know by acquittance. The nature of those object is such as they might be an effect of rational, divine act of creation. The creation contradicts nothing we know of the empirical world of objects that rise, change and finally perish. However, it is simply illegitimate to draw any conclusion from it, concerning existence of a totally different nature, the nature of a necessary being.

The second premise is grounded in a similar base like those of Al Farabi and Aquinas. Contingent beings, if they exist, must have been created. Hence a creator must exist, provided some beings are contingent in the Persian sense.

But turn now to the last premise, the Anselmian one. God’s existence is either necessary or impossible. It is necessary, if God exists, otherwise it is impossible. That is clearly a claim concerning God’s nature, and it is a tautology: either there is a nature of God or not.

Now, the possibility of God’s existence in the first premise is clearly something completely different than the possibility of God’s existence in the last premise. The former is a characterization of the content of the world we know by acquittance, whereas the latter is a characterization of the concept of God. For the nature of the empirical world does not contradict being eternal or simply uncreated, and hence not contingent in the Persian sense, as well. So possibility of God’s non-existence is again as well confirmed as the possibility of God’s existence. When reading the first premise Duns Scotus simply comes back to conceptual possibility at a point. He understands the premise to the effect that God’s existence is consistent.

13 Conclusions.

Three leading medieval thinkers: Anselm, Aquinas and Duns Scotus aspire to prove God’s existence. They understand God as a necessary being, as opposed to contingent universe. Anselm argue to the effect that God could not exist contingently and his existence is not impossible, since God is conceivable. Aquinas argue to the effect that the existence of contingent objects entail the existence of a necessary one, which is exactly God. And Duns Scotus argued to the effect that God could not exist contingently,
and his existence is possible, since some objects can be contingent. However, the argument of Anselm and that of Aquinas, although they may be judged as valid, are neither sound nor convincing for their premises lack justification. Duns Scotus, who was clearly aware of the fact, attempts to construct an improved version. And yet, the argument delivered by him, although its premises might be easily accepted, lacks validity, due to ambiguity of the modal expressions involved. Actually, in the premises they all introduce modal terms with some meaning not fully definite. And among those modal concepts, most difficult, God is hidden to appear out of a hat in the conclusion.

Is the existence of God or of a necessary being provable? I hardly think so, if the term “proof” is to be understand strictly. All those proofs prove is that God’s existence is derivable within the confines of some ontologies. And the ontologies in question may be even more controversial than God’s existence itself. Careful analysis of premises – rather than train of thought – of proofs of God’s existence, as well as proofs of God’s non-existence, always reveal inadequacies. The real advantage of proving God’s existence consists in that they are excellent logical exercises, delivered by really brilliant thinkers, and they are really helpful in analysis of the concept of God. This means anything but not that God’s existence may not be established with no appeal to faith. For proofs are not the unique method of justification. On the contrary, in my view God’s existence may be found very well grounded. But not proven.

Bibliography


Three Versions of the Ontological Argument

PETER VAN INWAGEN

Suppose that each of two texts contains a philosophical argument. What is it for them to contain the same argument? In my view there are clear, straightforward cases of this—and cases that are not of the following trivial kind: the author of one of the texts intends to reproduce the argument of the other and does so competently. But there are cases in which the question whether the argument of one text is the same argument as that of another text has no straightforward answer.

Chapters 2 and 3 of Anselm’s Proslogion, on the one hand, and Book V of Descartes’s Meditations, on the other, constitute one of these cases. Kant invented the designation “the ontological proof [Beweis]” as a name for an argument he knew from Descartes (and from refinements of Descartes’s argument in the writings of Leibniz and Wolff). Later writers applied this name to Anselm’s argument. (I am not sure who was the first to do this; Hegel certainly did.) But to call Anselm’s argument and Descartes’s argument by the same name is to imply that they are the same argument. And is that so? And what does that question mean? Here is one way of understanding the latter question: To ask whether Anselm’s argument and Descartes’s argument are the same argument is to ask whether any possible objection to either argument is relevant to, and applies with equal validity to, the other. By that rather demanding criterion, the two arguments are not the same. And neither is the same as the “modal ontological argument” that one finds in various recent writers—I am thinking particularly of Charles Hartshorne and Alvin Plantinga—in the (broadly speaking) analytical tradition. Still, it must be conceded that these three arguments have a great deal in common. And that fact, presumably, is what
leads philosophers to speak of them as three “versions of the ontological argument”. (Most Anglophone philosophers, I among them, prefer ‘ontological argument’ to ‘ontological proof’.) That phrase, popular and useful as it is, probably doesn’t bear up very well under close logical scrutiny – for it seems to imply that there is some argument that is the ontological argument, an argument of which the eleventh-century argument, the seventeenth-century argument, and the twentieth-century argument, are “versions”. But what argument, exactly, is this argument that is the ontological argument – and what is it for Argument A to be a “version” of argument B? (If Argument A is a version of Argument B, is Argument B a version of Argument A? You may well ask.) Well, established usage laughs at logic, and it is certainly common enough to speak of versions of an argument – the ontological argument, the cosmological and teleological arguments, the Consequence Argument, or what have you. In speaking as if the three arguments I shall discuss were versions of some one “background” argument – call it by what name you will – I mean to imply nothing more than whatever is contained in the vague statement that they exhibit a family resemblance. Or here is a statement that is a little less vague: Granted, a valid objection to one of the three arguments is not necessarily a valid – or even a relevant – objection to either of the others; nevertheless, if one is interested in having at one’s disposal all the available objections to each of the three arguments, and if one knows of an objection that applies to one of them, one would be well advised to consider carefully the question whether that objection also applies (or can be adapted so as to apply) to the others.

I have so far identified the three arguments I mean to discuss by reference to the philosophers who propounded them. I now propose designations for each of the arguments that are based on their content; I will call the three arguments the Meinongian Argument, the Conceptual Argument, and the Modal Argument.

1 The Meinongian Argument

According to St. Anselm, there are two modes of being (or of existence), a weaker, less demanding one, and a stronger, more demanding one. (These two modes are not exclusive: a thing can enjoy both.) God – that is *aliquid quo nihil maius cogitari possit*, something than which nothing greater can be thought or conceived – uncontroversially enjoys the weaker of these two modes (it is uncontroversial that God *is* or *exists* in the weaker sense of
‘is’/‘exist’); the question to which the argument is addressed is: Does he also enjoy being in the more demanding mode? And what the argument attempts to show is that God cannot, as it were, be confined to the weaker mode of being; if he is in the weaker mode, it follows that he is in the more demanding mode. (And it is this stronger mode of being that the Fool means the word ‘est’ to express when he says, “Non est Deus”. That is to say, it is only the stronger or more demanding kind of being that atheists deny to God.)

This very abstract statement of some of the ideas that figure in Anselm’s argument is somewhat misleading in that it suggests that Anselm’s argument was concerned with God’s nature or essence. What I have said might be taken to imply that the argument had the following premise:

God’s essence is such as to preclude his enjoying only the weaker mode of being.

Both Gaunilo and Aquinas seem to have supposed that Anselm’s argument did depend on this premise, and have said that, although we Christians believe that God’s essence is incompatible with his non-existence, this article of faith cannot serve as a premise in a philosophical argument for his existence. For, like all creatures, we human beings are unable to grasp the divine essence, and we can therefore know that his essence includes his existence only by revelation. (An aside: almost all the commentators on Anselm I have read translate his ‘esse’ as either ‘be’ or ‘exist’ as the mood takes them. I will follow them in this practice for the moment, and will say more about ‘be’ and ‘exist’ presently.)

But this understanding of Anselm’s argument is a misunderstanding. The argument does not presuppose that we to whom the argument is addressed grasp or understand the divine nature in its entirety, but only that we understand a certain name of God, and have a sufficient partial understanding of the divine nature to know that that nature “authorizes” the application of that name to God. (The name, of course, is ‘something a greater than which cannot be conceived’.) Anselm is certainly no friend of apophatic theology. He would say that the statement that either a being grasps God’s nature in its entirety or else must find God utterly incomprensible – or comprehensible only through negation – is based on a false opposition. To adapt a figure he uses in his Reply to Gaunilo, if we cannot look directly at the sun, it does not follow that we cannot see daylight. And, in Anselm’s view, our partial grasp of God’s nature is sufficient to show us that the applicability of the name ‘something a
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greater than which cannot be conceived’ to God is a consequence of that nature.

I now turn to the argument. What I have said implies that the argument presupposes an ontology that is in a certain very loose sense Meinongian – hence my designation “the Meinongian Argument”. I want now to say something about this ontology and about what I mean by calling it Meinongian. I ask you to consider what Anselm says about a painter and his picture:

“... cum pictor praecogitat quae facturus est, habet quidem in intellectu, sed nondum intelligit esse quod nondum fecit. Cum vero iam pinxit, et habet in intellectu et intelligit esse quod iam fecit.”

“... when a painter thinks beforehand of what he is about to make, he has it in his understanding, but he does not yet understand it to be because he has not yet made it. When he has now actually painted it, he has it both in his understanding and understands it to be because he has now made it.” (Proslogion II)

Let me tell a story of a painter and his painting that contains a little more detail than Anslem’s. In the course of telling the more detailed story, I shall provide some commentary that lays out in very rough form the ontology – the theory of being – I ascribe to Anselm. (In presenting his argument, Anselm uses the verb ‘existere’ only once; his preferred verb is ‘esse’. Like most philosophers and theologians who have written on Anslem’s argument, I find no philosophical significance in Anselm’s preference for ‘esse’, and in the remainder of my discussion of his argument, I will use ‘exist’ and ‘be’ interchangeably.) In the story and commentary, I will use the word ‘item’ – the most “ontologically neutral” word I can think of – as the most general count-noun. That is, in my use everything, everything without qualification, is an item; an item is anything that can be referred to by a pronoun.

Velasquez has just completed his famous portrait of Prince Philip Prosper. It – this item – stands before him. It is an item that exists in re, in reality. A year earlier – long before Velasquez had received his commission and had as yet not given any thought to painting the young prince –, it did not exist in re. Possibly it did not exist in any mode, but this is a point on which Proslogion provides no guidance. However that may be, six months ago, after Velasquez had begun thinking seriously about it, but before he had mixed his pigments or stretched his canvas, this painting, this item, did not exist in re but did exist in intellectu – for Velasquez was
thinking about it, “precogitating” it, one might say. It was the object of his precognition. And it is one and the same item that now stands before Velasquez and which three months ago existed in solo intellectu (that is, it existed in intellectu but not in re) and was then the object of Velasquez’s thought. In fact, that very item that once existed in solo intellectu is in 2011 a visible, tangible presence in the Kunsthistorisches Museum in Vienna – although you would probably be arrested if you did touch it.

It is important to realize that when Anselm applies the phrase ‘exists in solo intellectu’ to something, he does not mean to imply that the thing of which he speaks is in any sense a mental object or thing or item. For all I know, mental images of the finished painting were a part of the furniture of Velasquez’s mind long before he put brush to canvas. But these images, if such there were, existed in re. Their existence was “mental” only in the sense that they existed and were mental items. To be more specific, they were mental representations, representations of a painting, and were, of course, not themselves paintings. But the item of which they were mental representations was strictly and literally a painting, albeit a painting that existed in solo intellectu. An item is not said to exist “in solo intellectu” because it is an inhabitant of a mental realm called the intellect. Rather – this is Anslem’s idea – for an item to exist in solo intellectu is for it to enjoy a mode of being or existence that is accessible by, and only by, intellectual apprehension. Arthur C. McGill, the author of one of the many English translations of Proslogion, has rendered ‘in intellectu’ as ‘in relation to the intellect’. In my view, this “translation” blurs the line between translation and commentary, but I think the commentary is correct: existence in the intellect is existence in a mode such that one can be aware of the items that exist in that mode as objects of thought. And one can be aware of items that exist in the intellect alone only as objects of thought. It can be true of an item that exists in re that it, that very item, used to exist in solo intellectu. It is also important to realize that existence in solo intellectu is not a private affair; it would be an ontological solecism to say that a painting exists in solo intellectu pictoris – or it would at any rate if those words were meant to imply that of metaphysical or logical or conceptual necessity, the painting could be the object of only the painter’s thought. If one is willing to say that at a certain moment Velasquez’s portrait existed in Velasquez’s intellect although not yet in reality, one should be equally willing to say that at that moment it existed in God’s intellect (assuming that one is a theist), and, depending on the factual aspects of the contemporary situation, it might well have been true at
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that time that the same painting that did not yet exist in reality but did
did exist in Velasquez’s intellect also existed in the intellects of the assistants
in his studio. But it is better not to qualify ‘in intellectu’ by reference
to particular intellects. It is better not to use phrases like ‘in Velasquez’s
intellect’ or ‘in God’s intellect’. It is better to leave ‘in intellectu’ as
impersonal as ‘in re’.

It can, moreover, be a meaningful and non-trivial question whether an
item that is the object of someone’s thought – and which therefore exists in intellectu – also exists in re. Suppose, for example, that Velasquez awakens
in a hospital bed and is told that he suffered a severe blow to the head three
days before. He finds that he has no memory at all of the last six months
of his life. He asks, “My painting of the Infante Filipe – I was about to
start work on it. Did I paint it? Does it exist?” Each of these questions, if
Anselm is right, has exactly the logical structure and semantical features
its syntax suggests: there is a particular item, a particular painting, that is
the object of Velasquez’s thought, and he wants to know whether it, that
very painting, exists in re or in solo intellectu.

As a painting is a painting – is one of the objects that falls within the
extension of the property being a painting – whether it exists in re or in
solo intellectu, so aliquid quo nihil maius cogitari possit is something than
which nothing greater can be conceived whether it exists in re or, as the
Fool supposes, in solo intellectu. These propositions might be described
as Anselm’s anticipation of Meinong’s principle of Außersein, of the inde-
pendence of predication and being, of his doctrine that the object stands
beyond being and non-being. (I must repeat my assertion that it is only in
a very loose sense that I contend that Anselm is a Meinongian. There are
certainly important differences between the unreflective philosophy of be-
ing that I contend is presupposed in Proslogion and Meinong’s philosophy
of being and non-being – which of course is at the farthest possible remove
from “unreflective”. For one thing, Meinong regarded Existenz as one of
two modes of Sein, whereas ‘existere’ and ‘esse’ are for Anselm stylistic
variants. And Meinong would certainly not say that an unpainted painting
enjoyed a weaker mode of existence or being than a painting that has been
painted; he would say rather that the unpainted painting was an inhabitant
of both the realm of non-being and its province the non-existent. Finally,
there is the fact that the population of Meinong’s realm of non-being is
not affected by what mental acts thinkers happen to have performed, and
it is possible that Anselm would have said – if the question had come up
– that a painting of the Holy Family that exists in solo intellectu would
not have existed even in that mode if the painter who is “precogitating” it had never considered making a painting of the Holy Family.)

And now the argument. We all know how it goes. The Fool says in his heart that there is no God. But if the Fool is really to be denying the existence of God, he must name God properly or he will not, if I may so express myself, be denying existence to the right item. That is to say, he must use the name ‘Something than which nothing greater can be conceived’ or some very similar name. In this exposition of the argument, I’ll abbreviate this name as ‘Aliquid’. And he is obviously not denying that Aliquid exists in intellectu, for Aliquid is an object of his, the Fool’s, thought. No, what he is saying is that Aliquid does not exist in re, that Aliquid exists in solo intellectu. Suppose, then, that he is right; suppose that Aliquid exists in solo intellectu. Then one can conceive of a being greater than Aliquid – one need only conceive of a being that is like Aliquid in every respect but one, namely that it exists in re. (“Which”, Anselm says, “is greater”. That is, existence in re is greater than existence in solo intellectu. Many commentators have made the point that it is not clear whether Anselm’s premise is that any entity that exists in reality is greater than any entity that exists in the understanding alone, or only that if x exists in the understanding alone and y is in every respect like x but for the fact that y exists in reality, then y is greater than x. It seems evident, however, that Anselm needs only the weaker premise.)

Anselm’s argument may therefore be seen as a deduction of the proposition:

It is possible to conceive of something that is greater than something than which nothing greater can be conceived.

from the proposition:

Something than which nothing greater can be conceived exists in solo intellectu.

And if this deduction is logically valid, then the latter proposition must be false (and indeed necessarily false), since the former is obviously self-contradictory. It follows that God exists in re – that is, that something than which nothing greater can be conceived exists in re. For to exist in solo intellectu is to exist in intellectu and not in re, and, therefore, to exist in intellectu but not in solo intellectu is to exist in re.

Anselm’s reasoning depends on many premises. (I am using ‘premise’ in the broadest possible sense.) It seems to me that there are five of them:
THREE VERSIONS OF THE ONTOLOGICAL ARGUMENT

The Thesis of the Suitability of the Name: That Anselm’s “name” was well chosen (that an argument for the existence of something than which nothing greater can be conceived is indeed an argument for the existence of God, and not, say, an argument for the existence of the neo-Platonic One).

The Meinongian Existence Thesis: That even if the Fool is right when he says that something than which nothing greater can be conceived does not exist in reality, it remains true that the phrase ‘something than which nothing greater can be conceived’ denotes a certain item – an item that enjoys a weaker, less demanding mode of existence than existence in reality, to wit, existence in intellectu. And that item is accessible to the intellect, is the object of various of our intentional states. It is the object of an intentional state of Anselm’s when he says, “You so truly are that you cannot be thought not to be”, and it is equally an object of an intentional state of the Fool’s when he says, “There is no God”.

The Meinongian Predication Thesis: That the item that is denoted by ‘something than which nothing greater can be conceived’ is – without qualification – something than which nothing greater can be conceived: it has this property whether it exists in re or in solo intellectu.

The Existential Greatness Thesis: That if x and y are exactly alike save that x exists in re and y exists in solo intellectu, then x is greater than y.

The Impossibility of Conception Thesis: That it is impossible to conceive of something that is greater than something than which nothing greater can be conceived.

If these five premises (or assumptions or presuppositions of the argument or whatever one wishes to call them) are granted, then it seems to me that the argument is sound. Perhaps indeed it deserves to be called a proof of the existence of God – although whether it has that status would depend not only on the truth-value of its premises but also on their epistemic status.

The argument, proof or not, raises various interesting and important questions that I will not discuss. The most important of all is this: can a parallel argument be used to “prove” the existence of, e.g., an island than which no greater island can be conceived? (And I will not discuss the question whether Proslogion contains a second, modal argument for the
existence of God – or whether the conclusion of that second argument is not ‘God exists’ but rather the conditional statement ‘If God exists, he so truly exists that he cannot be conceived not to exist’, a conclusion that an atheist might well accept.) I will do no more than make two brief remarks about one of the five propositions I have identified as “premises” of the argument, the Meinongian Existence Thesis.

It seems to me to be evident that Anselm’s argument really does depend on or presuppose this thesis. The Meinongian (or quasi-Meinongian) ontology whose tacit acceptance I have ascribed to Anselm seems to me to be inextricably bound up in his reasoning. That is, any attempt to reformulate the argument of Proslogion as an argument that does not presuppose two modes of existence must yield – if anything – an argument that is simply not Anselm’s.

And it seems to me to be evident that Meinongianism in any form is simply wrong – the classical Meinongianism of Meinong and his students is wrong; the present-day “neo-Meinongianism” of philosophers like Terence Parsons is wrong; the “primitive” Meinongianism of Anselm is wrong. But I have defended this thesis elsewhere, and will not defend it here. My only object has been to show that Anselm’s argument presupposes, and essentially presupposes, an ontology that is – in the qualified sense I have laid out – a Meinongian ontology.

2 The Conceptual Argument

I believe that in the fifth of Descartes’s Meditations one can find a “version of the ontological argument” that is not Meinongian, that does not presuppose anything like modes of being or existence. (Descartes’s correspondence and the Discourse on Method are also valuable sources for his thoughts about this argument.) This “conceptual” argument, as I shall call it, proceeds rather by attempting to show that a certain concept – the concept of a supremely perfect being – is such that there must, of necessity, be something to which it applies. I concede at the outset that the argument to which I allude cannot be described as “Descartes’s ontological argument” without, to borrow a well-known phrase, some risk of terminological inexactitude. I concede that Descartes’s writings on the ontological argument are shot through with language that strongly suggests a kind of Meinongianism. Indeed, Anthony Kenny in his book on Descartes argues very forcefully and cogently for the conclusion that Descartes’s ontological argument presupposes a Meinongian ontology, an ontology that is in fact
very similar to the one I have ascribed to Anselm.\footnote{See chapter VII of [2].} Let me, therefore, put forward only this modest thesis: One can in Descartes’s writings \textit{find the materials} for a conceptual version of the ontological argument. This argument does not rest on the assumption that the phrase ‘\textit{ens summe perfectum}’ denotes something – something that uncontroversially enjoys a certain relatively undemanding mode of existence and may, on investigation, prove to enjoy existence in some more demanding mode as well; much less does the argument presuppose that the referent of this phrase is a supremely perfect being no matter what mode or modes of existence it may enjoy.

In any case, my interest is in the argument itself, and not in the question whether the argument can properly be ascribed to Descartes. (I will, however, simply as a matter of literary convenience, occasionally ascribe the argument I shall be discussing to Descartes.) Since I am not trying to be true to Descartes’s text, I will take the liberty of employing in my statement of the argument a certain decidedly non-Cartesian technical term whose function is to force its users always to make a clear distinction between a concept and the thing or things that concept applies to – a virtue that is not an invariable feature the language Descartes uses in Meditation V.

And here, finally, is the argument:

(1) To each concept or idea, we may assign a proposition that is that idea’s “anti-existential proposition.” (This is that decidedly non-Cartesian technical term.) The concept of the anti-existential proposition of an idea is best explained by example:

The anti-existential proposition of the idea of a mountain is the proposition that there are no mountains.

The anti-existential proposition of the idea of a triangle is the proposition that there are no triangles.

.. and so on.

(2) Each concept or idea, moreover, \textit{includes} certain properties. The concept of an idea’s including a property is, again, best explained by example:

The idea of a mountain includes the property extension.

The idea of a [Euclidian] triangle includes the property “having interior angles whose sum is equal to two right angles”.

\footnote{See chapter VII of [2].}
... and so on.

(3) The idea or concept of a supremely perfect being is the idea of a being who possesses all perfections.

(4) Existence is a perfection.

(5) The idea of a supremely perfect being, therefore, includes existence.

(6) If an idea includes existence, then its anti-existential proposition is self-contradictory.

(7) The proposition that there is no supremely perfect being is therefore self-contradictory.

(8) There is, therefore, a supremely perfect being. QED

Although it is not strictly speaking a premise of his argument in the narrowest sense, Descartes explicitly makes the point that it is a rather, well, *exceptional* thing for a concept or idea to include existence. He explicitly says that the idea of a triangle does not include existence and that the idea of a mountain does not include existence, and that, therefore – if I may attempt to translate his assertion into my terminology – no reflection on either of these concepts could possibly show that their respective anti-existential propositions were self-contradictory. (What Descartes actually says is that the idea of a triangle does not include *its* existence and that the idea of a mountain does not include *its* existence – and that the idea of a supremely perfect being does include *its* existence. But if we ask what the antecedent of the possessive pronoun in such statements is, we can provide no sensible answer to this question that does not imply some sort of Meinongianism.)

What shall we say about this argument? Well, we might attempt to apply to it the famous thesis of Kant’s that was the core of the standard refutation of the ontological argument for two centuries: that existence is a “logical”, not a “real”, predicate. (Kant’s word is ‘Sein’ not ‘Existenz’, but the distinction between being and existence, if there is one, is not in play here.) And that course is certainly open to us, for whether the argument I have laid out can properly be ascribed to Descartes or not, it certainly shares with Descartes’s argument the premise at which Kant’s thesis was directed: that existence is a perfection.

I would gloss Kant’s statement that existence is a logical, and not a real predicate, as follows:
The word ‘exists’ is, of course, a grammatical predicate. A sentence like ‘Vladi-Putin exists’ is, from the point of view of the grammarian, as good an example of a subject-predicate sentence as is ‘Vladimir Putin weighs 60 kilos’. But the grammatical predicate ‘exists’ (unlike the predicate ‘weighs 60 kilos’) does not express a property – attribute, quality, feature, characteristic – of things. The predicate ‘weighs 60 kilos’ expresses the common property of all things that weigh 60 kilos, but ‘exists’ does not express the common property of all things that exist – that is, the property that is common to everything.

I will not discuss Kant’s arguments for this thesis. I will remark only that its application to Descartes’s argument is obvious: a perfection, however we may understand the idea of a perfection, must be a property. Hence, Descartes’s premise ‘Existence is a perfection’ is false. (I do not claim that this brief remark includes everything Kant had to say about the relations between the two theses ‘Existence is a logical not a real predicate’ and ‘Existence is a perfection’.)

I agree with Kant that Descartes’s argument is defective, and I agree that the defect is, very broadly speaking, a logical defect, but I do not believe that he has correctly identified the defect. For one thing, it seems evident to me that existence is indeed a property, and is, in fact, a property held in common by all things. It is the property expressed by the open sentence ‘there is something that is identical with \( x \)’. As wisdom is the property of being an \( x \) such that \( x \) is wise and “weighing 60 kilos” is the property of being an \( x \) such that \( x \) weighs 60 kilos, so existence is the property of being an \( x \) such that there is something that is identical with \( x \). And if this is not granted, if one insists that existence is not a property, the Kantian critique of Descartes’s argument nevertheless faces the following formidable difficulty: whether existence is a property or not, necessary existence is certainly a property (it may be, as many have believed it to be, an impossible property; but impossible properties are a kind of property – just as impossible propositions are a kind of proposition).

If, moreover, necessary existence is a property, it is no doubt a perfection (in fact a rather better candidate for the office “perfection” than mere existence). Given that necessary existence is a perfection, one can easily construct an argument for the necessary existence of a supremely perfect being, an argument very similar in its logical structure to Descartes’s argument. And this second argument, whatever difficulties it may face, cannot be refuted by a demonstration that existence is not a property.

Let all this be granted, however, and it still seems evident that both
Descartes’s argument and the structurally similar argument involving necessary existence to which I have briefly alluded are defective – and that the defect is essentially the same in both cases. It remains only to identify this common defect. I identify the defect in Descartes’s argument as follows: it has a false premise, to wit:

If an idea includes existence, then its anti-existential proposition is self-contradictory.

(The corresponding premise of the "necessary existence" argument is ‘If an idea includes necessary existence, then its anti-existential proposition is self-contradictory.’)

To see this, let us ask what it is for an idea or concept to include a property. (I’m going at this point to switch to the word ‘concept’ in order to avoid some stylistic problems that the word ‘idea’ sometimes raises.) In presenting Descartes’s argument, I sidestepped this question by saying that the relevant sense of ‘include’ was best explained by giving examples, two of which I proceeded to supply. But, as Socrates would remind us, we philosophers should not be content to explain a philosophically important concept by presenting a series of examples; we should rather strive to provide an account or definition of that concept.

I will not actually attempt to present a Chisholm-style definition of the phrase ‘the concept x includes the property F’. It will suffice for my purposes to present a condition that (I contend) is sufficient for a concept’s including a given property:

If it is logically demonstrable that everything the concept x applies to (or ‘everything that falls under the concept x’) has the property F, then x includes F.

(For example, the concept of an omnipotent being includes the property omnipotence.) Call this statement the sufficiency thesis. It seems evident to me that the sufficiency thesis is true. (Or perhaps I should say: if the sufficiency thesis isn’t true, I have no idea what it is for a property to be “included” in a concept.) Now some philosophers may raise questions about the sufficiency thesis; they may wonder whether it is indeed evident that, for example, the concept of a rock star includes the property of being either a rock star or a caterpillar. If anyone does indeed regard this consequence of the sufficiency thesis as problematic – I don’t – I’ll simply issue a promissory note: I could present a more elaborate version of the sufficiency thesis that did not have consequences of this sort; and
the conclusion I shall derive from the above statement of the sufficiency thesis would also follow from the more elaborate thesis.

And here is the conclusion: If the sufficiency thesis is indeed true, every concept includes existence: the concept of a mountain, the concept of a triangle, the concept of a unicorn, the concept of a round square – even the concept of a non-existent object. But let us consider only the concept of a mountain. Here is an instance of a theorem of standard logic:

\[ \forall x (x \text{ is a mountain} \rightarrow \exists y y = x). \]

If, as I suppose, ‘\(x\) exists’ and ‘\(\exists y y = x\)’ are identical in meaning, we have our conclusion: the concept of a mountain includes existence. Or here is an intuitive way to put the same point: the concept of a non-existent mountain is a contradiction in terms: it is logically impossible for there to be a non-existent mountain. (This result is not presented as a refutation of Meinong; it is presented simply as a consequence of rejecting Meinong – as a consequence of the thesis that ‘\(x\) exists’ and ‘\(\exists y y = x\)’ are identical in meaning.)

And we have our counterexample to Descartes’s premise: the concept of a mountain includes existence but the anti-existential proposition of that concept – the proposition that there are no mountains – is not self-contradictory. (And what about the corresponding premise of the “necessary existence” argument: ‘If an idea includes necessary existence, then its anti-existential proposition is self-contradictory’? Consider the concept of a necessarily existent round square. This concept obviously includes the property necessary existence; and the non-self-contradictoriness of the proposition that there are no necessarily existent round squares is conveniently attested by its truth.)

Here is a simple point way to put this point, or what is essentially this point: a non-existent unicorn is a contradiction in terms; and yet the non-existence of unicorns is – experience testifies – not a contradiction in terms. By exactly the same logical token, a non-existent supremely perfect being is a contradiction in terms, but the non-existence of a supremely perfect being is not a contradiction in terms. The non-existence of a supremely perfect being may be a metaphysically impossible state of affairs (that is my own conviction), but it is not a contradiction in terms.

3 The Modal Argument

As there are versions of the ontological argument, there are versions of versions of the ontological argument. At any rate, there is more than one
version of the modal argument. Here is the version I think is the clearest and most elegant.

A perfect being, let us say, is a being that possesses all perfections essentially. (That is to say, a being is perfect in a possible world $w$ if and only if it possesses all perfections in every world accessible from $w$.) Necessary existence, moreover, is a perfection. (A being possesses necessary existence in a world $w$ if and only if it exists in every world accessible from $w$.) Suppose that a perfect being (so defined) is possible. Suppose, that is, that there is a perfect being in some world $w$ accessible from the actual world ($\alpha$). But then some being $x$ that exists in $\alpha$ is a perfect being in $w$ – since there is a perfect being (and hence a necessarily existent being) in $w$, $w$ is accessible from $\alpha$, and the accessibility relation is symmetrical. Might $x$ exist only contingently in $\alpha$? No, for in that case there is some world $w_1$ accessible from $\alpha$ in which $x$ does not exist; and $w_1$ is accessible from $w$, since the accessibility relation is transitive.

But is $x$ a perfect being in $\alpha$? Yes, for consider any given perfection – say, wisdom. The being $x$ is essentially wise in $w$, and hence is wise in $\alpha$, since $\alpha$ is accessible from $w$. But might $x$ be only accidentally wise in $\alpha$? No, for in that case there is a world $w_2$, accessible from $\alpha$, in which $x$ exists but is not wise. But, owing to the transitivity of the accessibility relation, $w_2$ is accessible from $w$. And the point is perfectly general: given the symmetry and transitivity of the accessibility relation, $x$ will have a property essentially in $\alpha$ if it has it essentially in any world accessible from $\alpha$. There therefore actually exists a being that has all perfections essentially – that is to say, there actually exists a perfect being. (Might someone protest that we have shown that the being $x$ possesses necessary existence in $\alpha$ but have not shown that $x$ possesses necessary existence essentially in $\alpha$? Well, if the accessibility relation is transitive, then anything that is necessarily existent is essentially necessarily existent. But it is not necessary to include a demonstration of that thesis in our argument, for we know that $x$ possesses all perfections essentially in $w$, and hence is essentially necessarily existent in $w$; it therefore follows from what we have shown that $x$ is essentially necessarily existent in $\alpha$.)

There are two important differences between this argument and the two other arguments we have examined: (a) it does not presuppose any sort of Meinongianism; it makes no appeal to the idea of distinct modes of being or existence, and (b) it contains no logical mistake. It does, however, depend essentially on the assumption that the accessibility relation is both symmetrical and transitive. Loosely speaking, the modal logic of
the argument is S5, the strongest modal system.

This is not the case with every version of the modal argument. Some are valid in weaker modal systems, but those arguments require additional premises. In one sense, our argument has only one premise: that a perfect being (defined as we have defined ‘perfect being’) is possible. Consider, by way of contrast, the first of Hartshorne’s modal arguments. Let ‘G’ represent the conclusion of the argument – ‘A perfect being exists’, ‘God exists’, however you want to state Hartshorne’s conclusion. This argument had two premises:

\[ G \prec \Box G, \text{ i.e., } \Box (G \rightarrow \Box G), \]
\[ \Diamond G. \]

Hartshorne appealed to S5 in his deduction of \( G \) from these two premises, but it was soon pointed out that the argument was valid in the weaker system \( B \). (The validity of \( B \) is equivalent to the statement that the accessibility relation is symmetrical; it does not require that it be transitive.) Hartshorne, moreover, later offered a modal ontological argument that required almost no modal logic at all:

\[ \Box G \lor \Box \neg G, \]
\[ \Diamond G, \]

hence, \( \Box G \),

hence, \( G \).

This argument requires no modal logic beyond the interdefinability of the possibility and necessity operators and the validity of the principle \( \Box \phi \rightarrow \phi \). One could regard the first premise of both Hartshorne’s arguments as substitute for an appeal to the strong modal system S5. At any rate, both premises follow from the assumption that the accessibility relation is both symmetrical and transitive (reading ‘G’ as ‘There is a necessarily existent being that has all perfections essentially’).

The modal ontological argument – in any of its versions, for they all have a “possibility” premise not very different from the one I have stated – suffers from only one defect: there seems to be no a priori reason, or none accessible to the human intellect (perhaps none accessible to any finite intellect) to think that it is possible for there to be a necessarily existent being that has all perfections essentially. I myself think that this

\[ ^2 \text{See section VI (especially pp. 49 and 50) of chapter 2 of [1]} \]
premise of the argument is true – but only because I think that there in fact is a necessarily existent being who has all perfections essentially. And my reasons for thinking that are by no means a priori; they depend (so I suppose) on what that being has revealed about himself to humanity. And I do not mean simply that no conclusive reason for thinking that such a being is possible can be supplied by a priori human reasoning. I mean that human reason is impotent to discover by a priori reasoning any consideration whatever that should cause a human reasoner to raise whatever prior probability he or she may assign to the possibility of such a being.

And I would go further. I would say that, divine revelation apart, a human being should either assign a prior probability of 0.5 to the proposition that there is a necessarily existent being who possesses all perfections essentially or else refuse to assign it any probability at all. (Which of these would be the right thing to do depends on the resolution of some thorny questions in the philosophy of probability.)

My conviction that this is so rests in part on my conviction that no one has presented any cogent argument a priori for the conclusion that we ought to assign some probability lower than 0.5 to that proposition, a conviction that I will not defend here – since a defense could only take the form of successive examinations of each of the many arguments that has been offered for that conclusion. And, of course, it rests on my conviction that the arguments that have been offered (by Leibniz and Gödel, among others) for the conclusion that a perfect being is possible lend no support whatever to their conclusions. I will not defend this conviction either, since an adequate examination of these arguments is not possible within the scope of this paper (and since I have done so elsewhere).³

I conclude that whatever value the modal ontological argument may have, whatever philosophical rewards may attend a careful study of the argument, this value and these rewards are not epistemological: they will not provide the student of the argument with any sort of reason for believing that a perfect being exists. If a philosopher’s sole interest in the modal ontological argument is in that sense epistemological, he or she will find it of no more interest than the following argument (formally identical with Hartshorne’s second argument) for the truth of Goldbach’s Conjecture (that every even number greater than 2 is equal to the sum of two primes – abbreviate this statement as ‘G’):

$$\Box G \lor \Box \neg G,$$

³In my essay, “Some Remarks on the Modal Ontological Argument” [3].
Three Versions of the Ontological Argument

\( \Diamond G, \)

hence, \( \Box G, \)

hence, \( G. \)

This argument is indisputably valid and its first premise is indisputably true. It is equally indisputable, however, that this argument is not only not a proof of Goldbach’s Conjecture but provides no reason whatever for thinking that Goldbach’s Conjecture is true. And the reason for this can be simply stated: one could have no reason for thinking that Goldbach’s Conjecture was possibly true unless that reason were a reason for thinking that Goldbach’s Conjecture was true \textit{simply}. The point that this example illustrates may be generalized.

Let us say that a proposition is \textit{epistemically neutral} (for a certain person or a certain population at a certain time) if the epistemic status of that proposition and the epistemic status of its denial (with respect to that person or population at that time) are identical. If an example of an epistemically neutral proposition (epistemically neutral for us, now) is wanted, I offer the following: the proposition that the number of stars in the Milky Way galaxy with a mass greater than that of our sun is even.

And let us say that a proposition is \textit{non-contingent} if either that proposition or its denial is a necessary truth.

I contend that the “Goldbach” example is a special case of and illustrates the following general principle:

If a proposition \( p \) is non-contingent, and is known to be non-contingent by a certain person or certain population at a certain time, and if \( p \) is epistemically neutral for that person or population at that time, then the proposition that \( p \) is possibly true is also epistemically neutral for that person or population at that time.

(This principle would obviously not be valid if its application were not restricted to non-contingent propositions: consider the proposition that I offered as an example of a proposition that is epistemically neutral for us; I take it to be obvious that we are warranted or perfectly justified – insert your favorite term of epistemic commendation here – in believing that “the number of stars in the Milky Way galaxy with a mass greater than that of our sun being even” is a metaphysically possible state of affairs.) Any instance of this generalization I can think of is either obviously true or neither obviously true nor obviously false. Here is one that
is obviously true (even more obviously true than the “Goldbach’s Conjecture” instance). Consider some “vast” or “enormous” natural number – say Skewes’s Number, at one time said to be the largest finite number that had figured essentially in any important mathematical result. Or, rather, take the following powers-of-10 approximation of that number: \(10^{10^{10^{34}}} \). And consider the proposition that the number of primes smaller than that number is even. It is evident that this proposition is non-contingent, and I believe it to be epistemically neutral for us. (It is certain that its truth-value could not be established by an enumeration of the primes smaller than \(10^{10^{10^{34}}} \) in any reasonable amount of time. A computer the size of the Hubble universe that had been counting primes for a trillion years would have counted only a minuscule portion of the primes less than that number.) But it is certainly evident that there could not be a reason for thinking that this proposition was possibly true that was not a reason for thinking it true.

If the principle I have proposed is true, then – since the conclusion of any version of the modal ontological must be a non-contingent proposition, and since one of the premises of that argument must be the proposition that its conclusion is possibly true – no version of the modal ontological argument can serve as a vehicle by which one can pass from epistemic neutrality as regards its conclusion to warrant. Nor can it serve even as a vehicle that can transport its passengers from epistemic neutrality to some status that lies between epistemic neutrality and warrant.

I do not claim to have shown that the principle is correct. But I would propose that proponents of the thesis that that the modal ontological argument might have some epistemological value do at least this much: provide an example (an example that is at least somewhat plausible; I do not demand that it be indisputable) of a non-contingent proposition that is epistemically neutral for some population and is such that the proposition that it, the chosen proposition, is possibly true is not epistemically neutral for that population. In my view, the discovery of a proposition with those properties would be an important contribution to the study of the modal ontological argument.

Bibliography


Part III

New Ontological Proofs
More Modest Ontological Argument

Richard M. Gale

Traditional ontological arguments try to hit a home run by proving that there necessarily exists a greatest conceivable being, a being that essentially has every perfection. The aim of this paper is to present a more modest ontological argument, one that gets us on base by hitting a single, double or triple. This is quite helpful since it aids our effort to score a run. After a brief critique of some of these traditional arguments, a more modest ontological argument will be given that attempts to prove only that there actually exists a necessary being who has many God-like attributes and could very well prove to be a suitable candidate to play the role of God in the lives of working theists as a being who is eminently worthy of being loved, worshipped, and obeyed.

Anselm’s famous ontological argument begins with a conception of God as a being than which a greater cannot be conceived, that is, a being that essentially has every desirable attribute or perfection to an unlimited degree and thus essentially has omnipotence, omniscience, omnibenevolence, and the like. It does not matter whether this is how people ordinarily conceive of God. It can be a purely stipulative definition, for it is Anselm’s aim to show that from this definition of God it can be deduced that he exists. The next step in his argument is to get his Biblical fool opponent, the one who denied in his heart that God exists, to grant that it is at least possible that there exist some being that instantiates or is an instance of this concept. From this admission of possibility, he deduces that it is necessary that the concept is instantiated by the use of an indirect proof in which a contradiction is deduced from the assumption that it is not instantiated, that is, that the being than which none greater can be conceived does not actually exist. On the assumption that existence is a
More Modest Ontological Argument

great-making property – that, all things being equal, a being is greater in a circumstance in which it exists than it is in one in which it does not – it follows that if that than which none greater can be conceived does not in fact exist, then it could be greater than it is. And that’s a contradiction, thereby showing that the assumption is not just false but necessarily false, and thus it is necessarily true that God exists.

The most powerful objection to this argument, which we owe to David Lewis (see, [6]), questions whether it really is contradictory to say that that than which none greater can be conceived could be greater than it is. An individual, in addition to actually having certain properties, possibly has others which it does not in fact have. For each of the latter properties, there is a possible but nonactual world in which this individual exists and has this property. All human beings fail to realize certain possibilities that are such that they would have been greater if they had. Thus all of these people are greater in some merely possible world than they are in the actual world. Because the greatness of an individual can vary in this manner across worlds, we must specify which world is in question when we speak about the greatness of an individual. Superman, for example, realizes his greatest greatness in the Marvel comic book possible world but not in the actual world since he fails to exist in it and thus is at a significant disadvantage in fighting crime in the actual world.

The allegedly contradictory proposition – that the being than which a greater cannot be conceived could be greater than it is – fails to be contradictory because it suffers from incompleteness since it fails to specify which world is in question. Is it the actual world or some other possible world? There is an implicit free variable or blank space after the final “greater than it is”; and, until it is filled in with the specification of some particular world, no proposition, that is, something that is true or false, gets expressed. Not all values for this variable result in a contradiction. To see why this is so a perspicuous rendering must be given of the possibility premise. Lewis performs a great service to our understanding of Anselm’s argument by giving a clear analysis of it in terms of possible worlds. To say that it is possible that there exist a being than which a greater cannot be conceived, which we will call a maximally excellent being, means that there is some possible world, w, in which there exists a being, x, such that x has maximal excellence in w.

This being x realizes a greatness in world w that is not exceeded by the greatness of any being in any possible world. But from this it cannot be deduced that w is the actual world, just as it cannot be deduced that the
possible world in which you achieve your greatest greatness is the actual world. And, if $w$ is not the actual world, then it is true, in opposition to Anselm, that this being, $x$, than which a greater cannot be conceived, could be greater than it is, provided that the free variable that follows “greater than it is” is replaced by “in the actual world”, given that existence is a perfection or great-making property. Thus, from the possibility that the concept of that than which none greater can be conceived can be instantiated, which is perspicuously formulated above by Lewis, it does not follow that the concept is instantiated by some existent being, a being that exists in the actual world.

Contemporary ontological arguers, such as Hartshorne, Malcolm, and Plantinga, have presented a modal version of the ontological argument that is not vulnerable to Lewis’s objection. The underlying insight of their new version is that the greatness of a being in some possible world depends not just on how goes it with that being in that world but also on how goes it with that being in other possible worlds, the possibilities or logical space that surround the individual. A set of china that is essentially unbreakable, that is, does not break in any possible world in which it exists, is greater than one that is just plain old unbreakable, that is, one that does not break in the actual world but does break in some possible world in which it exists. The requirement that a maximally excellent being must possess all of its omni-perfections essentially takes note of this, since this assures that this being will be at its greatest greatness in every world in which it exists (which, unfortunately, does not entail that it exists in the actual world). The concept of God, accordingly, must be strengthened so that a being than which none greater can be conceived be is not just maximally excellent (essentially have all of the omni-perfections) but also has necessary existence as well, in which a being has necessary existence if and only if it is necessary that it exists. Let us call a being who has both maximal excellence and necessary existence an unsurpassably great being.

From the admission that it is possible that there exists an unsurpassably great being the actual existence of this being can be deduced. Although this new ontological argument is valid it faces the problem that, whereas the fool rightly was willing to grant that it is possible that the concept of a maximally excellent being be instantiated, he would have to be not just a fool but a complete schmuck to grant that it is possible that the concept of an unsurpassably great being be instantiated. For in granting that it is possible that there is an unsurpassably great being he is granting that it is possible that it is necessary that there is a maximally excellent being. But
More Modest Ontological Argument

if his consent to the latter is to be an informed one, he must know that
the nested modal operators, “It is possible that it is necessary”, is to be
subject to the axiom of the S5 system of modal logic according to which
whatever is possibly necessary is necessary, that is, if it is possible that it
is necessary that \( p \), then it is necessary that \( p \). This has the consequence
that a proposition’s modal status is world-invariant. Since it is possible
that the proposition that it is necessary that there is a maximally excellent
being, this proposition is true in some possible world. But given that a
necessary proposition is true in every possible world, it follows that it is
ture in the actual world that there is a maximally excellent being. The
fool is well within his rights to charge the S5-based ontological argument
with begging the question in its possibility premise.

The intelligent S5 arguer, such as Plantinga, would grant that the arg-
ument does not succeed as a piece of natural theology, but then would
point out that it nevertheless serves the purpose of showing that it is not
irrational or epistemically impermissible to believe that God exists; for the
argument is valid and has premises, including the possibility one, that are
just as likely to be true as not.

If a stalemate of intuitions is to be overcome, the opponent of the arg-
ument must give some good argument for why its possibility premise is
false. This could be done by finding some concept that intuitively seems to
have more likelihood of being instantiatable than does the concept of being
an unsurpassably great being and that is strongly incompatible with it in
that if either concept is instantiated in any possible world, the other is in-
stantiated in none. The concept could be that of being an unjustified evil,
meaning an evil that God does not have a morally exonerating excuse for
permitting. It is impossible that God coexist with such an evil. But since
an unsurpassably great being, if it possibly exists, exists in every possible
world, in no possible world is there an unjustified evil. But it certainly
seems more likely that it is possible that there be an unjustified evil than
that there be an unsurpassably great being. Even some theists seem to
grant the possibility of an unjustified evil when they exercise themselves,
and not just for pastoral purposes, in constructing theodicies that attempt
to show that the apparently unjustified evils of the world really have a
justification.

Another property that is strongly incompatible with being an unsurpass-
ably great being and intuitively seems to be a better candidate for being
instantiatable than it is is being a world in which creatures, of which there
are many, always freely do what is morally wrong. God, in virtue of has
middle knowledge, i.e. his knowledge of what would result from his instantiating various free creatures, would not permit this to happen; and, thus, if God exists in every possible world, the property of being a world in which creatures, of which there are many, always freely do what is morally wrong is not instantiated in any possible world. And this clashes with our gut intuition that this property could be instantiated.

Recently, Alexander Pruss and I in [3] have concocted a new, more modest version of the ontological argument, one that is not out to hit a home run but just get us on base so as to increase our prospects for scoring a run. It makes use of the semantics of possible worlds. A possible world is an abstract entity comprised of a maximal compossible conjunction of abstract propositions. It is maximal because for every proposition, \( p \), either \( p \) is one of its conjuncts or not-\( p \) is; and it is compossible in that all of its conjuncts could be true together. A proposition is possible if it is true in some possible world and necessary if true in every possible world. A contingent proposition is true in some but not all possible worlds. A cosmos is that which actualizes all of the contingent propositions that are contained in some possible world. Let us call all of the contingent propositions that are contained in some possible world that world’s “Big Conjunctive Contingent Proposition”. It is a world’s Big Conjunctive Contingent Proposition that distinguishes it from every other possible world since possible worlds share in common all of the necessary propositions.

Our argument begins with the necessary conceptual truth that some possible world is actualized, even if it be a possible world all of whose contingent propositions are false, thus resulting in an empty or null cosmos. Let us call the possible world whose Big Conjunctive Contingent Proposition is actually true “\( p \)”. The question that our argument addresses is whether \( p \) has an explanation and, if so, what kind of an explanation.

It might appear as if we are laying the foundation for Clarke-style cosmological argument in which it is demanded in the name of the Principle of Sufficient Reason (PSR) that \( p \) actually have an explanation. The bane of this argument is that it employs a version of (PSR) that is too strong to be granted by the atheistic opponent of the argument. For that every

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1Criticisms of the argument are found in essays by: Graham Oppy in [29] and by Kevin Davey and Robert Clifton in [2], with a response from Gale and Pruss in [4]; Mike Almeida and Neal Judisch in [1], with a response from Gale and Pruss in [8]. An ingenious extension of our argument is made by Jerome Gellman in “Prospects for a Sound Stage 3 of Cosmological Arguments” (see, [5]), in which it is proven that the necessary being that our argument proves to exist is omnipotent.
contingently true proposition actually has an explanation occupies a posi-
tion in one’s wish list that is almost as exalted as that God exists. What is
special about our argument is that it can make do with a weak version of
(PSR) that requires only that every contingently true proposition possibly
has an explanation, thereby making it harder for the atheistic opponent of
the argument not to grant us this as a premise. Recall that what is possible
is realized in some possible world. Thus, if it is possible that proposition p
has an explanation, then there is some possible world in which it does have
an explanation, that is, there is a possible world, w, such that w contains
the propositions p, q, and q explains p. Once our opponent has granted
the following weak version of the (PSR):

(W-PSR) For every contingently true proposition, p, there is a possible
world w that contains the propositions p, q, and that q explains p.

We are able to deduce from it the strong version of the (PSR), namely:

(S-PSR) For every contingently true proposition, p, there is a proposition
q and q explains p.

This deduction, which is due to Pruss, goes as follows:

1. For every contingently true proposition, p, there is a possible world
   w that contains the propositions p, q, and that q explains p. (The
   principle (W-PSR))
2. p is a contingently true proposition and there is no explanation of p.
   (Assumption for indirect proof)
3. There is a possible world w that contains the propositions: ‘p and
   there is no explanation of p’ and q, and that q explains ‘p and there
   is no explanation of p’. (From (1) and (2))
4. In w, q explains p. (From (3), because explanation distributes over a
   conjunction)
5. In w, the proposition p both does and does not have an explanation.
   (From (3) and (4))
6. It is not the case that p is contingently true and there is no explanation
   of p. (From (2) and (5))
7. It is not the case for any proposition p that p is contingently true and
   there is no explanation of p. (From (6))
There is another way to deduce that the actual world’s Big Conjunctive Contingent Proposition, \( p \), actually has an explanation from the mere possibility that it has an explanation. Again, let \( w \) be the actual world and \( p \) be its Big Conjunctive Contingent Proposition. According to (W-PSR) it is possible that \( p \) has an explanation.

\[(8)\] There is a possible world \( w_1 \) whose Big Conjunctive Contingent Proposition contains \( p \) and \( q \), and \( q \) explains \( p \).

It can now be deduced that \( w_1 \) is identical with \( w \) and thereby \( p \) actually has an explanation. To do so appeal must be made to the premise that holds a world’s Big Conjunctive Contingent Proposition to be unique to it and thereby individuative. Since, as premise (8) says, \( p \) is in \( w_1 \)’s Big Conjunctive Contingent Proposition, it follows that \( w_1 = w \). And, given that \( w \)’s Big Conjunctive Contingent Proposition also is \( p \), it follows by the principle of the identity of indiscernibles that \( p \) has an explanation in \( w \), the actual world.

The only question that remains is what sort of an explanation \( p \) can have. It might be contended in the name of David Hume that there need not an explanation for \( p \) if each of its conjuncts has an explanation in terms of some other one(s) of its conjuncts, such as would be the case if there is an infinite succession of contingent beings each of which is explained by its immediate predecessor. The principle is that if there is an explanation for each constituent in an aggregate or conjunction, be it finite or infinite, there thereby is an explanation for the entire aggregate or conjunction. This principle, however, is false. For it is possible for there to be a separate explanation for the existence of each constituent in an aggregate, say each part of an automobile, without there thereby being an explanation of the entire aggregate – the automobile. The explanation for the latter would be above and beyond these several separate explanations for the existence of its constituent parts, as for example one that invokes the assembling activity in a Detroit factory.

Proposition \( q \) in (8), which is the explanation for \( p \), cannot be a scientific type explanation of the deductive nomological or inductive-statistical sort, since these explanations employ a contingent proposition that reports some law of nature. These explanations, thereby, would suffer from vicious circularity since this proposition, being contingently true, is a conjunct in \( p \) and thus the explanation of \( p \) would contain a proposition that is supposed to be explained by the explanation.

It would appear that the explanation must be a personal explanation in terms of the intentional actions of some being. But what sort of being
could it be and how does it cause the actualization of \( p \)? It cannot be a contingent being, since the proposition that this contingent being exists is a constituent of \( p \), which would result again in a viciously circular explanation. Given that an impossible being’s action cannot explain anything, it must be a necessary being, one that exists in every possible world. The proposition that this being exists, therefore, is not a conjunct of \( p \), thus escaping the conceptual absurdity that it would have to cause its own existence. Since our explainer is a necessary being, there is no need to give an explanation for its existence; if there is explanation for its existence, it will take the form of an ontological argument.

There is a serious problem about the modal status of proposition \( q \) – that this necessary being causes the actualization of \( p \). It must be either necessary or contingent, given that an impossible proposition can’t explain anything, but a dilemma argument can be given to show that it can’t be either. If \( q \) is contingent, \( q \) is a conjunct in \( p \), thereby resulting in a viciously circular explanation. But if \( q \) is necessary, it results in the absurdity that there is only one possible world, namely the actual world, \( w \), whose Big Conjunctive Contingent Proposition is \( p \). The reason is that since \( q \) is necessary it is true in every possible world that a necessary being cause the actualization of \( p \) and thus \( p \) is a conjunct in every possible world. And since \( p \) is a Big Conjunctive Contingent Proposition and no two worlds can have the same Big Conjunctive Contingent Proposition, it follows that there is only one possible world, surely an absurdity in spite of what Leibniz and Spinoza would contend.

Fortunately for our modest ontological argument, the argument for the first horn of the dilemma can be refuted. To do so, it must be assumed that the necessary being, call it “\( G \)”, freely causes the actualization of \( p \) in the Libertarian sense of “free,” in which a free action is not causally determined. Proposition \( q \), that \( G \) causes the actualization of \( p \), is a conjunct in \( p \), but no vicious circularity results from having \( q \) explain \( p \). The reason is that \( q \) is a self-explaining proposition. It is so because the existence of \( G \) is self-explaining or not in need of an explanation, given that \( G \) is a necessary being, and that \( G \) causes the actualization of \( p \) also is self-explaining according to the Libertarian theory of a free act. To say that someone freely performed some action explains why this action occurred. And because \( q \), although contingent, is self-explaining, \( q \) is excused from having to explain \( q \).

Now for some objections to our modest cosmological argument, saving the most powerful one, the gap problem, for last. It has been contended
that once our opponent realizes that (W-PSR) logically entails (S-PSR), she might no longer grant us (W-PSR), charging it with begging the question. Whether an argument begs the question is relative to the epistemic circumstances of its opponent before the argument is given, not after it has been given. But this response would not silence Graham Oppy, for he claims that once you understand (W-PSR) properly, you can see that it entails (S-PSR); and (S-PSR) is something which non-theists have good reason to refuse to accept. Those non-theists who were willing to grant (W-PSR) before they heard the argument which Gale and Pruss give should then say that they didn’t fully understand what it was to which they were giving assent. So Oppy argues in [29]. Herein Oppy is demanding that in order to have a proper or full understanding of a proposition one must know all of its deductive consequences; and, thus, if you believe $p$ and $p$ entails $q$, then you believe $q$. This demand is completely contrived and has the unwanted consequence that every valid deductive argument, when its premises are fully understood, can rightly be charged with begging the question.

Although Oppy’s demand is unacceptably strong, it still is true that to have an adequate understanding of a proposition one must know some of its entailment relationships. One would not understand, for example, the proposition that this is a material object unless one were prepared to deduce from it that it occupies space. But, plainly, one can understand that this is a material object without being aware of the very complex propositions that it entails within mereological theory about the logical relations between a whole and its parts.

We are not able to give a precise criterion for distinguishing between those entailment relations that are constitutive of understanding a given proposition and those that are not, since the concept of understanding is a pragmatic one and thus context-sensitive. But this does not mean that we cannot identify clear-cut cases of someone understanding a proposition and those in which she does not. And certainly one can understand a proposition that uses a modal concept, such as that of possibility and impossibility, without knowing every theorem of modal logic, just as one can understand a proposition employing geometrical concepts without knowing every theorem of geometry.

The most challenging objection to our argument has been given by Kevin Davey and Rob Clifton in [2]. Their strategy is to find a proposition that is strongly incompatible with (W-PSR), in that if either is true in any possible world the other is true in none, and which is at least as
plausible a candidate for being logically possible as is (W-PSR). Their candidate for such a proposition is that there is a contingent proposition that lacks an explanation in the actual world, say that there are cats or the cosmos for that matter. This modal intuition seems at first blush to have as much *prima facie* plausibility as does our modal intuition that every contingent proposition possibly has an explanation. But it turns out that these plausible modal intuitions are strongly incompatible. For, as Pruss has shown, (W-PSR) entails (S-PSR) and thus in no possible world is there an unexplained contingent proposition, but the Davey-Clifton intuition entails that there is just such a world.

To break this tie in modal intuitions it must be shown that one of the two rival modal intuitions coheres better with other of our background modal intuitions. To begin with, our belief in (W-PSR) coheres better with our proclivity to seek an explanation for any contingently true proposition. That we seek such an explanation shows that we do accept (W-PSR), for we would not seek an explanation if we did not believe that it is at least logically possible that there is one. Second, we know what it is like to verify that a given proposition has an explanation, namely by discovering an explanation for it, but we do not know what it is like to verify that a given proposition does not have an explanation: There are just too many possible worlds for that to be accomplished. It is beside the point to respond that we know how to falsify the latter but not the former, since a propositions truth-conditions are directly tied to its conditions of verification, not those for its falsification. These two considerations lend credence to the claim that, in the epistemic order, (W-PSR) is more deeply entrenched than is the Davey-Clifton claim that it is possible that a given contingent proposition has no explanation. From this conclusion it is reasonable to infer that, in the logical or conceptual order, (W-PSR) is a better candidate than is the Davey-Clifton proposition for being possible. This, of course, does not end the dispute but it definitely places the onus on those who share the Davey-Clifton modal intuition to come up with something new in support of it.

Scientifically inclined philosophers would claim that the explanation of \( p \) in terms of \( q \) is a bogus explanation since it fails to satisfy the requirements for a scientific explanation. The necessary being that \( q \) invokes causes \( p \)'s actualization without there being any relation of statistical relevance between his free effort of will and its effect, the resultant cosmos. And since it does so freely in the Libertarian sense, it is impossible to make any predictions as to which world's Big Conjunctive Contingent Proposi-
tion will get actualized. And, finally, given that this necessary being is not a denizen of space and time, there will be no spatio-temporal continuity between what it does and its actualization effects.

That q’s mode of explanation of p is bogus because it fails to have these features of a scientific explanation employs the question-begging scientistic principle of “The Legislativeness of Scientific Contexts”. This principle holds that the features that inform the use of a concept in a scientific context are legislative for the use of this concept in every context, any use that does not incorporate them being unintelligible. One has only to state this principle in order to defuse this objection to q being a legitimate explanation of p. For the principle is not one that is vouchsafed by science. Rather it is a metaphysical thesis that fails to find adequate argumentative support and can rightly be charged by the theist with begging the question.

It would be dogmatic for the scientistic objector to dismiss the Libertarian theory of freedom that is involved in q’s explanation of p. Our argument has established that if it is possible, as (W-PSR) requires, that there is an explanation for p, it must be in terms of a necessary being’s libertarian-type free action. Thus, to reject the Libertarian Theory is, in effect, to reject (W-PSR), and this doesn’t seem reasonable.

The most powerful objection to our argument is that it leaves too big a gap between the necessary being it proves to exist and the God of traditional theism. This is a common problem for many theistic arguments, as for example St. Thomas Aquinas’ Five Ways, which was papered over by his glib comment at the end of each argument that “et hoc dicimus Deum”. The gap problem is especially virulent for our argument, since our argument works no matter which possible world’s Big Conjunctive Contingent Proposition is actualized. Among the infinitely many Big Conjunctive Contingent Propositions, there are many whose actualization would not attest to the existence of a necessary being who comes anywhere near having God-like perfections. To actualize a Big Conjunctive Contingent Proposition gets actualized our argument should not be called an “ontological argument” since it employs a contingent premise. The appearance of contingency, however, is misleading. For our arguments works equally well regardless of which Big Conjunctive Contingent Proposition is actualized. It is necessary that some Big Conjunctive Contingent Proposition is actualized, and it is necessary that “if a Big Conjunctive Contingent Proposition is actualized, then there is an explanation for its being actualized”. And thus, for any Big Conjunctive Contingent Proposition, p, that is actualized, it is necessary that there is an explanation for p being actualized.

\[2\text{It might be charged that because it is a contingent fact which world’s Big Conjunctive Contingent Proposition gets actualized our argument should not be called an “ontological argument” since it employs a contingent premise. The appearance of contingency, however, is misleading. For our arguments works equally well regardless of which Big Conjunctive Contingent Proposition is actualized. It is necessary that some Big Conjunctive Contingent Proposition is actualized, and it is necessary that “if a Big Conjunctive Contingent Proposition is actualized, then there is an explanation for its being actualized”. And thus, for any Big Conjunctive Contingent Proposition, p, that is actualized, it is necessary that there is an explanation for p being actualized.}\]
gent Proposition whose sole true positive proposition is that there exists a pencil, would not require a being of great power and intelligence to bring about its actualization. And, as was argued previously in the refutation of the S5 modal ontological argument’s possibility premise, there are possible worlds that could not be actualized by an omnibenevolent being, such as a world all of whose human inhabitants are endlessly stretched on a rack sans any post-mortem compensation.

It is obvious that our argument has not established that its necessary being is sufficiently powerful, intelligent, and good to be a suitable object of worship, adoration, and obedience for the working theist. To close or at least significantly lessen the gap, it must be shown that the actual world’s Big Conjunctive Contingent Proposition supplies us with the resources to mount a battery of good theistic arguments and theodicies. Important issues in the philosophy of religion invariably turn out to be global ones. Hopefully, it can be shown by appeal to some of the conjuncts in \( p \) that we know that our cosmos is incredibly complex and wondrous due to its law-like regularity and simplicity, fine-tuning of natural constants, beauty, and natural purposiveness that any designer-creator of it would have to be possessed of astounding power and intelligence, certainly sufficient to satisfy the everyday working theist. Herein we are supplied with the needed materials for mounting impressive theistic arguments, such as a variety of teleological and cosmological arguments, as well as arguments based on religious experiences.

Another aspect of the gap problem is that our argument does nothing to show that its God is one. It is reasonable, however, to infer that our God is one because of the law-like regularity and simplicity of the universe. Moreover, Ockham’s razor should come into play: multiple Gods are not to be posited where one will do.

The most serious part of the gap problem concerns God’s goodness. Whereas ordinary theists are willing to accept a God whose power and knowledge is neither essential nor infinite, they would balk at wanting to worship and obey a God whose goodness is second-rate. That our God is not shown to be essentially good – good in every world in which it exists – should not be an obstacle to their faith. In the first place, it has the advantage of saving his freedom, since to perform an action freely requires the possibility of doing otherwise. Second, what matters foremost to them is not whether it is *logically* possible that God do what is morally wrong, but whether God is capable of doing so in the actual world, in which *capable* is understood in terms of what a being has the capacity, knowledge, and
opportunity to do. God could be said to be incapable in the actual world of doing wrong in the sense that he could not get himself to do so, that he is above temptation, that we can place absolute confidence in him. And this sense of God being incapable of performing a wrong action in the actual world is consistent with his performing a wrong action in some merely possible world in which he exists.

The most serious problem with our argument is not whether its God is essentially benevolent but whether he is actually benevolent, given the prevalence of horrendous evils for which we can no offer no explanation that would render God blameless. And this is of primary concern to the working theist. It is here that our argument becomes quite vulnerable. To meet this problem we’ll have to marshal all of the extant theodicies for God’s permitting all of the known evils of the world. This battery of theodicies will still leave countless apparently gratuitous evils, and it is at this point that faith must enter in that God has morally exonerating reasons for permitting these evils, even if we cannot access these reasons.

Bibliography


A New Modal Version of the Ontological Argument

E. J. Lowe

The original version of the ontological argument for the existence of God is due to St Anselm (see Charlesworth [1]). He argued that God is, by definition, a being than which none greater can be conceived; that such a being exists at least in the mind of the conceiver, that is, in conception; that it is greater to exist in reality than to exist merely in conception; and hence that such a being does exist in reality. This argument speaks explicitly only of God’s existence, not of his necessary existence. Modal versions of the ontological argument speak of the latter. Typically, they run roughly as follows. God is, by definition, a necessary being – one that necessarily exists; but, given that such a being is possible, it exists at least in some possible world, \( w \); however, since it is thus true in \( w \) that this necessary being exists, it is true in \( w \) that this being exists in every possible world, including the actual world; hence, this being exists also in actuality (see Plantinga [7], ch. 10). There are many objections that can be and have been raised against both modal and non-modal versions of the ontological argument. Rather than try to rebut those objections or refine existing versions of the argument,\(^1\) what I shall do in this chapter is to develop a new kind of modal ontological argument for the existence of God – or, more precisely and somewhat less ambitiously, for the existence of a necessary concrete being. I shall explain in due course why, although this kind of argument differs from more familiar variants of the modal

\(^1\)I try to do both in Lowe [4], where I also say more about the history of the ontological argument.
ontological argument, it does still unquestionably qualify as a modal on-
tological argument. Above all, it is an \textit{a priori} argument which focuses on
\textit{necessary existence}, not just existence.

First, I need to present some definitions. For the argument to proceed, I
need to define both what is meant by a \textit{necessary} being and what is meant
by a \textit{concrete} being. My definition of the former is as follows:

\begin{itemize}
  \item[(D1)] \textit{x is a necessary being} \overset{\text{df}}{=} x \text{ exists in every possible world.}
\end{itemize}

In contrast to a necessary being, we have \textit{contingent} beings, defined thus:

\begin{itemize}
  \item[(D2)] \textit{x is a contingent being} \overset{\text{df}}{=} x \text{ exists in some but not every possible
world.}
\end{itemize}

Next, I define a \textit{concrete} being as follows:

\begin{itemize}
  \item[(D3)] \textit{x is a concrete being} \overset{\text{df}}{=} x \text{ exists in space and time, or at least in
 time.}
\end{itemize}

In contrast to a concrete being, we have \textit{abstract} beings, defined thus:

\begin{itemize}
  \item[(D4)] \textit{x is an abstract being} \overset{\text{df}}{=} x \text{ does not exist in space or time.}
\end{itemize}

Observe that, according to these definitions, a being cannot be both
concrete and abstract: being concrete and being abstract are \textit{mutually
exclusive} properties of beings. Also, all beings are either concrete or ab-
stract, at least on the plausible assumption that a being cannot exist in
space without also existing in time: the abstract/concrete distinction is ex-
haustive. Consequently, a being is concrete if and only if it is not abstract.
(And, in fact, it is only this \textit{consequence} of (D3) and (D4) that is crucial
for the argument that follows, so that any other pair of definitions with the
same consequence would serve its purposes just as well.) However, there
is no logical restriction on combinations of the properties involved in the
concrete/abstract and the necessary/contingent distinctions. In principle,
then, we can have \textit{contingent concrete} beings, \textit{contingent abstract} beings,
\textit{necessary concrete} beings, and \textit{necessary abstract} beings. An example of
a contingent concrete being would be a particular \textit{horse} or a particular

\footnote{For further discussion and defense of this and the following definition, see Lowe
[2]. I want to allow that immaterial souls, if they exist and lack all spatial properties,
may nonetheless be accounted concrete beings, by virtue of existing at least in time if
not also in space. Note, however, that – as I shall soon explain – my argument does
not \textit{require} me to adopt precisely these definitions.}
mountain. An example of a necessary abstract being would be a particular number or a particular geometrical form. An example of a contingent abstract being would be a set all of whose members are contingent beings, such as the set of all existing horses. An example of a necessary concrete being would be God, if indeed such a being exists. I concede that it is somewhat controversial to say that God, if he exists, exists in time, although many theologians have maintained that he does. One reason, however, why I take God, if he exists, to be a concrete being in this sense is that it is difficult to see how an abstract being could have any causal powers, including the power of creating contingent concrete beings, which God is supposed to have. To say that God exists in time is not to imply that he must change over time: he may still be eternal and immutable. Although I am not in a position to prove that there can be only one necessary concrete being, I think it is very plausible to suppose this. Of course, the traditional God of the philosophers is more than just a unique necessary being with creative powers, but is also omniscient, omnipotent, and perfectly good. Again, I am not in a position to prove that the necessary concrete being whose existence I hope to establish must have these further properties. However, since most objectors to any version of the ontological argument think that it fails at a much earlier hurdle than this, I shall still consider myself to have achieved something of no little significance if I can at least demonstrate the existence of a necessary concrete being.

The first two premises of my argument are these:

(P1) God is, by definition, a necessary concrete being.

(P2) Some necessary abstract beings exist.

As noted earlier, examples of necessary abstract beings include numbers — for instance, the natural numbers, 0, 1, 2, 3, and so on ad infinitum. Why should we suppose that these numbers exist? Simply because there are mathematical truths concerning them — such as the truth that $2 + 3 = 5$ — and these truths are necessary truths, i.e., true in every possible world. The natural numbers are the truthmakers of such truths — the entities in virtue of whose existence those truths obtain — and hence those numbers must exist in every possible world, in order to make those truths obtain in every possible world.\(^3\)

The third premise of my argument is this:

\(^3\)For more on truth and truthmakers, see Lowe and Rami [6], in which an essay of my own on the subject is included.
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(P3) All abstract beings are dependent beings.

By a dependent being, in this context, I mean a being that depends for its existence on some other being or beings. This kind of dependence can be called existential dependence and may be defined (at least to a first approximation) as follows:4

(D5) \( x \) depends for its existence on \( y \) \( \overset{\text{df}}{=} \) necessarily, \( x \) exists only if \( y \) exists.

(\(D5\)), however, only defines the existential dependence of one particular entity on another. We need also to speak of the existential dependence of one kind of entity on another, which we may define (again to a first approximation) as follows, where \( F_s \) and \( G_s \) are entities of different kinds (for instance, abstract beings and concrete beings):

(D6) \( F_s \) depend for their existence on \( G_s \) \( \overset{\text{df}}{=} \) necessarily, \( F_s \) exist only if \( G_s \) exist.

Now, if (P3) is true, it seems reasonable to draw the following conclusion:

(C1) All abstract beings depend for their existence on concrete beings.

Of course, it might be suggested that, contrary to (C1), at least some and possibly all abstract beings depend only on other abstract beings for their existence. But this would seem to be problematic, because it would imply that, where abstract beings are concerned, there can be either circles of existential dependence or infinite descending chains of existential dependence, with the consequence that the existence of some or all abstract entities is not properly grounded. Let us then rule this out explicitly, by invoking another premise, namely:5

4 For further discussion and some refinements, see Lowe [5]. Note, however, that the two definitions (D5) and (D6) presented below are not in fact formally called upon in the version of the ontological argument that I am now developing, so that in the remainder of this essay the notion of existential dependence may, for all intents and purposes, be taken as primitive. There is an advantage in this, inasmuch as finding a perfectly apt definition of existential dependence is no easy task, as I explain in Lowe [5]. In particular, for the purposes of the present essay existential dependence really needs to be understood as an asymmetrical relation, and neither (D5) nor (D6) secures this.

5 Elsewhere I call such a principle an ‘axiom of foundation’, by analogy with a similar principle in set theory: see Lowe [3], p. 158.
(P4) All dependent beings depend for their existence on independent beings.

This still allows that a dependent being may depend for its existence on another dependent being, provided that, via some finite chain of dependence, it *ultimately* depends for its existence on one or more independent beings. We need to bear in mind here that the relation of existential dependence is a *transitive* relation: if \( x \) depends for its existence on \( y \) and \( y \) depends for its existence on \( z \), then \( x \) depends for its existence on \( z \).

Alternatively, however, (P3) might be challenged, with the contention that at least some abstract beings are independent beings. Let me briefly consider this possibility here, though I shall return to the matter at the end of this essay, since it is connected with other issues of theological significance. According to one position in the philosophy of mathematics, numbers are to be understood as being *set-theoretical* entities. For instance, it may be proposed that the number 0 is identical with the so-called empty set, \( \emptyset \), that the number 1 is identical with the unit set of the empty set, \( \{\emptyset\} \), that the number 2 is identical with the unit set of the unit set of the empty set, \( \{\{\emptyset\}\} \), and so on *ad infinitum*. Now, on this view, it seems clear that each number except for the number 0 depends for its existence on all of the preceding numbers in the series of natural numbers, simply because any set depends for its existence on its members. But it may be contended that the number 0, here taken to be the empty set, does not depend for its existence on anything else at all. However, I think we ought to be sceptical about the very existence of the so-called empty set: I believe that it is a mere mathematical fiction (compare Lowe [3], p. 254). After all, how could there really be any such thing as a set with no members, when what a set *is*, according to our common understanding, is something that ‘collects together’ certain other things, these things being its members. How could *something* ‘collect together’ nothing?

Anyway, although there is plenty of room for further discussion about these matters, I am going to take it that (P3) and (P4) are both true and hence that (C1), which follows from them, is also true. Sufficiently many good metaphysicians would agree with me for these to qualify as relatively safe assumptions, for the purposes of the present argument.  

I do concede, however, that both assumptions have been challenged by good philosophers too. For a challenge to (P4), see Schaffer [8].

(C2) The only independent beings are concrete beings.

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[6] Note that from (P3), together with definitions (D3) and (D4), we can also infer:
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Now, from (C1) we can infer the following:

(C3) There is no possible world in which only abstract beings exist.

And from (P2) we can infer this:

(C4) There is no possible world in which no abstract beings exist.

But (C3) and (C4) together imply the following:

(C5) In every possible world there exist concrete beings.

What (C5) means is that there is no ‘empty world’ – a world devoid of anything existing in space and time, or at least in time (compare Lowe [3], pp. 252 - 255). However, there are two different reasons for which (C5) could be true. One possibility is that (C5) is true because there is a necessary concrete being – a concrete being which exists in every possible world. This, of course, is what I should like to prove to be the case. And, indeed, it is because (C5) is true that we have at least a prima facie reason to believe that the following is true (or, at least, not to reject it out of hand as false):

(C6) A necessary concrete being is possible.

But another possibility is that (C5) is true just because in every possible world there exist contingent concrete beings – different ones in different worlds. This may very well be the case, but it is obviously not sufficient to prove what I want to prove.

Fortunately, though, I think it can be compellingly argued that only if there is a necessary concrete being can the truth of (P2) be adequately explained. This is because I consider the following further premise to be very plausibly true:

(P5) No contingent being can explain the existence of a necessary being.

The reason why (P5) is so plausible is this. A necessary being is, by definition (D1), a being that exists in every possible world, whereas a contingent being is, by definition (D2), a being that exists in some but not every possible world. Suppose, then, that N is a certain necessary being (for example, the number 7) and that C is a certain contingent being (for example, Mount Everest). How could C explain N’s existence? After all, N exists in possible worlds in which C does not exist – so C evidently cannot explain N’s existence in those possible worlds. But how, then, can C explain N’s existence even in worlds in which C does exist? For what C
would have to explain is why \( N \), a \textit{necessary} being, exists in those worlds, that is, a being which exists in \textit{every} possible world. It would, apparently, thereby have to explain why \( N \) exists also in worlds in which \( C \) does \textit{not} exist, which we have already ruled out as impossible. Someone might try to reply that \textit{different} contingent beings could explain the existence of the \textit{same} necessary being in different possible worlds: that Mount Everest, say, explains the existence of the number 7 in \textit{this}, the actual world, while the Golden Mountain, say, explains the existence of the number 7 in some \textit{other} possible world. But that seems absurd. Surely, a contingent being, such as Mount Everest – even an \textit{abstract} contingent being, such as the set whose members are the Seven Hills of Rome – simply doesn’t have the \textit{power} to explain the existence of a necessary being, such as the number 7. Again, the problem is that what it would purportedly be explaining the existence of is something that exists in \textit{every} possible world and hence something whose existence far transcends its own. Furthermore, to contend that the existence of a necessary being, \( N \), is explained in different possible worlds by different contingent beings in those worlds threatens to undermine the very \textit{necessity} of \( N \)’s existence. For then it appears to be a mere cosmic accident that every possible world happens to contain something that is, allegedly, able to explain the existence of \( N \) in that world.

Here it might be objected that, even if \((P5)\) is true, we are not entitled to assume that, where necessary beings are concerned, their existence \textit{needs} to be explained at all. However, while I agree that there may be no need to explain the existence of a necessary being which is an \textit{independent} being, I think that the existence of \textit{dependent} beings does always call for explanation:

\[(P6)\] \textit{The existence of any dependent being needs to be explained.}\n
And recall that we have already agreed, by endorsing \((P2)\), that some necessary abstract beings – such as the numbers – exist and that, by \((P3)\), these are all \textit{dependent} beings. So, with the additional help of \((P6)\), we may now infer:

\[(C7)\] \textit{The existence of necessary abstract beings needs to be explained.}\n
Observe, next, that the fact that an entity \( x \) \textit{depends for its existence} on an entity \( y \) does not imply that \( y \) \textit{explains the existence of} \( x \). Similarly, the fact that \( F \)s depend for their existence on \( G \)s does not imply that \( G \)s explain the existence of \( F \)s. Existence-\textit{explanation} is not simply the inverse of existential \textit{dependence}. If \( x \) depends for its existence on \( y \), this
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only means that \( x \) cannot exist without \( y \) existing. This is not at all the same as saying that \( x \) exists because \( y \) exists, or that \( x \) exists in virtue of the fact that \( y \) exists. So the mere fact that, by (C5), necessary abstract beings cannot exist without concrete beings existing doesn’t imply that concrete beings of just any kind, necessary or contingent, can explain the existence of necessary abstract beings. Indeed, we have already seen good reason to uphold (P5), that no contingent being can explain the existence of a necessary being. At the same time, it is clear that only concrete beings of some kind can explain the existence of necessary abstract beings since the latter, being one and all dependent beings, cannot explain their own existence:

\[(P7) \quad \text{Dependent beings of any kind cannot explain their own existence.}\]

Furthermore, it seems clear:

\[(P8) \quad \text{The existence of dependent beings can only be explained by beings on which they depend for their existence.}\]

From (P7), (P8) and (P3), together with (C1), we may now conclude:

\[(C8) \quad \text{The existence of necessary abstract beings can only be explained by concrete beings.}\]

And from (C7), (C8) and (P5) we may conclude:

\[(C9) \quad \text{The existence of necessary abstract beings is explained by one or more necessary concrete beings.}\]

From (C9) we may finally infer our desired conclusion:

\[(C10) \quad \text{A necessary concrete being exists.}\]

To draw the whole argument together, I shall now set it out in a concise and consolidated form, in which we can ignore some of the subsidiary conclusions that we drew along the way. The key premises are as follows:

\[(P2) \quad \text{Some necessary abstract beings exist.}\]
\[(P3) \quad \text{All abstract beings are dependent beings.}\]
\[(P4) \quad \text{All dependent beings depend for their existence on independent beings.}\]
\[(P5) \quad \text{No contingent being can explain the existence of a necessary being.}\]
\[(P6) \quad \text{The existence of any dependent being needs to be explained.}\]
(P7) Dependent beings of any kind cannot explain their own existence.

(P8) The existence of dependent beings can only be explained by beings on which they depend for their existence.

From (P3) and (P4), together with definitions (D3) and (D4), we may conclude:

(C1) All abstract beings depend for their existence on concrete beings.

From (P2), (P3) and (P6) we may conclude:

(C7) The existence of necessary abstract beings needs to be explained.

From (C1), (P3), (P7) and (P8) we can conclude:

(C8) The existence of necessary abstract beings can only be explained by concrete beings.

From (C7), (C8) and (P5) we may conclude:

(C9) The existence of necessary abstract beings is explained by one or more necessary concrete beings.

And from (C9) we may conclude:

(C10) A necessary concrete being exists.

Setting aside, for the moment, any further doubts that this argument might provoke, I need at this point to address the question of whether it really qualifies as a version of a modal ontological argument. It will be observed that the argument does not appeal to claim (C6) – that a necessary concrete being is possible – although, of course, if our final conclusion (C10) is true then so too, a fortiori, is (C6), since whatever is actually the case is thereby also possibly the case. Standard versions of the modal ontological argument do appeal to something like (C6), from which it is then concluded that something like (C10) is true, on the grounds that whatever is possibly necessarily the case is thereby actually necessarily the case. However, arguing for the truth of (C6) without appealing to (C10) is notoriously difficult and, moreover, the principle of modal logic whereby ‘whatever is possibly necessarily the case is thereby actually necessarily the case’ is also controversial. The fact that our new argument does not appeal either to (C6) or to this principle is therefore to its advantage, but it may also lead some to object that it consequently doesn’t really qualify as a type of modal ontological argument.
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In response to such an objection, I would reply that the new argument is, in line with modal ontological arguments quite generally, (1) a wholly a priori argument and (2) an argument which focuses on the notion of necessary existence, rather than just on that of actual existence. By (1) I mean that all of the argument’s premises are advanced as being a priori truths and that its conclusion follows from these by valid deductive reasoning – in short, that it is an a priori proof of the existence of a necessary concrete being. As for (2), this should be clear from the fact that the argument purports to establish the existence of a necessary being, that is, a being which necessarily exists, not just one that actually exists. Of course, in establishing this, it also assumes the existence of other necessary beings, by adopting premise \( P2 \). But these are not necessary beings of the same kind as that whose existence the argument attempts to prove, being only abstract rather than concrete beings. Hence, the argument cannot fairly be accused of being circular or of begging the question. That necessary abstract beings exist, such as numbers, is certainly not entirely uncontroversial, but it is far less controversial than that a necessary concrete being exists. Indeed, I would urge that all of the premises of the new argument are individually considerably less controversial than its conclusion. They are also clearly mutually consistent. And this, really, is the most that one can generally hope to achieve in a philosophical argument: that its premises non-trivially entail its conclusion and that every one of those premises has considerable plausibility and is considerably less controversial than its conclusion. For an argument with these features has the merit of providing us with a persuasive reason to endorse an interesting conclusion which, considered merely on its own, might appear to be implausible. In general, the more mutually independent premises such an argument has, the more persuasive it is, because this enables each premise to be individually more plausible despite the initial implausibility of the conclusion. So it is actually an advantage of our new argument that it has no fewer than seven premises. Of course, as we have seen, not all of these premises are simply asserted without any attempt at justification. Indeed, for almost all of them, and certainly for all of the more controversial ones, some justification has been offered.

Despite this response, some philosophers might suspect that what I have really offered is some version of the cosmological argument for the existence of God or of a God-like being. In response to such an objection I would reply that I have nowhere appealed to the existence of the cosmos as something that needs to be explained by something ‘external’ to it, where by
‘the cosmos’ I mean the sum total of existing concrete beings. The only beings whose existence I have assumed, and whose existence I seek to explain, are necessary abstract beings. Moreover, I have nowhere appealed to causal considerations in my argument. When I talk about existence-explanation, I do not mean causal explanation, but only metaphysical explanation. This should be evident from the fact that I talk about explaining the existence of abstract beings which, as I have made clear, I do not regard as being capable of standing in causal relations to anything, since they do not exist in space or time. Consequently, even if my argument looks rather different from standard versions of the modal ontological argument, I believe that if it is to be classified as belonging to any traditional form of argument for a ‘supreme being’ at all, it can only be said to be a version of the modal ontological argument.

Now I want to return, as promised earlier, to the question of whether all abstract beings are indeed dependent beings, since it is crucial to my argument that at least all necessary abstract beings have this status. A clue here, however, is provided by the very expression ‘abstract’. An abstract being, it would seem, is one which, by its very nature, is in some sense abstracted – literally, ‘drawn out of, or away from’ – something else. To that extent, then, any such being may reasonably be supposed to depend for its existence on that from which it is ‘abstracted’. All of the most plausible examples of abstract beings are, interestingly enough, entities which are, in a broad sense, objects of reason – such entities as numbers, sets, and propositions. They are all objects which stand in rational relations to one another, such as mathematical and logical relations. Very arguably, however, it does not make sense to think of such entities as existing and standing in such relations independently of some actual or possible mind which could contemplate and understand them. But then we have a very good candidate for the sort of being ‘from’ which such entities may be supposed to be somehow ‘abstracted’: namely, a mind of some kind, upon which they would thereby depend for their existence. But if the main argument of this essay is correct, then in the case of necessary abstract beings like these, the being upon which they depend for their existence and which explains their existence must be a necessary concrete being. Putting these two thoughts together – (1) that necessary abstract beings, insofar as they are objects of reason, are ‘mind-dependent’ beings, and (2) that they are dependent for their existence on a necessary concrete being – we are led to the conclusion that the being in question must be a rational being with a mind and, indeed, with a mind so powerful that it can comprehend all
of mathematics and logic. Thus, despite my earlier warning that the argument of this essay does not directly establish the existence of a being with all of the traditional ‘divine attributes’, it does in fact go considerably further in this direction than might initially be supposed. It does, in short, speak strongly in favour of the existence of a necessary concrete being possessed of a rational and infinite mind – something very much like the traditional ‘God of the philosophers’. Seen in this light, my ‘new’ modal ontological argument even has a close affinity with St Anselm’s original argument. For, clearly, if it were to be suggested that the ‘necessary concrete rational being’ whose existence I claim to have established is itself merely an object of reason, not something existing in concrete reality, then it may be replied that this would reduce that being to something that is just another necessary abstract being, and thus to something once more requiring the existence of an infinitely ‘greater’ being, in the shape of the necessary concrete rational being whose existence my argument is designed to prove.

Bibliography


A Cosmo-Ontological Argument for the Existence of a First Cause – perhaps God

UWE MEIXNER

I should say a few words why I present in a book about “ontological proofs” ideas that are relevant for what has come to be known as “cosmological proofs”. First, aside from the special meaning Kant has given the designations “ontological proof” and “cosmological proof”, a so-called cosmological proof is at least as much an ontological argument as a so-called ontological proof. Both sorts of argument aim to establish the existence of God, and existence is, of course, a central – perhaps the central – ontological concept. Second, one might even say that so-called cosmological arguments, if they exclusively use ontological concepts, like the argument I am going to present here, are more ontological in kind than many so-called ontological arguments. After all, Anselm’s original version of what Kant was the first to call an ontological argument and many later versions involve an epistemic and an axiological concept: the being such that no greater being can be conceived. Third, in my view, the interest of so-called ontological proofs is mainly logical, not theological, and not metaphysical. – Well, may this suffice as my excuse for what follows.

There is a kind of causation where the cause is sufficient for the realization of the effect (that is, for the realization of what is caused), the effect being some event: an entity involving a finite – in fact, small\(^1\) – temporal region, particulars, and properties (relational and non-relational ones) had by these particulars within that temporal region:

\(^1\)This is, of course, a vague condition. But a non-vague condition in its place would be arbitrary. Nevertheless, it is a necessary condition. Goings-on that fill large
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(A1) Effects (i.e., what is caused) are always events.

In sufficient causation, the cause determines the coming-about of the effect-event; the cause does not make the effect-event merely probable, or more probable than it would be without the cause, and the cause is not merely an indispensable factor for the coming-about of the effect-event. In sufficient event-causation, the coming-about of an event determines the coming-about of the effect-event. In sufficient agent-causation, simply the agent determines the coming-about of the effect-event.

In what follows, the phrase “a cause of” will always mean the same as “a sufficient cause of”, and “to cause” will always mean the same as “to be a sufficient cause of”. And these phrases will always be understood to exclude self-causation: Nothing is a cause of itself. (One may wish to count this as axiom (A0).) The definition of the concept that is central to this paper is this:

(D) A first cause is a cause without a cause, in other, fully explicit words: a first sufficient cause is a sufficient cause (of some event), but a sufficient cause that itself has no sufficient cause.

It is easily seen:

(T1) If agents are not events, then every agent that is a cause is a first cause.

Suppose we have an agent that is a cause, i.e., that causes some event. If agents are not events, then that agent is not an event, hence it has no cause (for otherwise it would be an effect, and therefore an event, since effects are always events according to (A1)).

Now indeed:

(A2) Agents are not events, but substances.

And therefore:

(T2) Every agent that is a cause is a first cause.

Hence:

(T3) If there are agents that are causes, then there are first causes.

temporal regions are not events in the here intended sense (which is adopted mainly because I do not wish to exclude from the start – already for conceptual reasons – that every event is caused by another, earlier event).
But are there agents that are causes? That there are such items is doubted by many, even denied. Doubtless, however, there are events that are causes. And if one could find an event that is a cause, but has no cause, then this causal event – though not a causal agent – would also serve as a perfect first cause. But are there events that are causes without having a cause? We do not have purely scientific evidence for the existence of such events. What we do have purely scientific evidence for is merely this:

(A3) *Some physical events are causes, but there is no physical event that causes them.*

Now, at this point, there is a crucial decision to be made in causation-theory. It is not an empirical, it is not a scientific, it is not a conceptual decision; it is a *genuinely metaphysical* decision. A choice is to be made between two very plausible metaphysical principles. One of these two principles is known as the *principle of (sufficient) causation*:

(A4.1) *Every event has a cause.*

The other principle is one of the principles known as *principles of physical causal closure*:

(A4.2) *Every physical event that has a cause is caused by a physical event.*

One cannot adopt both principles – because, unfortunately, their conjunction is not compatible with (A3). On the other hand, each of the two principles under consideration has so many credentials on its side that it seems rationally inappropriate to reject both. Let’s see what would be the consequences if one accepted the one, or the other.

(A4.2) is the modernization of a materialistic, or physicalistic, principle that emerged as a metaphysical side-effect of the rise of modern physics. This original principle is the following:

(A4.2*) *Every physical event is caused by a physical event.*

This latter principle was adopted by all who, inspired by impressive scientific progress, considered a purely immanent world-view – a world-view without transcendence – to be the only rational world-view. The insertion of “that has a cause” after “Every physical event” – which is of no detriment to the original metaphysical motivation – became necessary due to the developments in physics in the 20th century; these developments make modern physics entail the falsity of (A4.2*). (A4.2), however, is left quite untouched by them.
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Now, obviously, the conjunction of (A3) and (A4.2) logically entails that there are physical events that are causes, but have no cause. Thus, if we add (A4.2) to our list of axiomatic principles (and not (A4.1)), then the existence of first causes is established. There are, then, first causes in the form of physical events that are causes without having a cause.

In contrast, it is a straightforward logical consequence of (A4.1) that no event is a first cause. For if an event is a cause, (A4.1) requires that it also be caused, that is: have a cause. (A4.1) is a principle that throughout the roughly 2500 years of the history of philosophy was almost universally accepted by the philosophers as an absolute requirement of rationality, comparable in this to a law of logic. And when (A4.2*) became prominent in the philosophical consciousness (roughly 300 years ago), it peacefully coexisted there with (A4.1); indeed, one could regard (A4.2*) as a mere specialization of (A4.1), as merely spelling out what it is that (A4.1) means for physical events. All that changed in the 20th century with the establishment of quantum physics and empirical cosmology, and hence of the scientific fact that is stated by (A3). (A3) refutes (A4.2*), and it also refutes the conjunction of the logically weaker (A4.2) with (A4.1). But (A3) neither refutes (A4.2) taken by itself, nor does it refute (A4.1) taken by itself. If, tentatively, we add (A4.1) to our list of axiomatic principles (and not (A4.2)), continuing thereby the very long and almost univocal philosophical tradition in favor of (A4.1), we get an interesting result: There are physical events that have a cause, though they are not caused by any physical event.

Given (A3), one cannot adopt (A4.1) and (A4.2) together, and it does not seem rationally right to reject both. A choice, therefore, has to be made between these two principles. There is no argument that would rationally force one to choose (A4.1) rather than (A4.2). But since (A4.1) involves much less of a metaphysical commitment than (A4.2); since, in other words, the rational appeal of (A4.1) is more general than that of (A4.2), and less dependent on the rationality of a specific metaphysical motivation, I herewith adopt (A4.1) as axiomatic, and as a consequence change its label from “(A4.1)” to simply “(A4)”:

(A4) Every event has a cause.

And with both (A4) and (A3) as axioms, we now have as a theorem:

(T4) There are physical events that have a cause, though they are not caused by any physical event.
But, of course, with (A4) as an axiom, there is no chance that an event is a first cause; if there are first causes, then they must be something else than events. In fact, they must be *agents*, since the following is true:

(A5) *Every cause is an agent or an event.*

(A5) makes it possible to derive:

(T5) *Every first cause is an agent.*

Assume that X is a first cause, and assume also that X is an event. But then, according to (A4), X has a cause, and is, therefore, not a first cause – contrary to the first assumption. Therefore (holding on to that assumption): X is *not* an event, and therefore: X is an agent (because of (A5), and because X is, qua first cause, also a cause).

I am well aware that some philosophers have proposed facts, or even properties, as causes. But causes must be causally effective, and a property, taken by itself, is not causally effective; a property is only then causally effective – in an analogical way – if it *is had, exemplified, instantiated* by an object in such a way that the resulting *fact* is causally effective. But a fact, in its turn, is only then causally effective – in a derivative, secondary way – if it is replaceable in this role by a causal event. Causation by facts, in other words, is reducible to causation by events. – There is, therefore, no substantial reason to reject (A5).

(T4) gives rise to the following considerations: Suppose E* is one of the physical events that – according to (T4) – have a cause, though there is no physical event that causes them. Thus:

(a) E* is a physical event.
(b) E* has a cause.
(c) There is no physical event that causes E*.

Hence, by making use of (A5), we have:

(d) E* has a cause that is a nonphysical event or an agent.

Assume now the following additional axiomatic principles:

(A6) *Every event that is caused by an event is also caused by an event that is not caused by any event.*

(A7) *For all x, y, and z: if x causes y, and y causes z, then x causes z.*
(A7) expresses the transitivity of (sufficient!) causation – one of the most uncontroversial principles in causation theory. (A6), in turn, is the Limit Principle for Causation by Events. This, to some, may seem like a very problematic principle; it actually is no such thing. Suppose (A6) is wrong, and E is an event that is caused by an event, but there is no event that causes E and is not caused by any event. It is easily seen (employing (A7)) that a consequence of this supposition is the following: all causal chains of events that end with E are infinite or incomplete.

Suppose C is a causal chain of events which ends with E and which is neither infinite nor incomplete. (Note that for a normal conception of a causal chain – i.e., for the exclusion of its being a loop – the truth of (A0) is necessary.) Since C is not an infinite causal chain of events, there is a first event in C, call it “E₁”. Since C is a complete causal chain of events, there is no event that causes E₁. Given the transitivity of causation (i.e., the truth of (A7)) and given that C ends with E, E₁ causes E. Thus there is an event (namely, E₁) that causes E and is not caused by any event – contradicting the supposition which introduced E in the first place.

Is this consequence of negating (A6) for some event E – the consequence that all E-ending causal chains of events are infinite or incomplete – more reasonable a priori than (A6)? I don’t think so. Is this consequence more reasonable on empirical grounds than (A6)? I don’t think it is, certainly not given today’s physics.

Using the two principles last introduced, we obtain from (d):

(e) E* is caused by an agent.

The first alternative in (d) leads to the result that E* is caused by an agent, just as does (trivially) the second alternative in (d). Suppose the first alternative in (d) is true: E* is caused by a nonphysical event. With (A6) we obtain: E* is caused by an event E’ that is not caused by any event.² But according to (A4): E’ has a cause, G. Since E’ is not caused by any event, G must be an agent (according to (A5)). Since G causes E’ and E’ causes E*, it follows according to (A7): G causes E*. Therefore: E* is caused by an agent.

Consequently we get on the basis of (T2):

(f) There is an agent that is a first cause.

And this result – since, ultimately, it is a logical consequence purely of the axiomatic principles (A0) to (A7) – is a theorem: a statement logically proven on the basis of those axioms:

²Note that E’ must be a nonphysical event. Otherwise, E’ would be a physical event that causes E* – contradicting (c).

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There is an agent that is a first cause.

This result chimes perfectly with the penultimate result of what has traditionally, since Kant, been called “the Cosmological Argument for the Existence of God”. But although Thomas Aquinas nonchalantly concludes from the penultimate conclusion of the Cosmological Argument – that there is a first cause (which Thomas certainly thought to be an agent) – its ultimate conclusion: that there is God, it must nonetheless be emphasized that this is a very problematic last step. Nothing in Thomas Aquinas’s argument, and nothing in the modernization of it here presented: nothing in (T6) and the axiomatic principles on which (T6) is based, justifies the conclusion that this agent which is a first cause is God or even a god.

But, of course, the modernized Cosmological Argument I have presented can be strengthened. In order to see just at what point it can be strengthened, consider first the compact presentation of the argument as it is now:

\[(A0)\] Nothing is a cause of itself.

\[(A1)\] Effects are always events.

\[(A2)\] Agents are not events, but substances.

\[(A3)\] Some physical events are causes, but there is no physical event that causes them.

\[(A4)\] Every event has a cause.

\[(A5)\] Every cause is an agent or an event.

\[(A6)\] Every event that is caused by an event is also caused by an event that is not caused by any event.

\[(A7)\] For all x, y, and z: if x causes y, and y causes z, then x causes z.

[together logically entail among other things]

\[(T6)\] There is an agent that is a first cause.

Replace now (A3) by (A3\(^*\)) (leaving the other axioms – or premises – just as they are):

\[(A3\(^*\))\] The Big Bang (in symbols, BB) is a physical event that is a cause, but there is no physical event that causes it.

\(^3\)“Ergo est necesse ponere aliquam causam efficientem primam: quam omnes Deum nominant” (S. Th. I, qu. 2, a. 3; see the conclusion of the secunda via).
The specific principle \((A3^*)\) is just as true from the point of view of modern physics as the unspecific \((A3)\). With it and the rest of the axioms as premises, one can logically deduce:

\((T6^*)\)  \textit{There is an agent that is a first cause of the Big Bang.}

From \((A3^*)\) and \((A4)\) we get: BB is a physical event that has a cause, but there is no physical event that causes BB. Let A be a cause of BB. According to \((A5)\), \textit{A is an agent or an event.}

\textit{In case A is an agent, A is not an event (according to \((A2)\)), and therefore A is not an effect (according to \((A1)\)), i.e., A is not caused, in other words: A has no cause. But A causes BB. Thus: there is an agent (namely, A) that is a first cause of BB.}

\textit{In case A is an event, it follows on the basis of \((A6)\) that BB is also caused by an event that is not caused by any event. Let E' be such an event. It follows on the basis of \((A4)\) that there is a cause of E', and on the basis of \((A5)\) it follows that that cause (any such cause) can only be an agent (it cannot be an event, since E' is not caused by any event). Let A' be such an agent. A' causes E', and E' causes BB, and therefore (according to \((A7)\)): A' causes BB. Moreover, since A' is an agent, it is not an event (see \((A2)\)), and therefore not an effect (see \((A1)\)), i.e., A' is not caused, in other words: A' has no cause. Thus we have again: there is an agent (namely, A') that is a first cause of BB.}

An agent that is a first cause of the Big Bang – that is: of the initial event of the Physical World – does seem to be godlike. By excluding the causation of the same event (any event) by several agents – which is a plausible theoretical step – we can even obtain that \textit{there is one and only one} agent that is a first cause of the Big Bang. Moreover, also in line with traditional theism, the agent that causes the initiation of space-time-energy-matter can hardly be denied to be nonphysical. However, nothing so far shows that this agent is different from, say, what Schopenhauer called “the Will”, different from a blind, irrational, and basically evil – but transcendent – source of the Universe. That the First Cause of the beginning of the Universe is different from such a being is a matter of faith. But, note, it is also a matter of faith that God Himself is different from such a being.

Neither \textit{the axioms} nor \textit{the theorems} in this paper seem to me utterly speculative, epistemologically irresponsible, or irrational. I certainly believe that they provide food for serious thought. Yet there are, of course, objections. I will consider three of them.

**Objection 1** (against \((A3^*)\): The Big Bang does not exist, because the Big Bang, if it is anything, is the total physical event which occurs at the first moment of time, and there is no first moment of time (as Stephen Hawking has famously held). \textit{Response:} Even if there is no first moment
of time, it does not follow that there is no initial physical event. Note that events, though they are required to be temporally finite according to the notion of event here employed (see the beginning of this paper), are not required by that notion to have a first or a last moment. An initial physical event is a physical event whose temporal region is the initial interval of time – and that interval may be an interval that is open on one side, even on both sides. The Big Bang, then, is the total physical event whose temporal region is the initial interval of time. One might further object that there is not only no first moment of time, but also no initial interval of time. But, by the lights of modern physics (which may be wrong of course, but there is no guide in these matters that is known to be better), the initial interval of time is simply the first interval of time whose duration is the Planck-time (that is, $10^{-43}$ sec). There certainly is such an interval of time (even if there is no first moment of time), and the corresponding event – the Big Bang – is, as far as we know, correctly described by (A3$^*$).

**Objection 2** (against (A3) being the entire scientifically warranted truth): There is purely scientific evidence not only for (A3) but also for the existence of physical events that are causes without having a cause. For it is a scientific principle that if a physical event is not caused by any physical event, then it is not caused by anything. *Response:* The objection relies on (A4.2) – which is a principle of causal closure – being a scientific principle. No doubt, many scientists employ that principle; but that, by itself, does not make it a scientific principle. In fact, (A4.2) is not a scientific, but a metaphysical principle – just like (A4.1), the principle of causation. It is a metaphysical principle because logical, mathematical, empirical, and methodological-esthetical considerations alone are not sufficient for warranting its assumption.

**Objection 3:** The notion of agent causation, which is necessary for obtaining (T6) and (T6$^*$), is an irremediably unclear notion. When, for example, does agent causation happen? *Response:* This is a stock objection, the merits of which are doubtful. For one thing, the notion of event causation is not so clear either (and yet we continue to use it, and could not well do without it). For another thing, I have offered a detailed analysis of agent causation in my books *Ereignis und Substanz* [1] and *The Two Sides*

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4Many scientists in the past have made successful use – within the very context of their *scientific* endeavors – of the hypothesis that God exists and has created the Universe (for example, Johannes Kepler in his arduous search for the laws of planetary motion). But that, of course, does not imply that the existence of God is a *scientific* principle.
of Being [3], in the former regarding both creatural and divine agency, in
the latter regarding only creatural agency. A comprehensive theory of
causation, both of event causation and agent causation, can be found in
my book Theorie der Kausalität [2], also containing extensive discussions
of the literature. Some of the main results of Theorie der Kausalität are
presented in my paper “Causation in a New Old Key” [4]. The emergence
of creatural agent causation in the course of natural history is defended in
several of my papers, for example, “The Emergence of Rational Souls” [6]
and “New Perspectives for a Dualistic Conception of Mental Causation”
[5]. – And when does agent causation “happen”? Instances of agent cau-
sation do not happen, since they – in contrast to the effects involved in
them – are not events (and only events can happen). But if one absolutely
wishes to assign a time to an instance of agent causation, then it is simply
the time of the effect that is involved in it.

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[6] —— , “The Emergence of Rational Souls”, in Emergence in Sci-
ence and Philosophy, edited by A. Corradini and T. O’Connor, New
A Gödelian Ontological Argument Improved Even More

ALEXANDER R. PRUSS

1 Introduction

Gödel’s ontological argument axiomatizes the notion of a positive property, and then argues, based on plausible further “non-formal” axioms about which properties are in fact positive, that there is a being that has at least some of the central attributes of God.

The formal axioms are:

(F1) If $A$ is positive, then $\neg A$ is not positive.

(F2) If $A$ is positive and $A$ entails $B$, then $B$ is positive.

There are several ways of metaphysically understanding the notion of a positive property.\(^1\) On the excellence view, a positive property is one that in no way detracts from its possessor’s excellence, but whose negation does. On the limitation view, a positive property entails no limitation in its possessor, but its negation does. According to Leibniz, there are basic properties, all subsets of which are mutually compatible. Leibniz would probably define a positive property as one that is a conjunction of basic

\(^1\) Pruss [3] also offers an account due to Maydole on which a property is positive provided that it is better to have that property than not to have it. However, Oppy [1] has rightly pointed out that it is far from clear whether the disjunction of a property that it is better to have than not to have with a property that it is better to not have than to have counts as something that it is better to have than not to have.
properties. But we can modify his view to be open to the possibility that some basic properties are not valuable, by saying that some basic properties are excellences and a positive property is one that is entailed by one or more basic properties that are excellences. Each of these accounts makes (F1) and (F2) very plausible.

Now consider the following two non-formal axioms:

\((N1)\) Necessary existence is positive.
\((N2)\) Essential omniscience, essential omnipotence and essential perfect goodness are positive properties.

We can then define \(A\) to be a strongly positive property provided that \(EA\), the property of having \(A\) essentially, is a positive property. (By (F2), strongly positive properties are positive.) Pruss [3] then assumes a modal logic including S5 and proves:

**Theorem 1.1** Given (F1), (F2) and (N1), if \(A\) is a strongly positive property, then there is a necessarily existing being that essentially has \(A\).

It follows from (F1), (F2), (N1) and (N2) that there is a necessarily existing being that is essentially omniscient. And one that is essentially omnipotent. And one that is essentially perfectly good. But Pruss [3] could not show, without making further controversial assumptions, that there is a being that has all these three essential properties.

This paper remedies this defect. Admittedly, we will make further non-formal assumptions, but they will be very plausible.

We will end up by discussing a reformulation of the arguments in terms of negative properties as well as Oppy’s parody of the Pruss [3] argument.

2 **Uniqualization**

There can be at most one being that has the property of being the tallest woman. If \(A\) is a property such that it is impossible that there exist \(x\) and \(y\) such that \(x\) and \(y\) each have \(A\), but \(x \neq y\), then we shall say that \(A\) is uniqualizing. Being the tallest woman is uniqualizing.

The following non-formal axiom is very plausible:

\((N3)\) There is at least one uniqualizing strongly positive property.

\(^2\)Along these lines, Pruss [3] makes the suggestion that one could take a positive property to be one that is entailed by one or more basic properties.

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In fact, it is very plausible that we can give an example of one. Axiom \((N3)\) follows from \((N2)\) and:

\((N4)\)  **Essential omnipotence is uniqualizing.**

Why should we think essential omnipotence is uniqualizing? Well, the idea of two essentially omnipotent, or even two contingently omnipotent, beings is deeply problematic. Omnipotence requires perfect freedom and an efficacious will. But there cannot be two beings with perfect freedom and an efficacious will. For if they are perfectly free, they will be able to will incompatible propositions to be true, and then one of their wills shall have to fail to be efficacious. (This argument assumes that we are individuating beings in such a way that distinct beings with will have their own will. If God is a Trinity, the persons of the Trinity do not have distinct wills, and hence will not count as distinct *beings* in our sense.)

Other plausible examples of an equalizing strong positive property are *being greater than every other being* as well as *being creator of every other being*. To work with the latter property, we can assume:

\((N5)\)  **Being essentially such that one is creator of every other being is a positive property.**

\((N6)\)  **If \(x\) is creator of \(y\), then \(y\) is not creator of \(x\).**

And then being creator of every other being will be strongly positive and uniqualizing, so that \((N3)\) will follow.

All in all, it does not appear that \((N3)\) is very controversial.

We can now add to the results from Pruss [3]:

**Theorem 2.1**  Given \((F1)\), \((F2)\), \((N1)\) and \((N3)\), it follows that there is a necessary being that essentially has every strongly positive property.

The following immediately follows:

**Corollary 2.1**  Given \((F1)\), \((F2)\), \((N1)\), \((N2)\) and \((N3)\), there is a necessary being that is essentially omnipotent, essentially omniscient and essentially perfectly good.

Now, define:

**Definition 2.1**  A *God* is a being that is essentially omnipotent, essentially omniscient, essentially perfectly good, and essentially creator of every other being.

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3 Pearce and Pruss [2] argue that omnipotence just is perfect freedom and efficacious will.
Given \((F1), (F2), (N1), (N2), (N5) and (N6)\), there necessarily exists a unique God.

For \((N5) and (N6) implies (N3)\), so that by Theorem 4.2 it follows that there is a God, and by \((N6) this God is unique, as he is creator of every other being.\(^4\)

The proof of Theorem 4.2 needs the following lemma from Pruss [3]:

**Lemma 2.1** Given \((F1) and (F2)\), any pair of positive properties is compossible.

(The proof is easy: if \(A\) and \(B\) are positive and not compossible, then \(A\) entails \(\neg B\), so that by \((F2)\), \(\neg B\) is positive, and by \((F1)\), \(B\) cannot be positive as well, so absurdity follows.)

To prove Theorem 4.2, let \(U\) be a unqualizing strongly positive property, by \((N3)\). By Theorem 4.1, there is a necessarily existing being that essentially has \(U\). Let’s say that Umberto is such a being. Let \(A\) be any strongly positive property. Then \(EA\), the property of having \(A\) essentially, is also positive. By Lemma 2.1, \(EA\) is compossible with \(U\). Thus there is a possible world \(w\) at which there is a being, \(x\), that has both \(EA\) and \(U\). But Umberto exists at every world and has \(U\) at every world. Thus, Umberto exists at \(w\) and has \(U\) there. Since \(U\) is unqualizing, it follows that \(x\) is identical with Umberto. Therefore, Umberto has \(EA\) at \(w\). Thus, it is true at \(w\) that Umberto necessarily has \(A\). By \(S5\), it follows that at the actual world it is also the case that Umberto necessarily has \(A\). Hence, Umberto essentially has \(A\). Thus, Umberto essentially has every strongly positive property.

It is interesting to note that the above argument also shows that for every positive property \(A\), even ones that are not strongly positive, Umberto at least possibly has \(A\). For by Lemma 2.1, \(A\) is compatible with \(U\), and

\(^4\)Some think (mistakenly, I believe) that God is not the creator of abstract entities, and indeed that it is impossible for any being to be the creator of abstract entities, but that there nonetheless necessarily exist abstract entities. Impossible properties aren’t going to be positive, since an impossible property entails non-positive properties like \(being\ crueld\), so if it’s not possible to be the creator of abstract entities, and there must be abstract entities, we will instead need to work with the property of being the creator of every other \(concrete\ being\). Plausibly this property is strongly positive, and if we assume the axiom that every creator is concrete, we conclude that this property is unqualizing.
hence at some world some being has both $A$ and $U$. But only Umberto has $U$ in that or any other world, and so Umberto has both $A$ and $U$ in that world.

3 Negative properties

There is, perhaps, something somewhat unnatural and gerrymandered about the notions of positive properties offered in the introduction. The notion of a positive property on all three accounts offered does not exactly correspond to any intuitive notion of an excellence or a property that’s worth having. For instance, since anything entailed by a positive property is positive, if knowing that $2 + 2 = 4$ is a positive property, so is being foolish or knowing that $2 + 2 = 4$. This disjunction is not a counterexample to (F2) given the three stipulative metaphysical accounts of positive properties offered in the introduction, but given the relative complexity of the stipulations, our intuitions about the non-formal axioms are liable to be less confident than we would like.

We might, however, proceed differently, by taking as our primitive the notion of a negative property, which is actually more natural than the Gödelian notion of a positive property. We can think of a negative property as one that limits a being in some way. The following two axioms then are intuitively plausible:

\[(F1^*) \quad \text{If } A \text{ is negative, then } \neg A \text{ is not negative.}\]

\[(F2^*) \quad \text{If } B \text{ is negative and } A \text{ entails } B, \text{ then } A \text{ is negative.}\]

Axiom $(F1^*)$ tells us that to lack something that limits one is not limiting, i.e., that limitation is avoidable, while $(F2^*)$ tells us that a property that entails a limitation is limiting.

We can then stipulate a positive property as one whose negation is negative. It is easy to see that under this stipulation the conjunction of $(F1)$ with $(F2)$ is equivalent to the conjunction of $(F1^*)$ and $(F2^*)$. But it is better to work with the more natural notion of a negative or limiting property as in $(F1^*)$ and $(F2^*)$.

We can define a strongly negative property as a property $A$ such that $PA$ is negative, where $PA$ is the property of possibly having $A$. A property is strongly negative if and only if its negation is strongly positive. We can say that a property is nearly universal provided that it must be had by all

\[5\]Cf. Oppy’s criticism of Maydole’s definition of positivity ([1], p. 359).
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except at most one being. A property is nearly universal if and only if its negation is uniqualizing.

We still need some non-formal axioms. These will be direct translations of the $N$-axioms.

(N1*) Possible non-existence is negative.

(N2*) Possible non-omniscience, possible non-omnipotence and possible lack of perfect goodness are negative properties.

(N3*) There is at least one strongly negative nearly universal property.

(N4*) Being possibly non-omnipotent is nearly universal.

(N5*) Being possibly such that there is something distinct from one that one is not a creator of is a negative property.

These axioms are still plausible. To substantiate (N3*) we just use the negations of our examples of candidates for strongly positive uniqualizing properties (or small variants on them, if desirable for stylistic or intuitive reasons). Thus coexisting with an entity one did not create or not being omnipotent are properties that, plausibly, all beings except at most one can have.

Moreover, given (F1*) and (F2*), the starred versions of the N-axioms are equivalent to the unstarred versions. Hence we get equivalent starred versions of our results: Theorem 4.2, Corollaries 6.1 and 6.2. For instance, we have:

Corollary 3.2 Given (F1*), (F2*), (N1*), (N2*), (N5*) and (N6), there necessarily exists a unique God.

The only difference between (C4*) and (C4) is that the argument behind (C4*) did not rely on a gerrymandered concept of positivity.

Let us go back to the initial counterintuitiveness of the idea that being foolish or knowing that $2 + 2 = 4$ is positive. The parallel claim on the side of negativeness is that not being foolish and not knowing that $2 + 2 = 4$ is negative, i.e., limiting. And that certainly is true – it is limiting through its second conjunct.

4 Oppy’s parody

Oppy begins by stipulating the notion of a natural property (not in the sense in which in the preceding section I talked of some notions as more natural than others) as
a property whose instantiation in no way entails the existence of any supernatural entities, or the holding of any supernatural states of affairs, or the like, but the instantiation of whose negation does in some way entail the existence of supernatural entities, or the holding of supernatural states of affairs, or the like. ([1], pp. 360 - 361)

Oppy then offers two formal axioms about naturalness. The first parallels ($F1$), and the second parallels a plausible generalization of ($F2$). Then Oppy offers a non-formal axiom:

The following property is natural: having no world-mate that is a necessarily existent, essentially omnipotent, essentially omniscient$^6$, essentially perfectly good being. ([1], p. 361)

Oppy argues that his axioms entail that there is no necessarily existent, essentially omnipotent, essentially omniscient and essentially perfectly good being. And he’s right about that.

One problem with Oppy’s parody is that there his notion of a natural property is quite unnaturally gerrymandered, arguably even more so than the three versions of the notion of a positive property considered in the introduction, and much more so than the one in the preceding section which was defined in terms of negativity. One way in which Oppy’s notion of a natural property is gerrymandered is that it is defined in an extrinsic way – a natural property is one whose negation entails that there are supernatural states of affairs, rather than entailing something about the entity that has it. This gerrymandering makes the intuitions behind his non-formal axiom less reliable.

A second problem is that given Oppy’s stipulative definition of a natural property, the mere assumption that there are any natural properties is incompatible with classical theism. For according to classical theism, God is a necessarily existent and essentially supernatural being.$^7$ But if $A$ is a natural property in Oppy’s sense, then the instantiation of $A$ does not entail the existence of any supernatural entities, and hence the proposition

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6Oppy’s text has a reduplicated “essentially omnipotent” in several places. I assume one of the occurrences is always meant to be “essentially omniscient”.

7One might worry whether God counts as supernatural in worlds in which there is no nature. But one might reasonably respond that in those worlds, God is trivially supernatural, being trivially beyond the realm of the natural, because there are no realm of the natural there.
that \( A \) is instantiated does not entail the existence of God. But if God exists, then every proposition entails his existence. So the very existence of a natural property is incompatible with theism.\(^8\)

This gives the classical theist a simple reason to reject Oppy’s non-formal axiom that his complex property about lacking divine world-mates is natural, because the classical theist is committed to there not being any natural properties in Oppy’s sense.

But there does not appear to be a parallel point to be made about the notions of positive and negative properties, since it is highly intuitive, whether or not God exists, that there are some positive properties, such as knowing that \( 2 + 2 = 4 \) or such as not being cruel, and some negative properties, such as not knowing that \( 2 + 2 = 4 \) or such as being cruel. The very idea that there is a positive or a negative property does not by itself appear to commit one to anything that an atheist rejects. Thus Oppy’s parody is dialectically inferior to these Gödelian arguments.

5 Conclusions

The improved Gödelian arguments of Pruss [3] can be improved some more by introducing the notion of a unicaqualizing property. Moreover, instead of running the arguments with the somewhat gerrymandered notion of a positive property, one can instead take as primitive the more natural notion of a negative or limiting property. Finally, Oppy’s [1] parody is not parallel to these theistic arguments, because Oppy’s notion of a “natural property” is such that the mere assumption that there is a natural property is incompatible with classical theism, which makes Oppy’s parody dialectically inferior to the argument that it is a parody of.

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\(^8\) What if the natural property were non-instantiable? But my argument did not assume instantiability. Moreover, there cannot be any non-instantiable natural properties, because if \( A \) is a non-instantiable property, then the proposition that \( A \) is instantiated entails every proposition, and in particular entails the proposition that there is a supernatural being, contrary to Oppy’s definition of naturalness.

Part IV

Semantics for Ontological Proofs
Logic of Existence, Ontological Frames, Leibniz’s and Gödel’s Ontological Proofs

SERGIO GALVAN

1 Introduction

The aim of this article is twofold. First, I intend to present the fundamental outlines of two conceptions of ontological frame. The first one connected to Kant’s concept of possible object and the second one related to Leibniz’s. Leibniz maintains that the source of possibility is the mere logical consistency of the notions involved, so that possibility coincides with analytical possibility. Kant, instead, argues that consistency is only a necessary component of possibility. According to Kant, something is possible if there is a cause capable of bringing it into existence; to this end consistency alone is not sufficient. Thus, while the Leibnizian notion of consistency is at the root of the concept of analytical possibility, the Kantian notion of possibility is the source of real possibility. This difference underlies the distinction between the ontological structure (which is characterized by Kant’s interpretation) and the Leibnizian ontological structure. Both structures are presented as the semantic basis for two systems of logic of existence (PE and PEL), the second of which is the Leibnizian extension of the first. The distinction between the two different conceptions of ontological structure plays an important role in the discussion of Gödel’s ontological proof, that can be formally interpreted on the ontological frame of the pure perfections. While this proof, under some emendation condition, is conclusive in the context of Leibniz’s ontological structure, it is not so within the Kantian
Logic of Existence, Ontological Frames, ...

one. Then, in the second part of this paper I’ll say something about their relations to Leibniz’s and Gödel’s ontological argument.

2 Logic of Existence

The logic of existence, **PE**, is an extension of the **S5** first-order modal logic. On the basis of this extension it is possible to deal, from the logical point of view, with the existential predicates, i.e., the predicate of existence and its modalizations (non real predicates in Kantian terminology), and the essential or non existential predicates (real predicates in Kantian terminology).

Besides the usual signs for modal first-order language, the set of logical signs of **PE** includes the new sign for the existence monadic predicate \(E\). The set of well-formed formulas is extended in the obvious way.

In order to avoid confusion between language and metalanguage, the following metalinguistic symbols shall henceforth be used: \(\land, \lor, \Rightarrow, \iff\) and \(\neg\) (conjunction, disjunction, implication, equivalence and negation), \(\Box\) and \(\Diamond\) (necessity and possibility), and \(\forall\) (**universal quantifier**) and \(\exists\) (**existential quantifier**).

**PE** includes the **axioms** and **rules** of **S5** first-order logic. In particular:

Axiom **T**: \(\Box \alpha \rightarrow \alpha\),

Axiom **5**: \(\Diamond \alpha \rightarrow \Box \Diamond \alpha\),

Necessitation **N**: \(X \vdash \alpha \Rightarrow \Box (X) \vdash \Box \alpha\), where \(\Box (X) = \{\Box \beta \mid \beta \in X\}\).

Further, we have:

Axiom **NE**: \(\alpha \rightarrow \Box \alpha\), **provided** \(E\) **does not occur in** \(\alpha\),

Axiom **NI**: \(\Box \exists x E(x)\).

If we add also:

Leibnizian Axiom **PL**: \(\forall x \Diamond E(x)\),

we obtain the **Leibnizian logic of existence**, **PEL**, i.e., the Leibnizian extension of the basic existence logic **PE**.

Hereafter we list several **derivable rules** and **theorems** in the system **PE**:

T1. a) \(\Box (\alpha \land \beta) \vdash \Box \alpha \land \Box \beta\),

b) \(\Box \alpha \land \Box \beta \vdash \Box (\alpha \land \beta)\),

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T2. a) $\Box \alpha \vdash \neg \Diamond \neg \alpha,$  
b) $\Diamond \alpha \vdash \neg \Box \neg \alpha,$

T3. a) $\Box \Diamond \alpha \vdash \neg \Diamond \Box \neg \alpha,$  
b) $\Diamond \Box \alpha \vdash \neg \Box \Diamond \neg \alpha,$

T4. $\alpha \vdash \beta \Rightarrow \Diamond \alpha \vdash \Diamond \beta,$

T5. $\alpha \vdash \Box \Diamond \alpha,$

T6. $\Diamond \Box \alpha \vdash \alpha,$

T7. $\Diamond \alpha \vdash \beta \Rightarrow \alpha \vdash \Box \beta,$

T8. Barcan Rule (BR): $\forall x \Box \alpha \vdash \Box \forall x \alpha,$

T9. Converse Barcan Rule (CBR): $\Box \forall x \alpha \vdash \forall x \Box \alpha,$

T10. $\Diamond \forall x \alpha \vdash \forall x \Diamond \alpha,$

T11. $\exists x \Box \alpha \vdash \Box \exists x \alpha,$

T12. $\exists x \alpha \vdash \exists x \Diamond \alpha \vdash \Diamond \exists x \alpha,$

T13. $\Diamond \exists x \alpha \vdash \exists x \Diamond \alpha \vdash \exists x \alpha,$ provided $E$ does not occur in $\alpha.$

3 Ontological Frames

The central concept of semantics for $\textbf{PE}$ is that of ontological model, which is based in its turn on the concept of ontological frame. We will first present the concept of ontological frame, and then that of model. Finally, after introducing some basic semantical notions, the soundness of the principal axioms of $\textbf{PE}$ will be proved.

Informally, an ontological structure is a set of possible worlds correlated and determined according to the objects existing in them. And formally, by an ontological structure we mean a quad-tuple of the form $\mathcal{S} = \langle W, R, \mathcal{U}, E \rangle,$ where:

- $W$ is a non empty set of ontologically possible worlds;
- $R$ is a total relation over $W$;
- $\mathcal{U}$ is a universal structure formed by a non empty set $U$ of possible objects and a set $P$ of attributes (properties and relations), i.e., $\mathcal{U} = \langle U, P \rangle;$
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– $E$ is a function from possible worlds to the power set $2^U$.

An ontologically possible world is an analytically possible world – that is a simply consistent and maximal set of states of affairs – where at least one possible is actualized, that is exists. The relation $R$ of accessibility between worlds is a total relation. It establishes that every world is a possible alternative to every other. The relation $R$ expresses the metaphysical and, hence, unconditional nature of the notion of possibility inherent in the structure that we are presenting. We use the symbols $u, v, w, \ldots$ as variables for worlds.

A possible object is an analytically possible object, i.e., an object that necessarily satisfies the only requirement of coherence. The set of analytically possible objects is, in turn, subdivided into two disjoint subsets: the subset of really possible objects and the subset of purely possible objects (simply consistent). This distinction is introduced below. Now it is important to stress two issues. Firstly, that the set $U$ is the same in every world. The reason for this is that the objectual domain is a set of possible objects and these are present (as possible), although not (as actual existent), in all worlds. Secondly, it is worth noting that the objects are individual, i.e. completely determined with respect to all properties. The reason for this is the fact that an object can only exist as complete.

The elements of $U$ are denoted in the semantic metalanguage by $\bar{x}, \bar{y}, \bar{z}, \ldots$. We assume, however, that the individual variables, when they are not quantified, perform the function of individual names. In this way, the individuals of the domain can also denoted by the signs $x, y, z, \ldots$, which, once an interpretation or evaluation function $I$ of the language has been established, stand for specific objects in the individual domain. It should be noted that the interpretation function for the individual names is independent of the worlds because, as we shall shortly see, individual names are taken to be rigid designators.

Given the modal context, the attributes – elements of the set $P$ – can be understood either intensionally or extensionally. The intension is given as a function which establishes the extension of the attribute for every world. In our semantic apparatus, however, it is important to establish that for all non-existential attributes, the intension of the attribute fixes the same extension in each world, because the essential attributes are conceived in rigid manner. The reason for this is that the individuals of $U$ are possible objects, and possible objects do not vary with respect of non-existential attributes but do so solely because they exist or otherwise in a world, that is, because they are actualized or otherwise in that world. Attributes
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will be indicated in the semantic metalanguage by means of the signs \( \bar{P}^n_i \), where \( i \) is the index of the order of the attribute and \( n \) is the number of its places. Of course, \( \bar{P}^n_i \subseteq U^n \), i.e., \( \bar{P}^n_i \in 2^{U^n} \). In accordance with the rigid character of the non-existential attributes, the interpretation function for non-existential predicates is independent of the worlds, so that also the non-existential predicates are rigid designators. The attribute of existence will be discussed separately since it is the only attribute considered as non-rigid.

A basic idea of the ontological structure that we are presenting is the distinction – of Kantian origin – between essential predicates, real in Kants terminology, and the existence predicate, a non real (or non essential) predicate. A real predicate states how the object is determined (order of \( \text{sosein} \)). Conversely, the existence predicate states if it is actual in one world or another (order of \( \text{dasein} \)). Hence the different behaviour of real and existence predicate. An object can be actual in one world and non-actual in an alternative possible world, despite being identical with respect to real predicates.

As previously stated, the objectual domain is constant for all possible worlds. However, the possibilia actualised in each of the possible worlds are not the same. The possibilia actualized in a world are only the possibilia existing in that world, i.e., the possibilia for which the property of existence \( E \) is valid in it. So, the possible denoted by \( x \) is actualized in the world \( u \) if and only if \( E(x) \) is true in \( u \). This amounts to saying that the extension of \( E \) varies from world to world, i.e., that the intension of \( E \) is the function \( E : W \rightarrow 2^U \) and that this function is not constant.

**Remark:** Two are the general approaches to the semantics of quantified predicative modal systems. The first one is the approach based on constant domain models (see, Fitting and Mendelsohn [7], p. 95) and the second one approach based on varying domain models (see, Fitting and Mendelsohn [7], p. 101). The first approach depends on a possibilist interpretation of quantification, according to which quantification regards all the possible objects (regardless of whether such objects are existing in this or in another world). The reference to existing objects (and thus the identification of the domains of existing objects with respect to different worlds) is obtained, then, by the predicate of existence \( E \) taken as primitive. It characterizes, in fact, those objects that are existing in the single worlds. The second approach depends, instead, on an actualist interpretation of quantification, according to which a quantifier ranges over what
Logic of Existence, Ontological Frames, ... actually exists, that is, on objects existing in any individual world. Given certain conditions, however, the two approaches are equivalent. It is sufficient that each formula of the predicative modal language is existentially relativized (that is, that the condition $E(x)$ is given for each quantification of $x$ occurring in the formula), for obtaining that this formula is true in every varying domain model if and only if the same formula existentially relativized is true in every constant domain model (see, Fitting and Mendelsohn [7], Proposition 4.8.2, p. 106).

Our approach relies on constant domain models with existence predicate. However, it is not a standard approach, insofar as its quantified language does not satisfy the condition of existence relativization for each formula. Quantification can concern variables not restricted by the existence predicate. And the reason for this is the assumption that the truths concerning the essential structure (expressed by real Kantian predicates) of objects hold for non-existing objects too. Thus our approach rejects the so-called serious actualism ($A(x)$ implies $E(x)$, where $A$ is a predicate letter), according to which an object cannot possess a property without existing. In fact, the existence predicate is conceived of as a genuine predicate, and not as an existence determinator, which plays the role of determining referents of variables to be members of particular domains (as, for example, it is maintained in Szatkowski [37], p. 481).

The function $E$ meets two requirements:

(i) (Existence Condition): $\forall u(\emptyset \neq E(u) \subseteq U)$.

The existence condition ensures that the extension of the predicate of existence is not empty in every possible world. This means that in every world at least one possible must be actualized. The rationale of this condition is obvious. A world, where at least one possible does not exists, would be really impossible, because in it the nothing would exist, which is impossible.

(ii) (Limited Condition of Exhaustiveness): $\neg \Box \forall \exists u (\bar{x} \in E(u))$.

$E(u)$ satisfies the limited condition of exhaustiveness because it is not necessarily true that for every element $\bar{x}$ of $U$ there exists a possible world in which $\bar{x}$ is actual. Included in $U$, in fact, are also purely possible individuals, which are not actual in any possible world. In other words, the exhaustiveness condition is not generally satisfied because possible existence coincides with real possibility, not with pure analytical possibility. If possible existence coincided with analytical possibility, one could correctly assume, for every possible (= non-contradictory) object, a possible
(= non-contradictory) world in which the object is actualized. Instead, once it has been posited that possible existence coincides with real possibility – not with purely analytical possibility –, we may not exclude that a non-contradictory object may not be actualized in any world.

**Remark:** But, what is the ground of the distinction between really possible entities and purely analytically possible entities? In other terms, why the *existence* in some world does not necessarily pertain to all analytically possible entities but only to a part of these, i.e., to real possible entities? The answer lies in the non-essentialist nature of the theory of being, formalized in the $\mathcal{G}$ structure. In any ontological theory based on the distinction between essential predicates and existence, existence cannot be deduced from the essential order of things. Therefore, affirming that the really possible entities are analytically possible amounts to saying that existence is a property that can be banally derived from the essential order of things, in the sense that existence can be attributed to any analytically possible entity just because of its essential consistency. On the contrary, the real possibility requires, besides consistency, a foundation, based on a power to make be.

The ontological frame $\mathcal{G}$ provides the semantic apparatus for the PE system (characterized by Kant's interpretation). For it to be extended to the Leibnizian system PEL, the limited exhaustiveness condition is to be replaced by the following

**(Unlimited (Global) Exhaustiveness Condition):** $\forall \bar{x} \exists u (\bar{x} \in E(u))$.

An *ontological model* $\mathcal{M}$ consists of the ontological frame $\mathcal{G}$ plus an interpretation $\mathcal{I}$ of the PE language on $\mathcal{G}$, i.e., $\mathcal{M} = \langle \mathcal{G}, \mathcal{I} \rangle$.

An *interpretation* $\mathcal{I}$ is a function which associates elements of the modal structure to signs of the PE language. This association depends on the worlds only for the existence predicate. In fact, in accordance with the nature of the elements of the modal structure introduced above, only the existence predicate is conceived as a non-rigid designator. The other designators – i.e., the individual variables and the non-existential predicates – are all rigid. Specifically,

(i) $\mathcal{I}$ associates possible objects to individual variables: $\mathcal{I}(x) = \bar{x}$ (with $\bar{x} \in U$).

Of course, the function $\mathcal{I}$ could be introduced as a two-argument function on pairs $(x,u)$, where the first argument is a individual variable and the
second argument is a sign for worlds. By means of this formulation, the rigidity of individual names can be explicitly expressed by the formula:

\[(\text{Rig1}) \quad \forall x \forall u \forall v (I(x, u) = I(x, v)).\]

Clearly, the rigidity of individual names presupposes the invariance of objectual domain.

(ii) \(I\) associates attributes to the non-existential predicative constants:

\[I(P^n_i) = \bar{P}^n_i \quad (\text{with } \bar{P}^n_i \subseteq U^n).\]

Here, too, the function \(I\) could be introduced as a two-argument function \(I\) on pairs \((P^n_i, u)\), where the second argument is a sign for worlds. Also here, by means of this formulation, the rigidity of the non-existential predicates can be explicitly expressed by the formula:

\[(\text{Rig2}) \quad \forall P^n_i \forall u \forall v (I(P^n_i, u) = I(P^n_i, v)).\]

That the non-existential predicates are rigid is due to the fact that possible objects are invariably characterized by the same attributes in every possible world.

(iii) \(I\) associates the existence property to the predicate \(E\):

\[I(E, u) = E(u).\]

The existence predicate is the only non-rigid designator. Different possible worlds are distinct precisely because different possible individuals are actual in them. It has even been said that one possible world is not determined by anything other than the set of possible individuals that are actual in it. If one modifies the set of actual individuals, one modifies the possible world in question.

If \(I\) is an interpretation, then the symbol \(I^o_x\) denotes the interpretation, called \textit{reinterpretation} of \(I\) with respect to \(x\) on \(o\), defined by:

\[I^o_x(y, u) \overset{\text{def}}{=} \begin{cases} o & \text{if } y = x, \\ I(y, u) & \text{if } y \neq x. \end{cases}\]

Of course, \(o\) is tacitly assumed to be an entity suitable to the variable \(x\), and both \(I\) and \(I^o_x\) are assumed to be interpretations in the same ontological structure \(S\). We say that interpretations \(I, \mathcal{R}\) agree apart from \(x\) (symbolically: \(I \equiv^x \mathcal{R}\)) if for some \(o\), \(I^o_x = \mathcal{R}\). Note that \(\equiv^x\) is an equivalence relation on the set of all interpretations of an ontological structure \(S\). The equivalence class of \(I\) with respect to \(\equiv^x\) will be further denoted
by \{\mathcal{I}_x^2\}. Given an ontological model \(M = \langle \mathcal{G}, \mathcal{I} \rangle\), the symbol \(\{\mathcal{M}_x^2\}\) denotes the class of all ontological models \(N = \langle \mathcal{G}, \mathcal{K} \rangle\) such that \(\mathcal{K} \in \{\mathcal{I}_x^2\}\). Formally, \(\{\mathcal{M}_x^2\} = \{\langle \mathcal{G}, \mathcal{K} \rangle \mid \mathcal{K} \in \{\mathcal{I}_x^2\}\}\).

Let \(M = \langle \mathcal{G}, \mathcal{I} \rangle\) be an ontological model. The satisfiability relation \(\models_u\) is defined as usual, for any possible world \(u \in W\), by the following conditions:

(i) \(M \models_u P^n_i(x_1, ..., x_n)\) iff \(\langle \mathcal{I}(x_1, u), ..., \mathcal{I}(x_n, u) \rangle \in \mathcal{I}(P^n_i, u)\),
(ii) \(M \models_u E(x)\) iff \(\mathcal{I}(x, u) \in \mathcal{I}(E, u)\) iff \(\mathcal{I}(x, u) \in E(u)\),
(iii) \(M \models_u \alpha \land \beta\) iff \(M \models_u \alpha\) and \(M \models_u \beta\),
(iv) \(M \models_u \neg \alpha\) iff not \(M \models_u \alpha\) (symbolically: \(M \not\models_u \alpha\)),
(v) \(M \models_u \forall x \alpha\) iff \(M \models_u \alpha\) for every \(N \in \{\mathcal{M}_x^2\}\),
(vi) \(M \models_u \Box \alpha\) iff \(M \models_w \alpha\) for every \(w \in W\) such that \(uRw\) (i.e., for every \(w \in W\)).

We say that a formula \(\alpha\) is true in an ontological model \(M = \langle \mathcal{G}, \mathcal{I} \rangle\) (symbolically: \(M \models \alpha\)) iff \(M \models_u \alpha\), for every world \(u \in W\). And a formula \(\alpha\) is true in an ontological structure \(\mathcal{G}\) (symbolically: \(\mathcal{G} \models \alpha\)) iff \(M \models_u \alpha\), for every ontological model \(M = \langle \mathcal{G}, \mathcal{I} \rangle\). The sets of all true formulas in an ontological model \(M\) and an ontological structure \(\mathcal{G}\) will be denoted by \(\text{Th}(M)\) and \(\text{Th}(\mathcal{G})\), respectively. If \(X\) is a set of formulas, then we write \(M \models X\), \(\mathcal{G} \models X\) if \(X \subseteq \text{Th}(M)\), \(X \subseteq \text{Th}(\mathcal{G})\), respectively.

4 Soundness Theorem

Let us consider only the typical axioms of PE and PEL. We have to prove the validity of the axioms NE, NI and PL on respective structures \(\mathcal{G}\).

**Theorem 4.1** Let \(\mathcal{G}\) be an ontological structure. Then, the axiom NE is true in \(\mathcal{G}\). Formally, \(\alpha \rightarrow \Box \alpha \in \text{Th}(\mathcal{G})\), provided \(E\) does not occur in \(\alpha\).

**Proof:** Let \(M = \langle \mathcal{G}, \mathcal{I} \rangle\) be an ontological model and \(u \in W\). The proof processes by induction on the complexity of \(\alpha\).

**Basis:** \(\alpha \equiv P^n_i(x_1, ..., x_n)\)

1. \(M \models_u P^n_i(x_1, ..., x_n)\) hypothesis
2. \(\langle \mathcal{I}(x_1, u), ..., \mathcal{I}(x_n, u) \rangle \in \mathcal{I}(P^n_i, u)\) 1, def. \(\models\)
3. $\forall x_1, \ldots, x_n \forall u, w(\langle \mathcal{I}(x_1, u), \ldots, \mathcal{I}(x_n, u) \rangle = \langle \mathcal{I}(x_1, w), \ldots, \mathcal{I}(x_n, w) \rangle)$ (Rig1)

4. $\forall P^n_i \forall u, w(\mathcal{I}(P^n_i, u) = \mathcal{I}(P^n_i, w))$ (Rig2)

5. $\forall w(\langle \mathcal{I}(x_1, w), \ldots, \mathcal{I}(x_n, w) \rangle \in \mathcal{I}(P^n_i, w))$ 2, 3, 4

6. $\forall w(M \models_w P^n_i(x_1, \ldots, x_n))$ 5, def. $\models$

7. $M \models_u \Box P^n_i(x_1, \ldots, x_n)$ 6, def. $\models$

Step: $\alpha \equiv \beta \land \gamma$

1. $M \models_u \beta \land \gamma$ hypothesis

2. $M \models_u \beta$ and $M \models_u \gamma$ 1, def. $\models$

3. $M \models_u \Box \beta$ and $M \models_u \Box \gamma$ 2, inductive hypothesis

4. $\forall w(M \models_w \beta)$ and $\forall w(M \models_w \gamma)$ 3, def. $\models$

5. $\forall w(M \models_w \beta \land \beta)$ 4, the property of $\forall$

6. $\forall w(M \models_w \beta \land \gamma)$ 5, def. $\models$

7. $M \models_u \Box (\beta \land \gamma)$ 6, def. $\models$

Step: $\alpha \equiv \neg \beta$

1. $M \models_u \neg \beta$ hypothesis

2. $M \not\models_u \beta$ def. $\models$

3. $M \models_w \beta \Rightarrow M \models_w \Box \beta$ inductive hypothesis

4. $M \models_w \beta \Rightarrow \forall u(M \models_u \beta)$ 3, def. $\models$

5. $M \models_w \beta \Rightarrow M \models_u \beta$ 4, the property of $\forall$

6. $M \not\models_u \beta \Rightarrow M \not\models_w \beta$ 5, the classical logic

7. $M \not\models_u \beta \Rightarrow \forall w(M \not\models_w \beta)$ 6, the propety of $\forall$

8. $\forall w(M \not\models_w \beta)$ 2, 7

9. $\forall w(M \models_w \neg \beta)$ 8, def. $\models$

10. $M \models_u \Box \neg \beta$ 9, def. $\models$

Step: $\alpha \equiv \forall x \beta$

1. $M \models_u \forall x \beta$ hypothesis

2. $\forall x(M \models \beta)$ 1, def. $\models$

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3. $\forall x(M \models_u \square \beta)$
4. $\forall x\forall w(M \models_w \beta)$
5. $\forall w\forall x(M \models_w \beta)$
6. $\forall w(M \models_w \forall x \beta)$
7. $M \models_u \square \forall x \beta$

Step: $\alpha \equiv \square \beta$

1. $M \models_u \square \beta$ hypothesis
2. $M \not\models_u \square \square \beta$ hypothesis
3. $\forall w(M \models_w \beta)$ 1, def. $\models$
4. $\exists w(M \not\models_w \square \beta)$ 2, def. $\models$
5. $\exists w(M \not\models_w \beta)$ 4, def. $\models$
6. contradiction 3, 5

Theorem 4.2 Let $\mathcal{G}$ be an ontological structure. Then, the axiom $NI$ is true in $\mathcal{G}$. Formally, $\square \exists x E(x) \in \mathcal{Th}(\mathcal{G})$.

Proof:

1. $M \not\models_u \square \exists x E(x)$ hypothesis
2. $\exists w \forall \mathcal{M} \in \{\mathcal{M}_x^w\} (\mathcal{M} \not\models_w E(x))$ 1, def. of $\models$
3. $\exists w \forall \mathcal{K} \in \{\mathcal{I}_x^w\} (\mathcal{K}(x, w) \notin \mathcal{I}(E, w))$ 2, def. of interpretation
4. $\forall u(\emptyset \neq E(u) \subseteq U)$ Existence Condition
5. $\forall w \exists \bar{x}(\bar{x} \in E(u))$ 4, def. of the set $\emptyset$
6. $\forall w \exists \mathcal{K} \in \{\mathcal{I}_x^w\} (\mathcal{K}(x, w) \in \mathcal{I}(E, w))$ 5, def. of interpretation and of $E(u)$
7. contradiction 3, 6

Theorem 4.3 Let $\mathcal{G}$ be an ontological structure, which satisfies the unlimited (global) exhaustiveness condition. Then, the axiom $PL$ is true in $\mathcal{G}$. Formally, $\forall x \diamond E(x) \in \mathcal{Th}(\mathcal{G})$.

Proof:
Philosophically, the fact that there are models of the ontological structure $S$ in which not all analytical possibilia are existing in some world is very important. It means that the analytical possibility of a concept is not a sufficient condition for that concept to be exemplifiable (actuable). To be precise this element underlies the insurmountable component of the Kantian criticism of the modal ontological proof.

In fact, the aim of the second part of this paper is to analyze specifically this question in order to show that also the last version of ontological proof, Gödel’s one, falls under the Kantian criticism of the purely analytical notion of possibility. We will consider, first, the Leibnizian formulation of modal proof and then Gödel’s one. For both formulations we shall present only the relevant aspects from the Kantian versus Leibnizian point of view about possibility.

5 Leibniz’s formulation of modal ontological proof

Leibniz’s version of the ontological argument stands out for being endowed with modal structure. This is quite evident in all formulations Leibniz assigns to the argument along every step of his reasoning. The same is found, in a less developed fashion and with a few features of its own, also in Descartes’ Meditation V. The advantage of the modal formulation of the argument is that the modal structure of it allows the narrowing down of all premises to one, more precisely, the one stating the possibility of the maximally perfect Being.

Leibniz’s version of the ontological argument (see, Leibniz [19] and [20]) is based on two premises: (Strong or Weak) Descartes’ Principle and Main Premise. They are as follows:
Strong Descartes’ Principle (SDP): \( \forall x (G(x) \land E(x)) \rightarrow \Box (G(x) \land E(x)) \)

Informally: For every \( x \) it is necessary that if \( x \) is \( G \) and \( x \) really exists then it is necessary that \( x \) is \( G \) and \( x \) really exists.

Weak Descartes’ Principle (WDP): \( \forall x (G(x) \rightarrow \Box G(x)) \)

Informally: For every \( x \) it is necessary that if \( x \) is \( G \) then it is necessary that \( x \) is \( G \).

The symbol DP stands for one of the principles SDP or WDP.

Main Premise: \( \exists x (G(x) \land E(x)) \)

Informally: There exists \( x \) such that it is possible that \( x \) is \( G \) and \( x \) really exists.

Preparing to showing that the formula \( \exists x (G(x) \land E(x)) \) or \( \exists x G(x) \) is provable in PE – depending on the use of the axiom (SDP) or (WDP), respectively – we first prove the Leibniz’s Rule (LR), namely: \( X, \alpha \vdash \Box \alpha \Rightarrow \Box(X), \Diamond \alpha \vdash \alpha \).

\[ \begin{align*}
1. & \quad X, \alpha \vdash \Box \alpha \quad \text{hypothesis} \\
2. & \quad X, \neg \Box \alpha \vdash \neg \alpha \quad 1, \text{contraposition} \\
3. & \quad X, \Diamond \neg \alpha \vdash \neg \alpha \quad 2, \text{Transf } \Box \Diamond \\
4. & \quad \Box(X), \Box \Diamond \neg \alpha \vdash \Box \neg \alpha \quad 3, \text{necessitation N} \\
5. & \quad \Box(X), \Diamond \neg \alpha \vdash \Box \neg \alpha \quad 4, \text{by axiom 5} \\
6. & \quad \Box(X), \neg \Box \neg \alpha \vdash \neg \Diamond \neg \alpha \quad 5, \text{contraposition} \\
7. & \quad \Box(X), \Diamond \alpha \vdash \Box \alpha \quad 6, \text{Transf } \Box \Diamond \\
8. & \quad \Box(X), \Diamond \alpha \vdash \alpha \quad 7, \text{and axiom T} \\
\end{align*} \]

And for \( \vdash \exists x (G(x) \land E(x)) \), the proof is as follows:

\[ \begin{align*}
1. & \quad G(x) \land E(x) \rightarrow \Box(G(x) \land E(x)), G(x) \land E(x) \vdash \Box(G(x) \land E(x)) \quad \text{modus ponens} \\
2. & \quad \Box(G(x) \land E(x)) \rightarrow \Box(G(x) \land E(x))), \Diamond(G(x) \land E(x)) \vdash G(x) \land E(x) \quad 1, \text{LR} \\
3. & \quad \forall x \Box(G(x) \land E(x) \rightarrow \Box(G(x) \land E(x))), \exists x (G(x) \land E(x)) \vdash \exists x (G(x) \land E(x)) \quad 2, \forall I, \exists I, \exists I \\
4. & \quad \exists x \Diamond(G(x) \land E(x)) \vdash \exists x (G(x) \land E(x)) \quad 3, \text{SDP} \\
\end{align*} \]
Some comments about the particular principles may be here useful.

(Strong or Weak) Descartes’ Principle: For Leibniz this principle is plausible since it is utterly befitting that the most perfect Entity, provided it does exist, be necessary. An entity that possesses perfections or existence contingently cannot be most perfect. Gödel too believes that the necessary existence is a perfection. Note that the affirmation of the positivity of necessary existence is not axed even by the Kantian criticism. As a matter of fact, as for the existence, this criticism is justified for two reasons. Not only does the existence add nothing to the perfection of an object but neither does it indicate any perfection of the essence. All essences (the possibilia) are prone to existence; hence the fact that an essence is exemplified does not mean that this essence is more perfect than another. Neither does necessary existence add, of itself, any perfection to the essence of a thing. However, as it cannot be equally attributed to all essences, it is an indicator of greater perfection of the essences to which it is indeed attributed. In other words, not any essence may be endowed with necessary existence, but only that privileged one, which is essence able to exist necessarily. There is, then, a strong reason for considering necessary existence susceptible to evaluation. This does not mean, though, that necessary existence may be considered a perfection to the same extent as essential perfections. Indeed, as stated above, existence (hence necessary existence, too) belongs to a different modal level from the essential layer. What truly allows to count necessary existence as a highly plausible requisite for the maximally perfect Entity is that necessary existence is most certainly a more perfect form of existence than mere existence. Hence, the former is not assessed as such against essential properties, but against existence. Finally, viewing divine perfection not only in terms of its essential properties but – being existing – also in terms of its modality of being, its perfection requires necessary existence (with reference also to such layer).

Main Premise: Leibniz’s attempt to justify the premise, as featured in his [20] writing, consists of proving that pure perfections are compatible with one another, and to the extent that it is possible to postulate that the intersection of all perfections – that is, the maximally perfect being, as the bearer of all perfections – is itself possible. In our language that means
to obtain \( \exists x (G(x) \land E(x)) \) (some really possible \( x \) is \( G \)). The argument consists of two parts:

**Part 1:** The starting point is the definition of positive absolute perfection:

"I call a perfection every simple quality which is positive and absolute, i.e., which expresses whatever it expresses without any limitation." (Leibniz [20], p. 261)

More formally, let \( A, B, C \ldots \) be positive properties; let \( G(x) \) be \( A(x) \land B(x) \land C(x) \ldots \); and let \( F(x) \) be a conjunction of a finite number of positive properties. Then, by definition, \( \text{Cons}F(x) \), i.e., \( F(x) \) is consistent. In fact, from the above definition it can be easily derived that the perfections cannot be incompatible. As a matter of fact, since they are simple, they do not result from the composition of other perfections and, consequently, neither can they be the negation of any other. Therefore, the conjunction of any finite number of perfections is consistent. But, at this point, Leibniz’s principle comes into play according to which every consistent property is exemplifiable, i.e. the principle stating that there is at least one possible in which that property inheres. In formal terms, \( \text{Cons}F(x) \Rightarrow \exists x F(x) \), because: *consistency = satisfiability = analytical possibility*. And, by passage to infinity, \( \exists x G(x) \).

**Part 2:** At this point, to achieve \( \exists x (G(x) \land E(x)) \) requires the use of Leibniz’s axiom \( \text{PL} \), \( \forall x \Diamond E(x) \). In fact:

1. \( G(x), E(x) \vdash G(x) \land E(x) \) \hspace{1cm} by assumption
2. \( \Box G(x), \Diamond E(x) \vdash \Diamond (G(x) \land E(x)) \) \hspace{1cm} 1, possibilitation
3. \( \Box G(x), \Diamond E(x) \vdash \exists x (G(x) \land E(x)) \) \hspace{1cm} 2, \( \exists \)
4. \( \Box G(x), \forall \Diamond E(x) \vdash \exists x (G(x) \land E(x)) \) \hspace{1cm} 3, \( \forall \)
5. \( \Box G(x) \vdash \exists x (G(x) \land E(x)) \) \hspace{1cm} 4, by \( \text{PL} \)
6. \( G(x) \vdash \exists x (G(x) \land E(x)) \) \hspace{1cm} 5, by \( \text{WDP} \)
7. \( \exists x G(x) \vdash \exists x (G(x) \land E(x)) \) \hspace{1cm} 6, \( \exists \)
8. \( \vdash \exists x (G(x) \land E(x)) \) \hspace{1cm} 7 and \( \exists x G(x) \)

Leibniz’s process contains two problem areas. The first concerns the passage to infinity dealt with at the end of Part 1. The second regards the use of Leibniz’s principle \( \text{PL} \) on analytical possibility.
1. The passage to infinity, which takes place at the end of the first part of the argument, is formally expressed by the following implication: if (for any $F$) $\exists x F(x)$ then $\exists x G(x)$. Now, this very implication is not guaranteed if the logic that regulates the relations between the concepts is not complete. It should be noted that the passage from the satisfiability of all finite subsets of an infinite set to the satisfiability of the latter is legitimate only if the theorem of semantic finiteness (or compactness) is valid. However, this theorem is not unconditionally valid; as a matter of fact, the theorem of semantic finiteness is guaranteed only by completeness.

2. The second objection regards the use of Leibniz’s principle, that is refuted by Kant. It is not guaranteed that from the analytical possibility of $G (\exists x G(x))$ follows the real possibility of $G (\exists x (G(x) \land E(x)))$, unless it has accepted the Leibnizian axiom $\mathbf{PL}$ on identity between analytical and real possibility.

6 Gödel’s modified version of the ontological proof

Gödel’s version of the ontological proof is interesting because, compared to that of Leibniz, it is a new attempt to overcome the objection to Leibniz’s unjustifiable passage to infinity. This objective is pursued by showing that the system of pure perfections has a principal ultrafilter structure. The objection to the passage is overcome under rather strong conditions. However, the proof is incapable of overcoming the second objection based on Kantian criticism. To show this, it is convenient to adapt Gödel’s ontological proof to the semantics of the structure $\mathfrak{S}$. This forces to make some change, but allows two kinds of advantages:

1. Firstly, in order to obtain the main premise of the proof, it is possible to use only part of Gödel’s argument, reducing the number of the necessary axioms, i.e. to only three.

2. Secondly, the semantics of the structure $\mathfrak{S}$ allows us to avoid some problems associated with Gödel’s concept of existence. There are five important issues in this regard.

   a. Gödel’s notion of perfection is independent, as Gödel says, from the accidental features of the world. This means that the extensional meaning of a property would have to be referred to sets of possible
entities and not to sets of actual entities. In other words, the range of variables would be a domain of possible and not of actual individuals.

b. As a consequence of point a, it is difficult to conceive the existence as exemplification in some world. According to extensional interpretation, all the possibles are by definition exemplified in every world.

c. If the entities are interpreted as possibles, the domain of possibles is the same in all the worlds. Therefore, a semantics based on a fixed domain is more appropriate than a semantics based on a variable domain.

d. It is convenient to conceive the essential predicates of the individual possibles as rigid predicates, i.e., predicates that hold for the same individual in every world.

e. Consequently, it is also convenient to introduce a new predicate of existence, as it is in PE and in the ontological frame $\mathcal{G}$.

Therefore, Gödel’s ontological proof may be presented in the semantic context of the structure $\mathcal{G}$ enriched with the deontic component, i.e., in the ontological structure of the pure perfections $\mathcal{G}\mathcal{P}$. Gödel’s proof is set out in two parts. The first part demonstrated the possible existence of a substance that, according to the definition provided, is God (similarly to main premise). The second part demonstrated that God necessarily exists, based on the fact that maximum divine perfection requires divine existence to be necessary (similarly to principle DP). For our purpose, we shall consider only the first part as the other is already covered by Leibniz’s formulation presented above. We follow Gödel in presenting the proof (although the numbers of the axioms differ) as for $A1$ and $A2$. As for $A3$, we keep to the formulation provided by Fitting [6], p. 148, where $A3$ coincides with the Axiom 11.10. The reasons underlying this change will become clear later on.

6.1 Gödel’s modified version of the ontological proof, viewed syntactically

The language of Gödel’s modified version of ontological proof (GMV) is obtained by extending the first order language of the theory PE to the third order language, where $x, y, z, ...$ are first order variables for objects; $X, Y, Z, ...$ are second order variables for properties (intensions) or sets
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(extensions) of possibilia; and \( X, \ Y, \ Z, \ldots \) are third order variables. So, \( X(x) \) is to be read: \( x \) is an element of the set \( X \), or, intensionally, the property \( X \) holds of \( x \). Similarly, \( X(X) \) is to be read: the property \( X \) is an element of the set of properties \( X \). To signs for properties can be applied the usual boolean operators: \( \land \) and \( \neg \), where if \( x \) belongs both to set \( X \) and to set \( Y \), we can write \( (X \land Y)(x) \) and say that \( x \) belongs to the intersection of \( X \) and \( Y \). Similarly, \( \neg X \) indicates the negation of \( X \) (intension) or the complement of \( X \) (extension). So \( \neg X(x) \) is to be read: \( x \) is not an element of the set \( X \), that is, \( x \) is an element of the complement of \( X \).

Additionally, the language of GMV contains, insofar as extension of PE, the second order specific predicate \( E \) and a specific third order sign. \( P \) is this sign for a third-order property. Intuitively, it is the sign of positivity concerning the essential properties of the elements of \( U \), i.e., the sets corresponding to the properties of possible objects (according to the extensional interpretation), that, in Gödel’s view, represent pure perfection. A perfect property is a property expressing unlimited value.

Yet it should be noted that in the semantics of GMV: 1. Only real properties are susceptible of being positive. 2. Being possible (analytically possible) is positive (compared to not being possible) but being actual (what we mean through the predicate \( E \)) cannot be considered positive if compared to being possible; neither is the property of being necessary \( (E(x) \rightarrow \square E(x)) \) assessable through predicate \( P \), since it is not a real property. 3. In Leibniz’s wake, Gödel deems it appropriate to speak of positivity of necessary existence rather than contingency. The positivity of necessary existence is determined through Axiom 4, where the property \( P \) is attributed directly to necessary existence. This seems to be justified in Gödel’s language, in that it does not make any difference between existence properties and real properties. As pointed out above, though, this triggers a number of issues related to Gödel’s semantics, which will be analyzed shortly. 4. The fact that necessary existence may not be the subject of assessment in terms of \( P \), does not imply that it is not a perfection with respect to simple existence, hence it does not justify the assumption of Descartes’ principle DP. Indeed, in this particular instance the comparison takes place within the modality of being and not within the modality of essence or between the former and the latter. The reasons for that have already been explained.

The following abbreviations may also be used (see, Fitting [6], p. 148):

1. \( pos(Z) \overset{\text{df}}{=} \forall X(Z(X) \rightarrow P(X)) \);
2. \( X \text{ intersection of } Z \equiv \forall x (X(x) \leftrightarrow \forall Y(Z(Y) \rightarrow Y(x))) \).

The Gödel’s modified version of ontological proof (\textbf{GMV}) is obtained by adding to \textbf{PE} the following three additional axioms:

\textbf{A1.} \( \forall X (P(X) \leftrightarrow \neg P(\neg X)) \)

(Exactly one of a property or its complement is positive),

\textbf{A2.} \( \forall X \forall Y (P(X) \land X \subseteq Y \rightarrow P(Y)) \)

(The properties entailed by positive properties are positive),

\textbf{A3.} \( \forall Z (pos(Z) \rightarrow \forall X (X \text{ intersection of } Z \rightarrow P(X))) \)

(The conjunction of any collection of positive properties is positive).

I am not interested in explaining the details of these axioms. I want just to present some brief remarks about them and then a further remark about their formal structure.

The first axiom is true because contradictory perfections cannot be both positive and because either a property or its negation is positive. It should be noted that \( X \) can be not positive, not because it does not express value, but because the value it expresses has a constitutive limit: this is true, for instance, for the concept of human being. In this regard, it is also understandable why, conversely, not being a human being is positive. It is positive because it excludes the limit.

The second axiom is true because if \( X \subseteq Y \) then being \( Y \) is a prerequisite for being \( X \), so if \( X \) is positive, \( Y \) must also be positive.

The third axiom is a generalization up to infinite of the principle stating that if two properties are positive their intersection is also positive. The importance of this principle will be illustrated below.

Let us notice that the additional axioms do not contain any occurrence of necessity operator. The reason lies in the fact that these axioms are formulas in which there are no occurrences of existence predicate, thus they only contain essential predicates. Any modal operators present in the standard formulation can therefore be omitted under the \textbf{PE} axioms \textbf{NE} and \textbf{T}.

6.2 Gödel’s modified version of the ontological proof, viewed semantically

The semantics of \textbf{GMV} is obtained by extending the structure \( \mathfrak{S} \) to the structure of pure perfections and by interpreting the language of \textbf{GMV}
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on that enlarged structure. We denote the *structure of pure perfections* by \( \mathcal{SP} \).

\( \mathcal{SP} \) differs from \( \mathcal{S} \) for the following reasons:

1. Besides the domain of possible \( U_1 \) (coinciding with the domain \( U \) of \( \mathcal{S} \)), there are the properties domain \( U_2 \subseteq 2^{U_1} \) and the properties of properties domain \( U_3 \subseteq 2^{U_2} \). First-order variables are interpreted on \( U_1 \), first-order predicate variables are interpreted on \( U_2 \) and third-order predicate variables are interpreted on \( U_3 \). It is not necessary to distinguish between properties of objects existing in a world and properties of simply possible objects, because the properties which are taken into consideration are the properties of simply possible objects. According to our approach, it makes sense to talk about positivity of these properties and only of these.

2. Within \( \mathcal{SP} \) is singled out a principal ultrafilter defined over \( U \), on which the predicate \( P \) is interpreted. We call this ultrafilter \( \text{UP} \). This means:

   - 2.1. \( \text{UP} \subseteq 2^U \),
   - 2.2. \( X \in \text{UP} \) and \( Y \in \text{UP} \Rightarrow X \land Y \in \text{UP} \),
   - 2.3. \( X \in \text{UP} \) and \( X \subseteq Y \subseteq U \Rightarrow Y \in \text{UP} \), that is, \( \text{UP} \) is a filter,
   - 2.4. \( \emptyset \notin \text{UP} \), that is, \( \text{UP} \) is a proper filter,
   - 2.5. \( X \in \text{UP} \) or \( \neg X \in \text{UP} \), that is, \( \text{UP} \) is an ultrafilter,
   - 2.6. \( \exists x(x \in U \text{ and } \forall X(X \in \text{UP} \Rightarrow x \in X)) \), that is, \( \text{UP} \) is a principal filter;
     
     the singleton of element \( x \) is denominated, for obvious reason, the generator of the ultrafilter.

As can be easily noted, \( \text{UP} \) is a set of properties subject to certain constraints. These constraints imply that this set is a principal ultrafilter. To say that \( X \) is a positive property is to say that \( X \) is an element of the ultrafilter \( \text{UP} \).

Let us try to show, at this point, the correctness of \( \text{GMV} \) with respect to \( \mathcal{SP} \). Ceteris paribus (that is, without considering the standard clauses of the model relation \( \models \)), to show the correctness of \( \text{GMV} \) with respect to \( \mathcal{SP} \) is to show that for each model \( \mathcal{M} = (\mathcal{SP}, \mathcal{I}) \), \( \mathcal{M} \models A1 \land A3 \iff \mathcal{I}(P) = \text{UP} \).

**Part 1:** \( \mathcal{I}(P) = \text{UP} \Rightarrow \mathcal{M} \models A1 \land A3 \):
ad A1: \( \forall X (P(X) \leftrightarrow \neg P(\neg X)) \).
According to 5, \( \mathbf{U} \) is a maximal set, that is, it cannot contain \( X \) and \( \neg X \) at the same time and, on the other hand, if it does not contain \( X \) it must contain \( \neg X \). Now, \( \forall X (P(X) \leftrightarrow \neg P(\neg X)) \) is equivalent to the conjunction of \( \forall X (P(X) \rightarrow \neg P(\neg X)) \) and \( \forall X (\neg P(X) \rightarrow P(\neg X)) \), which exactly corresponds to maximality.

ad A2: \( \forall X \forall Y (P(X) \land X \subseteq Y \rightarrow P(Y)) \).
The axiom is made true immediately by 3.

ad A3: \( \forall Z (pos(Z) \rightarrow \forall X (X \text{ intersection of } Z \rightarrow P(X))) \).
The axiom is made true, by 6 and 3. Let \( Z \) be any conjunction of positive properties whatsoever. Moreover, let \( X \) be the intersection of such positive properties. Then \( X \) must be positive. Indeed, according to 6, there is a generator of all positive properties. It coincides with the singleton of the entity which is characterized by all the perfections. This generator will therefore be included to \( X \), since it is included to all positive properties belonging to \( Z \). But, then, by virtue of 3, \( X \) is also positive.

Part 2: \( \mathfrak{M} \models A1 - A3 \Rightarrow \mathcal{I}(P) = \mathbf{U} \):
\( \mathcal{I}(P) \) is a principal ultrafilter in virtue of the following statements:

ad 2.1.: \( \mathcal{I}(P) \subseteq 2^U \).
In fact, the perfections are subsets of \( U \).

ad 2.2.: \( X \in \mathcal{I}(P) \) and \( Y \in \mathcal{I}(P) \Rightarrow X \land Y \in \mathcal{I}(P) \).
The proof is articulated in two parts: (i). the first part shows that given two positive properties, their intersection is not void, and (ii). the second shows that the intersection of any pair of positive properties is itself positive. Ad (i): Let us assume that \( P(X) , P(Y) \) and \( X \cap Y = \emptyset \). Hence, by A1, \( \neg P(\neg X) \) and \( Y \subseteq \neg X \). But then, by A2, \( P(\neg X) \) - a contradiction. Ad (ii): It follows from A3.

ad 2.3.: \( X \in \mathcal{I}(P) \) and \( X \subseteq Y \subseteq U \Rightarrow Y \in \mathcal{I}(P) \).
It follows from A2.

ad 2.4.: \( \emptyset \notin \mathcal{I}(P) \).
\( \mathcal{I}(P) \) is a proper filter, because \( \emptyset \) is not a positive property: the proof, that follows from A1 and A2, is well-known (see, for example, Fitting [6], pp. 147 - 48).

ad 2.5.: \( X \in \mathcal{I}(P) \) or \( \neg X \in \mathcal{I}(P) \).
It follows from \textbf{A1}.

\textbf{ad 2.6.:} \( \exists x (x \in U \text{ and } \forall X (X \in \mathcal{P} \Rightarrow x \in X)) \).

The demonstration follows from \textbf{A3}. The intersection of all perfections is therefore positive. But positive properties cannot be empty. Therefore, the generator of the ultrafilter exists. This coincides with the singleton of the entity characterized by all perfections.

\textbf{Remark:} Gödel’s original system contains, instead of \textbf{A3}, the weaker axiom \( \forall X \forall Y (P(X) \land P(Y) \rightarrow P(X \land Y)) \) (\textit{If two properties are positive, their combination is also positive}). However, this axiom is not sufficient to ensure the differentiation of \( \exists x G(x) \). As stated by Szatkowski [36], p. 319, a system of axioms such as \textbf{A1} - \textbf{A2} plus Gödel’s axiom, which was just mentioned in place of \textbf{A3}, could be interpreted on a non-principal ultrafilter containing all co-finite subsets of \( U \). In such an ultrafilter, the proposition \( \exists x G(x) \) would be false since in it the intersection of all co-finite sets is empty. It is therefore necessary to reinforce the system with an axiom such as \textbf{A3} (corresponding to Fitting’s axiom 11.10). An alternative axiom to this could be the assertion of the positivity of \( G \) (which is equivalent to \textbf{A3}, as shown by Fitting [6], p. 153), an axiom that was also proposed by D. Scott in place of the original one (see, Sobel [35], p. 145). Of course, the reinforcement of the third axiom diminished the meaning of Gödel’s proof. In fact, the meaning of a proof is determined by the greater accessibility of the axioms compared to the conclusion and, in our case, there is hardly any difference between the effort to reach the axiom and the effort to reach the conclusion, which is not far from circularity. In the context of these reflections, in order to guarantee the principal character of the ultrafilter and, consequently, to ensure the truth of the conclusion, it could be useful to require the presence of at least one finite positive property. It is well known, in fact, that the principality of an ultrafilter is equivalent to the existence of at least one finite subset of the ultrafilter domain (see, Bell and Machover [4], p. 140). This would mean that at least some perfections could not be shared by an infinite number of subjects. In this case, the perfection would lie in the exclusivity of the attribution, of which unicity would be the highest expression. Hence, to assume axiomatically that a positive property owned by a single subject exists would imply that that subject owns them all. After all, the property of being One is traditionally one of the divine attributes and, therefore, its being positive appears highly plausible. In conclusion, an axiom stating that being One is a perfection or, in general, an axiom stating the existence of
a perfection which can be shared only by a finite number of subjects would have a greater founding meaning than the axiom of divine perfection or equivalents.

6.3 Conclusions and critical remarks

We finish the section with two conclusions and four critical remarks.

**Conclusion 1:** Within the scope of the ontological structure $\mathcal{SP}$, axioms $A_1$, $A_2$ and $A_3$ (with the emendation of the third axiom) provide that the system of perfections $\mathcal{SP}$ contains a principal ultrafilter. Not only does this mean that the intersection of any two (hence $n$) perfections does exist, but also that the intersection of all perfections, that is $\bigcap X_P(X)$, in virtue of the infinite passage (being perfections infinite) also exists. Now, it should be noted that this passage is entirely sound. As a matter of fact the model that satisfies the existence of non-empty intersections of any two (or a finite number of) perfections is the same that satisfies the existence of the intersection of all perfections. No property of compactness should be resorted to.

We have not obtained:

*For every finite set $F$ of perfections there exists a model $\mathcal{M}$ of $F$ such that $M$ satisfies $F$,*

but

*There exists a model based on the frame $\mathcal{SP}$ with ultrafilter $\mathcal{UP}$ such that for every finite or infinite set $A$ of perfections, $\mathcal{SP}$ satisfies $A$ – hence the afore mentioned result.*

Despite the severe limitations of the above mentioned observations, Gödel’s proof allows for the first flaw of the Leibnizian proof to be overcome.

**Conclusion 2:** Gödel’s proof, too, features the second problem to Leibniz’s proof: $\exists x \Diamond (G(x) \land E(x))$, does not follow from $\exists x G(x)$ unless one accepts Leibniz’s principle about the reality of analytically possibilia $\text{PL}$.

In conclusion, Gödel’s proof, too, is affected by the same basic flaw that was detected in Leibniz’s proof. There is no guarantee that the analytical possibility is a real possibility.
**Remark 1:** The manuscript left by Gödel features two other axioms. These are instrumental for the second part of the proof, that is, proving Descartes’ Principle DP. Indeed, one states that perfections are rigid; the other that the necessary existence is positive. However, the second one is problematic for the above-mentioned reasons.

**Remark 2:** As has already been mentioned before, the language used to formulate our version of Gödel’s ontological proof is similar to that used by Fitting [6] (see, also Hájek [14]). As within our language, in Fitting’s, too, the quantification is construed in a possibilist way, that is, the domain of quantifiers is the set of possibles and actual existence is expressed by existence predicate E. In other words, the objectual domain is fixed and the existence predicate defines the extensions of existent objects in each possible world. However, as we have explained in the note on predicate E, there is a deep difference between our approach and the standard possibilist approach followed by Fitting. For us, the expression \( \exists x P(x) \) states that a possible object is characterized by the property P, whilst if we want to affirm that this object is actually existing, then it is necessary to also attribute the existence predicate E to it. In the standard approach, instead, the use of quantifiers is always existentially relativized, so that \( \exists x P(x) \) should always be understood as \( \exists x (E(x) \land P(x)) \) and \( \forall x P(x) \) as \( \forall x (E(x) \rightarrow P(x)) \). This difference makes the standard approach incapable of accounting for the necessity character of perfections positivity. In fact, it would be fair to assume that according to Gödel himself, the properties that may be evaluated from the standpoint of their perfection were essential properties, that is, the properties rigidly defined in an extensional manner on the set of possible – and not of existent – beings. On the other hand, that does not appear to stem from Fitting’s formulation (which can be found in Hájek’s work, too) of Axiom 2: \( \forall X \forall Y (P(X) \land \Box \forall x (E(x) \rightarrow (X(x) \rightarrow Y(x)))) \rightarrow P(Y) \) (Axiom 11.5), where the inclusion relation among positive properties is restricted to the respective extensions defined on the set of existent beings. This, though, implies unacceptable consequences, as it will be clear in the two following remarks.

**Remark 3:** In our interpretation on \( \mathfrak{SP} \), the Fitting’s Axiom 11.5 lacks plausibility, since the logic of real existence does not obey any deontic principle. It is possible, then, that among the existing beings of all really possible worlds there be inclusion relations that are not compatible with the logic of perfections. For the sake of argument, it would be fair to
postulate that in all really possible worlds every existing honest individual is an existing farmer. Hence, being a farmer would be a pure perfection, which is absurd.

**Remark 4:** The above note voids Fitting’s proof of Gödel’s Theorem 1 $(P(X) \vdash \Box \exists x (E(x) \land X(x)))$ of all pertinence. It may be easy to note that this theorem is obtained in Fitting [6], p. 147, by virtue of Axiom 11.5 in Fitting’s wording disputed above.

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Fully Free Semantics for some Anderson-like Ontological Proofs

MIROSŁAW SZATKOWSKI

1 Introduction

It seems necessary first to clarify the notions in the title of this article. And so, in general, the phrase: *ontological proof* – often used interchangeably with the phrase: *ontological argument* – means a proof, for the conclusion that *God* (*necessarily*) *exists*, from premisses which are independent from the observation of the world. Since the starting point for ontological proofs presented in this paper is Anderson’s modification of Gödel’s ontological proof, hence – the term: *Anderson-like ontological proofs*. More technically speaking, Gödel’s ontological proof – see, [5] – requires 2nd order modal language with a predicate of *positiveness* over properties, and depends on three definitions of: *God-being* (*A God is any being that has every positive property*), *essence* (*A property A is an essence of an object x if and only if A entails every property of x*) and *necessary existence* (*An object x has the property of necessarily existing if and only if its essence is necessarily exemplified*), and on six axioms: (g1). A conjunction of positive properties is also positive; (g2). A property or its complement is positive; (g3). If a property is positive, then its complement is not positive; (g4). If a property is positive, then it is necessarily positive; (g5). The property of necessary existence is a positive property; (g6). Any property entailed by a positive property is positive. The modification of Gödel’s ontological proof proposed by A. Anderson consists in offering new definitions of *God-being* (*A God is any being that has necessarily all and only positive properties*) and *essence* (*A property A is an essence of an object x if and only if A*
entails all and only the properties that \( x \) has necessarily), and the axiom: The property of God-being is a positive property – instead of \((g1)\) and \((g2)\).

The present paper is a natural continuation of our earlier one [15]. And as their titles may already suggest, each of Anderson-like ontological proofs presented in both these works adopts at least one of the two free principles of quantification: of the 1\(^{st}\) order and/or of the 2\(^{nd}\) order. More precisely, all partly free Anderson-like theories of the previous paper make use of free universal specification of the 1\(^{st}\) order: \( \forall x \phi \land E(t) \rightarrow \phi(x/t) \) where \( t \) is a term of the 1\(^{st}\) order (to be read: If \( \phi \) holds for every object \( x \) of the domain of quantification and \( t \) is an object of this domain, then \( \phi(x/t) \) also holds) and of the principle: \( \forall x E(x) \) (to be read: Every object of the domain of quantification is its element) instead of the classical universal specification of the 1\(^{st}\) order: \( \forall x \phi \rightarrow \phi(x/t) \) (to be read: If \( \phi \) holds for every object \( x \), then \( \phi(x/t) \) also holds). In contrast to this approach, the classical universal specification of the 2\(^{nd}\) order: \( \forall \alpha \phi \rightarrow \phi(\alpha/\tau) \) where \( \tau \) is a term of the 2\(^{nd}\) order (to be read: If \( \phi \) holds for every property, then \( \phi(\alpha/\tau) \) also holds) is an axiom of these theories. The 1\(^{st}\) order fragments of fully free Anderson-like theories presented in this paper are the same as in [15], only the 2\(^{nd}\) order fragments make the difference. Namely, the classical universal specification of the 2\(^{nd}\) order is now replaced by the free universal specification of the 2\(^{nd}\) order: \( \forall \alpha \phi \land E(\tau) \rightarrow \phi(\alpha/\tau) \) (to be read: If \( \phi \) holds for every property of the domain of quantification and \( \tau \) is a property of this domain, then \( \phi(\alpha/\tau) \) also holds) and by the principle: \( \forall \alpha E(\alpha) \) (to be read: Every property of the domain of quantification is its element). That is exactly our reason for the use of the phrases “partly free” or “fully free” in relation to Anderson-like theories – some Anderson-like theories are named partly free ones because they are free only on the 1\(^{st}\) level, while other Anderson-like theories are named fully free ones because they are, in addition, free on the 2\(^{nd}\) order level.

Now, a small comment on Barcan formulas, converse Barcan formulas and principles of exportation seems in place. A simple situation, their 1\(^{st}\) order case, i.e., Barcan formulas – \( \forall x L\phi \rightarrow L\forall x \phi \) (to be read: If for every object the formula \( \phi \) holds necessarily, then necessarily for every object the formula \( \phi \) holds), converse Barcan formulas – \( L\forall x \phi \rightarrow \forall x L\phi \) (to be read: If necessarily for every object the formula \( \phi \) holds, then for every object the formula \( \phi \) holds necessarily) and principles of exportation - \( L\exists x \phi \rightarrow \exists x L\phi \) (to be read: If necessarily there exists an object such that \( \phi \) holds, then there exists an object such that \( \phi \) holds necessarily), are refused as
theorems in all Anderson-like theories of our both papers. But, what is important, the formula: $L\exists xG(x) \leftrightarrow \exists xLG(x)$ (to be read: *If necessarily some object is a God, then some object is necessarily a God*) is a theorem in them. Unlike the $1^{st}$ order case, in all Anderson-like theories of our both papers theorems are: 2$^{nd}$ order Barcan formulas – $\forall\alpha L\phi \rightarrow L\forall\alpha\phi$ (to be read: *If for every property the formula $\phi$ holds necessarily, then necessarily for every property the formula $\phi$ holds*), 2$^{nd}$ order converse Barcan formulas – $L\forall\alpha\phi \rightarrow \forall\alpha L\phi$ (to be read: *If necessarily for every property the formula $\phi$ holds, then for every property the formula $\phi$ holds necessarily*) and 2$^{nd}$ order principles of exportation – $L\exists\alpha\phi \rightarrow \exists\alpha L\phi$ (to be read: *If necessarily there exists a property such that $\phi$ holds, then there exists a property such that $\phi$ holds necessarily*). We can appreciate it from a semantic point of view – the semantics validates the 2$^{nd}$ versions of Barcan formulas, converse Barcan formulas and principles of exportation, even though they invalidate their 1$^{st}$ order versions, since the intensions over which the second-order quantifiers range are restricted to those that for each world deliver a subset of the first-order domain of that world as the extension.

The aim of this paper is to prove strong completeness theorems for fully free Anderson-like ontological systems with respect to corresponding classes of model structures.

Model structures contain families of world-varying objectual domains of the 1$^{st}$ order and families of world-varying objectual domains of the 2$^{nd}$ order. Members of these 1$^{st}$ order domains are called existing objects of the world, whereas members of these 2$^{nd}$ order domains are called existing properties of the world in question. Terms of the 1$^{st}$ order receive only rigid (i.e., world-independent) extensions, which are elements of the union of all 1$^{st}$ order domains. Terms of the 2$^{nd}$ order receive non-rigid (i.e., world-dependent) extensions and rigid intensions. Extensions of the 2$^{nd}$ order terms are elements of the union of all 2$^{nd}$ order domains, but intensions of the 2$^{nd}$ order terms are chosen among members of a conceptual domain of the 2$^{nd}$ order, which is the set of all functions from the set of worlds to the family of all subsets of the union of all 1$^{st}$ order domains such that if the value of a function at some world belongs to a 2$^{nd}$ order domain connected with this world then every value of this function belongs to a 2$^{nd}$ order domain connected with its argument; these functions are called conceptual properties. According to these notions, 1$^{st}$ order quantification at a world is over the 1$^{st}$ order domain associated with that world, and
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not over the union of 1st order domains, i.e., one can think of a 1st order domain associated with a world as the set of things actually existing at that world. And the domain of the 2nd order quantification is the set of conceptual properties. One particularly relevant question is how to treat the complementation of a 2nd order term semantically, if its intension at a given world does not belong to the 2nd order domain connected with this world? For instance, let the value \((a(A))(w)\) of the intension \(a(A)\) of the term \(A\) in the world \(w\) not belong to the 2nd order domain connected with the world \(w\), but \((a(A))(w)\) belong to two different 2nd order domains connected with the world \(w_1\) and \(w\), respectively. How should \(a(-A)\) be defined? We decide that the intention \(a(-A)\) of a 2nd order term \(-A\) (the complementation of \(A\)) with respect to an assignment \(a\) is defined as follows: for any possible world \(w\), \((a(-A))(w)\) is the complement of \((a(A))(w)\) with respect to the union of all 1st order domains, if \((a(A))(w)\) does not belong to the 2nd order domain indexed by the world \(w\); and \((a(-A))(w)\) is the complement of \((a(A))(w)\) with respect to the 1st order domain indexed by the world \(w\), if \((a(A))(w)\) belongs to this domain. And finally, the component \(G\) of model structures is a distinguished property, intended to denote the God-likeness property, which is a member of every domain of the 2nd sort.

One can of course state that these semantic choices are not incidental, but completely motivated (cf. also [3] and [4]). The point left undiscussed here is that 1st order terms receive rigid extensions in world-varying objectual domains. We will only attend to choices concerning the treatment of 2nd order terms and of 2nd order domains of quantification. And so, if terms of the 2nd order received only rigid extensions in world-varying objectual domains of the 2nd order, all formulas of the form \(L\phi \leftrightarrow \phi\) would be verified, trivializing modal operators. One way to avoid this difficulty could be the requirement that 2nd order terms receive world-relative extensions in the union of all 2nd order domains. Unfortunately, a serious problem of this approach would be that the 2nd order universal specification would be valid in such model structures, in direct conflict with our intentions. This principle would also be preserved if 2nd order terms received world-relative extensions in 2nd order domains corresponding to particular worlds and quantifications of 2nd order variables at a world would be over the 2nd order domain of that world. There is still the possibility that 2nd order terms would receive world-relative extensions in the union of all 2nd order domains, while quantifications of 2nd order variables at a world would be over the 2nd order domain of that world. As a result, the 2nd order uni-
universal specification would be no longer valid, but the new difficulty with this proposal is that the principle of exportation of the 2\textsuperscript{nd} sort, which we adopt as an axiom schema, can easily be falsified. And it should be noted that if 2\textsuperscript{nd} order terms received intensions in the set of all functions from the set of worlds to the union of all 2\textsuperscript{nd} order domains and this set would be interpreted as a 2\textsuperscript{nd} domain of quantification, then the 2\textsuperscript{nd} order universal specification would be valid in this semantics. It would be also valid if intensions of 2\textsuperscript{nd} order terms were members of the set of all functions from the set of worlds to the union of all 2\textsuperscript{nd} order domains such that values of these functions at a given world belong to that world and this set would be the 2\textsuperscript{nd} domain of quantification. We believe that our solution is the best – if not the only – response to all these difficulties.

2 Anderson-like Theories, Viewed Syntactically

The formal language $\mathcal{L}$ of fully free Anderson-like theories is equipped with a 2\textsuperscript{nd} order unary predicate $P$, an existence determinator $E$, a necessity operator $L$, two sorts of variables: $x, y, z, \ldots$ (1\textsuperscript{st} order), $\alpha, \beta, \gamma, \ldots$ (2\textsuperscript{nd} order), Boolean operator: $\neg$ (complementation), logical symbols: $\land, \neg$ (conjunction, negation) and $\forall$ (universal quantifier) for both sorts of variables. Terms of the 1\textsuperscript{st} sort are variables of the 1\textsuperscript{st} sort and terms of the 2\textsuperscript{nd} sort are formed from variables of the 2\textsuperscript{nd} sort by applying complementation any finite (possibly zero) numbers of times. Thus, the set of terms of the 2\textsuperscript{nd} sort and the set of formulas are given by the grammar, respectively:

$$A \xrightarrow{df} \alpha \mid -A$$

$$\phi \xrightarrow{df} A(x) \mid E(x) \mid E(A) \mid P(A) \mid \phi \land \psi \mid \neg \phi \mid L \phi \mid \forall x \phi \mid \forall \alpha \phi.$$ 

The remaining propositional connectives: $\lor, \rightarrow, \leftrightarrow$ as well as the strict implication $\prec$, the existential quantifier $\exists$, the possibility operator $M$, the identity $\overset{1}{\approx}$, the inequality $\not\overset{1}{\approx}$ for terms of the 1\textsuperscript{st} sort and the existence determinator $E$ of the 2\textsuperscript{nd} order are introduced as follows:

$$\phi \lor \psi \xrightarrow{df} \neg(\neg \phi \land \neg \psi), \quad \phi \rightarrow \psi \xrightarrow{df} \neg \phi \lor \psi, \quad \phi \leftrightarrow \psi \xrightarrow{df} (\phi \rightarrow \psi)$$

$$\land(\psi \rightarrow \phi), \quad \phi \prec \psi \xrightarrow{df} L(\phi \rightarrow \psi), \quad \exists \xi \phi \xrightarrow{df} \neg \forall \xi \neg \phi \quad \text{where } \xi \text{ is a variable of any sort,} \quad M \phi \xrightarrow{df} \neg L \neg \phi, \quad (x \overset{1}{\approx} y) \xrightarrow{df} \forall \alpha(\alpha(x) \leftrightarrow \alpha(y))$$
and \((x \not\approx y) \overset{\text{df}}{=} \neg(x \approx y)\).

According to this grammar, although logical forms of expressions \(E(x)\) and \(E(A)\) might suggest ascribing the status of the 1\(^{\text{st}}\) and 2\(^{\text{nd}}\) sort predicate to the symbol \(E\) in \(E(x)\) and \(E(A)\), respectively, they are crucially different from predicates in the way they work. Because, if predicates of the 1\(^{\text{st}}\) order are understood to be properties of individuals and predicates of the 2\(^{\text{nd}}\) order - to be properties of properties, then in our treatment - \(E\) is neither a property of individuals, nor of properties, nor of anything. And so, for example, \(\forall \alpha \phi \rightarrow \phi(\alpha/E)\), \(P(E)\) and \(E(E)\), where the symbol \(E\) inside the parentheses is an existence determinator of the 1\(^{\text{st}}\) order, and the symbol \(E\) outside the parentheses is an existence determinator of the 2\(^{\text{nd}}\) order, are not formulas of our language.

Interestingly enough, even those who give \(E\) the name: \emph{existence predicate}, don’t always treat it as a predicate (cf. K. Lambert [9], pp. 159 - 160). But, even more motivation for treating the symbol \(E\) in \(E(x)\) and \(E(A)\) in the way dissimilar to predicates can be induced after introducing the semantical machinery, which is done in the next section.

Further three definitions to be assumed in fully free Anderson-like theories are borrowed from Anderson [2]:

\[
G(x) \overset{\text{df}}{=} \forall \alpha (P(\alpha) \leftrightarrow L\alpha(x)) \tag{2.1}
\]

\(G(x)\) is read: \textit{x is God-like} or simply \textit{x is a God}

\[
A \text{Ess } x \overset{\text{df}}{=} \forall \beta (L\beta(x) \leftrightarrow L\forall y(A(y) \rightarrow \beta(y))) \tag{2.2}
\]

\(A \text{Ess } x\) is read: \textit{a property A is an essence of entity x}, where A is a term of the 2\(^{\text{nd}}\) sort

\[
NE(x) \overset{\text{df}}{=} \forall \alpha (A \text{Ess } x \rightarrow L\exists y\alpha(y)) \tag{2.3}
\]

\(NE(x)\) is read: \textit{x necessarily exists}.\(^1\)

To increase readability, we will occasionally use symbols: \(\forall, \exists, \iff, \implies, \iff\) for quantifiers and propositional connectives of metalanguage.

The \textit{axioms} and the \textit{axiom schemas} of fully free Anderson-like theories are divided into two groups:

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\(^1\)For comparison, Gödel’s definitions of \emph{God-like being} and \emph{essence} are, respectively, as follows: \(G(x) \overset{\text{df}}{=} \forall \alpha (P(\alpha) \rightarrow \alpha(x))\) and \(A \text{Ess } x \overset{\text{df}}{=} \forall \beta (\beta(x) \rightarrow L\forall y(A(y) \rightarrow \beta(y)))\); while both authors adopt the same definition of \emph{necessary existence}.
(I). *Obligatory* axioms and axiom schemas (we assume that $\phi, \psi$ are formulas and $\xi$ is a variable of any sort):

*All that is needed for classical propositional logic*,

$$\forall \xi (\phi \rightarrow \psi) \rightarrow (\forall \xi \phi \rightarrow \forall \xi \psi) \quad (2.4)$$

**Informally:** Universal quantifiers of the $1^{st}$ and $2^{nd}$ order are distributive,

$$\phi \leftrightarrow \forall \xi \phi \text{ if } \xi \text{ is not free in } \phi \quad (2.6)$$

**Informally:** Universal quantifiers of the $1^{st}$ and $2^{nd}$ order are vacuous, if they bind variables which are not free,

$$\forall \xi \phi \land E(A) \rightarrow \phi(\xi/A) \text{ if } A \text{ is a term of the same sort as } \xi \quad (2.7)$$

**Informally:** If every object (property) of the domain of quantification is so and so and $A$ is a member of this domain, then $A$ is so and so,

$$\forall \xi E(\xi) \quad (2.8)$$

**Informally:** Every object (property) of the domain of quantification is its member,

$$\forall \alpha (\alpha(x) \rightarrow E(x)) \quad (2.9)$$

**Informally:** For every property $\alpha$ of the $2^{nd}$ order domain of quantification related to a world $w$, if an object $x$ has the property $\alpha$ then $x$ is an element of the $1^{st}$ order domain of quantification related to $w$,

$$E(A) \leftrightarrow E(\neg A) \quad (2.10)$$

**Informally:** A property $A$ is a member of the domain of quantification iff the complementation of $A$ is a member of this domain,

$$M E(A) \rightarrow E(A) \quad (2.11)$$

**Informally:** If it is possible that a property $A$ is a member of the domain of quantification, then it indeed is a member of this domain,

$$L(\phi \rightarrow \psi) \rightarrow (L\phi \rightarrow L\psi) \quad (2.12)$$

**Informally:** The necessity operator is distributive,

$$L\phi \rightarrow M\phi \quad (2.13)$$

**Informally:** What is necessary is also possible (or, Necessity implies possibility),

$$\forall \alpha L\phi \rightarrow L\forall \alpha \phi \quad \text{(Barcan formula of the } 2^{nd} \text{ sort)} \quad (2.14)$$
Informally: If every property is necessarily such and such, then necessarily every property is such and such,
\( \forall \alpha \phi \rightarrow \forall \alpha L \phi \) (converse Barcan formula of the 2\(^{nd}\) sort) \hspace{1cm} (2 .15)

Informally: If necessarily every property is such and such, then every property is necessarily such and such,
\( \exists \alpha \phi \rightarrow \exists \alpha L \phi \) (principle of exportation of the 2\(^{nd}\) sort) \hspace{1cm} (2 .16)

Informally: If necessarily there exists a property which is such and such, then there exists a property which necessarily is such and such,
\( \exists \alpha \phi \rightarrow L \exists \alpha \phi \) (principle of converse exportation of the 2\(^{nd}\) sort) \hspace{1cm} (2 .17)

Informally: For every property \( \alpha \): if an object \( x \) has the property \( \alpha \), then there exists an object with this property,
\( \forall x \exists \alpha \alpha(x) \) \hspace{1cm} (2 .18)

Informally: Every object has some property,
\( (x \approx y) \rightarrow (\phi(z/x) \rightarrow \phi(z/y)) \) where \( z \) does not occur within the scope of a modal operator, and \( x, y \) are free for \( z \) in \( \phi \) \hspace{1cm} (2 .20)

Informally: Identical elements are substitutable “salva veritate”,
\( \forall \alpha, \beta (\forall x (\alpha(x) \leftrightarrow \beta(x)) \rightarrow (\phi(\gamma/\alpha) \leftrightarrow \phi(\gamma/\beta))) \) where \( \gamma \) does not occur within the scope of a modal operator, and \( \alpha, \beta \) are free for \( \gamma \) in \( \phi \) \hspace{1cm} (2 .21)

Informally: For any two properties: if they have the same elements, then they are substitutable “salva veritate”,
\( G(x) \rightarrow (L(x \approx y) \rightarrow (\phi(z/x) \leftrightarrow \phi(z/y))) \) where \( x \) and \( y \) are free for \( z \) in \( \phi \) \hspace{1cm} (2 .22)

Informally: If an object is God-like, then all what concerns it concerns also an object which is necessarily identical with it,
\( G(x) \rightarrow ((x \neq y) \rightarrow L \exists \alpha (\alpha(x) \land \neg \alpha(y))) \hspace{1cm} (2 .23) 

Informally: If an object is God-like, then necessarily there exists a property which separates it from another object under the proviso
that this both objects are unequal,
\[ \forall \alpha (P(\alpha) \rightarrow \neg P(\neg \alpha)) \]  
(2.24)

Informally: For every positive property, its complement is not positive,
\[ \forall \alpha, \beta (P(\alpha) \land \forall x (\alpha(x) \rightarrow \beta(x)) \rightarrow P(\beta)) \]  
(2.25)

Informally: Every property that is entailed by any positive property is, itself, positive,
\[ P(A) \rightarrow E(A), \]  
(2.26)

Informally: Every positive property is a member of the domain of quantification,
\[ LG(x) \rightarrow G(x) \]  
(2.27)

Informally: If necessarily an object has the property of Godlikeness, then it indeed is God-like.

Every Anderson-like theory must be equipped with an axiom saying that the property of being God-like is positive and therefore it must be legitimate to treat the property of being God-like as a term of the 2\textsuperscript{nd} sort. Thus, the following two axioms are obligatory for all Anderson-like theories:

\[ \exists \alpha (\alpha \approx G) \text{ or shortly: } E(G), \]  
(2.28)

Informally: The Godlikeness is a member of the 2\textsuperscript{nd} order domain of quantification,

\[ \exists \beta (P(\beta) \land (\beta \approx G)) \text{ or shortly: } P(G) \]  
(2.29)

Informally: The property of Godlikeness is a positive property,

where, of course, the symbol \(\approx\) stands for the relation of identity of objects of the 2\textsuperscript{nd} sort i.e. properties.

However, the relation \(\approx\) can be introduced in Anderson-like theories in two different ways by the following optional definitions:

\[ A \approx B \overset{\text{df}}{=} \forall x(A(x) \leftrightarrow B(x)) \]  
(2.30)

Informally: Two properties are identical iff they have the same elements,
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\[ A \overset{2}{\approx} B \overset{df}{=} \forall x (A(x) \leftrightarrow B(x)) \quad (2.31) \]

**Informally**: Two properties are identical iff necessarily they have the same elements,

and it is clear that the translation of (2.28) and (2.29) to the original language depends on which optional definition of \( \overset{2}{\approx} \) has been applied. And it is the case that one of (2.30) and (2.31) must be adopted in every Anderson-like theory.

Moreover, the choice of definition of \( \overset{2}{\approx} \) affects another obligatory axioms of Anderson-like theories. Those axioms take the form:

\[ (-\alpha)(x) \rightarrow -\alpha(x) \quad (2.32) \]

**Informally**: If an object has a property of being the complement of a property, then it does not have the property,

\[ \forall \alpha \forall x (-\alpha(x) \rightarrow (-\alpha)(x)) \quad (2.33) \]

**Informally**: For every property \( \alpha \) of the 2\textsuperscript{nd} order domain of quantification and every object \( x \) of the 1\textsuperscript{st} order domain of quantification, both related to the same world: if \( x \) does not have the property \( \alpha \), then \( x \) has the property of being the complement of \( \alpha \),

\[ G(x) \rightarrow ((x \overset{1}{\approx} y) \rightarrow L(x \overset{1}{\approx} y)) \quad (2.34) \]

**Informally**: If an object is God-like, then any object identical with it is necessarily identical

or

\[ L((-\alpha)(x) \rightarrow -\alpha(x)) \quad (2.35) \]

**Informally**: It is necessary: if an object has a property of being the complement of a property, then it does not have the property,

\[ L\forall \alpha \forall x (-\alpha(x) \rightarrow (-\alpha)(x)) \quad (2.36) \]

**Informally**: It is necessary that for every property \( \alpha \) of the 2\textsuperscript{nd} order domain of quantification and every object \( x \) of the 1\textsuperscript{st} order domain of quantification, both related to the same world, the following holds: if \( x \) does not have the property \( \alpha \), then \( x \) has the
property of being the complement of \( \alpha \),
\[
G(x) \rightarrow L((x \approx y) \rightarrow L(x \approx y)) \quad (2 \cdot 37)
\]
Informally: If an object is God-like, then necessarily, any object identical with it is necessarily identical,

depending on which one of (2.30), (2.31) has been adopted.

What is important is to note that neither of two optional definitions of \( \approx \) provides what one might have expected of an identity relation. Indeed, the formula: \((\alpha \approx \beta) \land E(x) \rightarrow (\alpha(x) \rightarrow \beta(x))\) is unprovable on the basis of the definition (2.31). However, it can be proved if (2.30) is applied. On the other hand, the formula: \((\alpha \approx \beta) \rightarrow (P(\alpha) \rightarrow P(\beta))\) is unprovable on the basis of (2.30) but it can be proved if (2.31) is applied. Both formulas are some forms of the principle of the indiscernibility of identical properties. Thus, the choice between the two versions of the identity between properties should not be seen as a technical exercise, but it plays a pervasive role in philosophical thinking. The difference between these two notions of identity is brought out by the difference between the two non-equivalent groups of axioms: (2.32), (2.33) and (2.34) on the one hand, and (2.35), (2.36) and (2.37) on the other. The second group of axioms necessitate axioms of the first group, respectively. As we shall see, every Anderson-like theory is closed with respect to the inference rule of necessitation, so given the axioms of the first group, the axioms of the second group can be easily deduced.

The status of particular axioms is also worth commenting on here. And so, in comparison with our previous paper [15], instead of the axioms \(A(x) \rightarrow E(x)\) and \(\alpha(x) \rightarrow \exists y \alpha(y)\) we have now \(\forall \alpha(\alpha(x) \rightarrow E(x))\) (see, (2.9)) and \(\forall \alpha(\alpha(x) \rightarrow \exists y \alpha(y))\) (see, (2.18)), respectively. But, what about the significance of these axioms? By means of these axioms, the formula \(\forall \alpha(\exists x L \alpha(x) \rightarrow L \exists x \alpha(x))\) and the much significant - from the "ontological proof" point of view - formulas \(\forall x NE(x)\) and \(P(NE)\) (see, the proofs of the theorems T14 and T18 in Appendix) are provable. In Section 4 we will give examples of model structures demonstrating that the formulas \(\alpha(x) \rightarrow \exists y \alpha(y)\) and \(\exists x L \alpha(x) \rightarrow L \exists x \alpha(x)\) are not theorems of any fully free Anderson-like ontological proofs – although the subcase \(\exists x LG(x) \rightarrow L \exists x G(x)\) of the last formula is provable in any fully free Anderson-like theory and in all partly free Anderson-like theories introduced in [15].
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The special role of the formula $\exists x L(G(x) \rightarrow L \exists x G(x))$ as well as of the formulas $L(G(x) \rightarrow G(x)$ (see, (2.27)) and $L \exists x G(x) \rightarrow \exists x L(G(x))$, which is provable in all fully free Anderson-like theories, was already emphasized in [15]. Notice that the general principles $\forall \alpha (L \exists x \alpha(x) \rightarrow \exists x L \alpha(x))$ and $L \exists x \alpha(x) \rightarrow \exists x L \alpha(x)$ are not true in semantics, which will be introduced in the next section; see, the footnote 5 in the subsequent part of the paper. Further, the need for introduction of the axiom $\forall \alpha \exists \alpha(x)$ (see, (2.19)) to theories is directly proportional to the presence or absence of the principle saying that every object has the property of being self-identical in the theories. More precisely, (2.19) is easily provable by means of this principle and the axiom schema (2.7).

The status of the axiom $G(x) \rightarrow ((x \not\approx y) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ (see, (2.23)) is also worth thinking about in this passage. The reader may wonder at the use of this axiom. It might seem a more natural choice to use either the formula $G(x) \rightarrow ((x \not\approx y) \rightarrow L(x \not\approx y))$ or, as we have done in [13] and [15], $G(x) \rightarrow (\exists \alpha(\alpha(x) \land \neg \alpha(y))) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ as an axiom. To answer, let us note that both formulas $G(x) \rightarrow ((x \not\approx y) \rightarrow L(x \not\approx y))$ and $G(x) \rightarrow (\exists \alpha(\alpha(x) \land \neg \alpha(y))) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ are easily derivable from $G(x) \rightarrow ((x \not\approx y) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ in the context of other axioms of fully free Anderson-like theories, and the converse deducibility does not hold. On the other hand, it would be difficult to find model structures for fully free Anderson-like theories in which the formula $G(x) \rightarrow ((x \not\approx y) \rightarrow L(x \not\approx y))$ would be valid and both formulas $G(x) \rightarrow (\exists \alpha(\alpha(x) \land \neg \alpha(y))) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ and $G(x) \rightarrow ((x \not\approx y) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ would be invalid (or, both formulas $G(x) \rightarrow ((x \not\approx y) \rightarrow L(x \not\approx y))$ and $G(x) \rightarrow (\exists \alpha(\alpha(x) \land \neg \alpha(y))) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ would be valid and the formula $G(x) \rightarrow ((x \not\approx y) \rightarrow L \exists \alpha(\alpha(x) \land \neg \alpha(y)))$ would be invalid).

It should be clear why the axiom $\forall \alpha \forall x (\neg \alpha(x) \rightarrow (\neg \alpha)(x))$ (see, (2.33)) is quantified while $(\neg \alpha)(x) \rightarrow \neg \alpha(x)$ (see, (2.32)) is not. The justification of this uneven treatment lies in our semantics. In the next section we shall present semantics in which the formulas $\forall x (\neg \alpha(x) \rightarrow (\neg \alpha)(x))$ and $\forall \alpha (\neg \alpha(x) \rightarrow (\neg \alpha)(x))$ are not true; see, the footnote 6 in the subsequent part of the paper. There is a conflict about what we need to insure the
truthfulness of the formulas \( \neg \alpha(x) \rightarrow (-\alpha)(x) \), \( \forall x(\neg \alpha(x) \rightarrow (-\alpha)(x)) \) and \( \forall \alpha(\neg \alpha(x) \rightarrow (-\alpha)(x)) \) and what we need to ensure the truthfulness of other axioms. Similar remarks may be made about \( L\forall \alpha \forall x(\neg \alpha(x) \rightarrow (-\alpha)(x)) \) (see, (2.36)) and \( L((\neg \alpha)(x) \rightarrow \neg \alpha(x)) \) (see, (2.35)).

The presence of the axiom \( G(x) \rightarrow ((x \approx y) \rightarrow L(x \approx y)) \) (see, (2.34)) or \( G(x) \rightarrow L((x \approx y) \rightarrow L(x \approx y)) \) (see, (2.37)) in every Anderson-like theory on the one hand, and the absence of the formulas \( (x \approx y) \rightarrow L(x \approx y) \) and \( L((x \approx y) \rightarrow L(x \approx y)) \) on the other hand, say that there are two sorts of identity between objects: necessary identity and contingent identity. More precisely, an identity is necessary if it cannot be avoided, and a contingent identity just happens to be the case. Thus, any object which is identical with a God-like object is necessarily identical with it, and by contrast, the identity between other objects does not imply that they are necessarily identical.

Throughout this paper, we will consider different fully free Anderson-like theories, which will be denoted by appropriate acronyms. The first symbol of each acronym will be \( \mathcal{V}^A \). Any Anderson-like theory employing the definition (2.31) will be given an acronym ending with the symbol \( \star \) and thus, the theories employing (2.30) can be easily recognized by their \( \star \)-less acronyms.

(II). Optional axioms of fully free Anderson-like theories are chosen according to the following criteria:

(i) treatment of the property of necessary existence,

(ii) treatment of properties abstracted from expressions of the form \( I_x(y) \) defined by:

\[
I_x(y) \overset{df}{=} (x \approx y),
\]

(2.38)

(iii) characterization of modal operators,

(iv) treatment of the so-called permanence.

As to (i), if we intend to treat the property of necessary existence as a term of the 2\(^{nd}\) sort we should adopt the optional axiom:

\[
\exists \alpha(\alpha \approx^2 \text{NE}) \text{ or shortly: } E(\text{NE}),
\]

(2.39)

Informally: The necessary existence is a member of the 2\(^{nd}\) order domain of quantification,
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and augment the acronym of theory with the symbol n.

As to (ii), if we intend to treat expressions of the form $l_x(y)$ as terms of the 2$_{nd}$ sort we should adopt the following optional axiom:

$$\exists \alpha (\alpha \equiv l_x) \text{ or shortly: } E(l_x),$$

2.40

Informally: $l_x$ is a member of the 2$_{nd}$ order domain of quantification,

and augment the acronym of theory with the symbol s.

Once again we wish to emphasize that the form of optional axioms (2.39) and (2.40) depends on which definition ((2.30) or (2.31)) of the relation $\equiv$ has been chosen. And it is easy to prove, applying (2.7) and (2.40), that the axiom (2.19) is dependent on the remaining ones in all Anderson-like theories with the acronym s.

As to (iii), we choose one of the following:

5) $\text{ML} \phi \rightarrow \text{L} \phi$

Informally: If it is possible that $\phi$ necessarily holds, then $\phi$ necessarily holds,

(b) $\text{ML} \phi \rightarrow \phi$

Informally: If it is possible that $\phi$ necessarily holds, then $\phi$ holds,

(c) $\text{ML} \phi \rightarrow \text{L} \phi$, $\text{ML} \phi \rightarrow \phi$ and $P(\alpha) \rightarrow \text{LP}(\alpha)$

$P(\alpha) \rightarrow \text{LP}(\alpha)$ is read: If a property is positive, then it is necessarily positive,

(d) $\text{ML} \phi \rightarrow \text{L} \phi$, $\text{L} \phi \rightarrow \text{LL} \phi$ and $P(\alpha) \rightarrow \text{LP}(\alpha)$

$\text{L} \phi \rightarrow \text{LL} \phi$ is read: If $\phi$ necessarily holds, it is necessary that $\phi$ necessarily holds,

and augment the acronym of theory by symbol 5, b, c or d indicating the choice that has been made.2

The status of the axiom schema $\text{ML} \phi \rightarrow \text{L} \phi$ is worth stressing here. Its presence allows - by using definition: (2.1), axiom schemas or axioms:

---

2Anderson’s ontological proof is based on the propositional modal logic S5. However, its variants proposed by P. Hájek in [6] and [7] and by us in [14] were already grounded on propositional modal logics weaker than S5.

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(2.7), (2.12), (2.13), (2.28) and (2.29), and inference rules: R1 and R2 - to prove the ontological formula \( \text{LG}(x) \rightarrow G(x) \), which functions in all Anderson-like theories either as an independent axiom or as a provable formula. The question naturally arises why this axiom, as a particular case of the axiom schema \( L\phi \rightarrow \phi \), is restricted only to \( G \), while the weaker axiom schema \( L\phi \rightarrow M\phi \) applies more generally. There is no general philosophical answer to this question; the reason for this discrimination is rather technical - the axiom schema \( L\phi \rightarrow M\phi \) is indispensable for proving the necessary existence of God while the axiom schema \( L\phi \rightarrow \phi \) is not such an indispensable constituent.

As to (iv), we simply add the axiom schema

\[ \forall x \text{LE}(x) \tag{2.41} \]

Informally: Every object of the domain of quantification is necessarily its element,

and augment the acronym of the theory with the symbol \( p \).

Each fully free Anderson-like theory has the inference rule: modus ponens, necessitation and generalization, respectively:

\[
\begin{align*}
R1: & \quad \frac{\phi, \phi \rightarrow \psi}{\psi} \\
R2: & \quad \frac{\phi}{L\phi} \\
R3: & \quad \frac{\phi}{\forall \xi \phi}
\end{align*}
\]

and the following ones (resembling in their form the corresponding Thomason’s rules R4 - R7 introduced in [16]):

\[
\begin{align*}
R4_0 & : \quad \frac{\phi \rightarrow L\chi}{\phi \rightarrow L\forall x\chi} \\
R4_n & : \quad \frac{\phi \rightarrow .\psi_1 < \ldots < .\psi_n < L\chi}{\phi \rightarrow .\psi_1 < \ldots < .\psi_n < L\forall x\chi}
\end{align*}
\]

where \( x \) is not free in \( \phi \),

\[
\begin{align*}
R5_0 & : \quad \frac{\phi \rightarrow (x \not\approx y)}{\neg \phi}
\end{align*}
\]

where \( x \) is not free in \( \phi \),

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\[
R5_n : \phi \rightarrow .\psi_1 \prec \ldots \prec .\psi_n < (x \nless y) \\
\phi \rightarrow .\psi_1 \prec \ldots \prec .\psi_n-1 \prec L\neg \psi_n
\]

where \( x \) is not free in \( \phi, \psi_1, \ldots, \) or \( \psi_n, n > 0 \).

By \( \vdash_{\text{Th}} \) we denote the inference relation determined by axioms and rules of the fully free Anderson-like theory \( \text{Th} \). Thus, for a set of formulas \( X \) and a formula \( \phi \) we write: \( X \vdash_{\text{Th}} \phi \) to mean that there exists a \( \text{Th} \)-

In [16] R. H. Thomason proved the strong completeness theorem for the 1st order free modal logic \( \text{Q3} \) (using his nomenclature) with identity and definite descriptions. \( \text{Q3} \) is determined except modus ponens, necessitation and generalization (precisely, the inference rule deductively equivalent with generalization) additionally by \( R4 - R7 \). The inference rules \( R4 - R7 \) differ from \( R4_0, R4_n, R5_0 \) and \( R5_n \), \( n > 0 \), only in the use of term \( t \), which can be variable, constant or definite description, in place of the variable \( y \) as in \( R5_0 \) and \( R5_n \), \( n > 0 \). As Thomason stated, the inference rules \( R4 - R7 \) are needed for the proof of semantic completeness of \( \text{Q3} \), but he did not known whether they are redundant or not.

It is not difficult to prove that in Anderson-like theories with the axiom (b) the inference rules \( R4_0 \) and \( R4_n, n > 0 \), are redundant. The question is to decide whether these inference rules are dependent or independent in Anderson-like theories without the axiom (b). The solution to this problem is far from being trivial. The situation does not look any better, when the status of the inference rules \( R5_0 \) and \( R5_n, n > 0 \), in Anderson-like theories (with or without the axiom (b)) is considered. We do not investigate these questions, because we have no space for them here.

For the reader interested in the dependence or independence of the inference rules \( R4_0, R4_n \) and \( R5_0, R5_n, n > 0 \), it should be helpful what Parsons in [12] says:

We show that \( R4 \) is independent in \( T \) or \( S4 \) given the free variables axiom \( \text{ME}(x) \rightarrow \text{E}(x) \), even given \( R6 \) and \( R7 \). \( R6 \) is easily seen independent even in \( S5 \) with \( R4 \) and \( R5 \), given the free variables axiom \( \text{E}(x) \) (in neither case is substitution of terms for free variables allowed, but generalization and necessitation are allowed).

On the other hand, we show that \( R4 - R7 \) are redundant in purely logical proofs and a fortiori in proofs from closed axioms. To show this we set up a system \( \text{Q3}^* \) for cut-free deductions of finite sets of formulae, for which cut is admissible. \( \text{Q3}^* \) is shown complete with respect to validity in the above semantics. It can then be shown that Thomason’s \( \text{Q3} \) is complete in this sense without \( R4 - R7 \). \( \text{Q3}^* \) does contain a negative \( \forall \)-introduction rule: from \( \Gamma \cup \{ \text{E}(t) \} \) and \( \Gamma \cup \{ y \neq t, \neg \phi(y/x) \} \) we can infer \( \Gamma \cup \{ \neg \forall x \phi \} \) (\( y \) not free in \( t \) or conclusion) whose reduction to a more usual kind of such rule requires \( R6 \). This application of \( R6 \) is reducible to cut.
derivation of $\phi$ from $X$. Such a derivation is a finite sequence of formulas (derivation steps) each of which has to be justified in an appropriate manner. Each step of derivation is therefore required to be an axiom of $\text{Th}$ or an element of $X$ or a result of applying an inference rule to the preceding step (or steps). Moreover, applying inference rules is subject to the following important restriction:\footnote{Some comments on this restriction may be found in our [13] and [15].}

rules other than $R1$ are applicable only to steps which are obtained without using elements of $X$.

The results below will find many applications in the paper, but their proofs are left to the reader as an exercise.

**Proposition 2.1** For any fully free Anderson-like theory $\text{Th}$:

(i) $X \cup \{\phi\} \vdash_\text{Th} \psi$ iff $X \vdash_\text{Th} \phi \rightarrow \psi$,

(ii) if $X \vdash_\text{Th} \phi$ then $\{L\psi : \psi \in X\} \vdash_\text{Th} L\phi$,

(iii) if $X \cup \{\phi(\xi/\zeta)\} \vdash_\text{Th} \psi$ and $\zeta$ is a variable not occurring in $\psi$ or in any member of $X$, then $X \cup \{\exists \xi \phi\} \vdash_\text{Th} \psi$.

3 Anderson-like Theories, Viewed Semantically

By a fully free model structure we mean a quintuple of the form $\mathfrak{W} = \langle W, R, \mathcal{D}_1, \mathcal{D}_2, G \rangle$ where $W \neq \emptyset$ is the set of possible worlds, $R \subseteq W^2$ is the relation of accessibility, $\mathcal{D}_1$ is the family $(D_w)_{w \in W}$ of the $1^{st}$ sort domains - members of $\bigcup_{w \in W} D_w$ are called existing objects -, $\mathcal{D}_2$ is the family $(D_w)_{w \in W}$ of the $2^{nd}$ sort domains $D_w \subseteq 2^{D_w}$, $w \in W$ - members of $\bigcup_{w \in W} D_w$ are called existing properties. Apart from existing properties we also consider so called conceptual properties of the structure, by which we mean functions $f \in W \mapsto 2^{\bigcup_{w \in W} D_w}$ satisfying the following condition: if $f(w) \in D_w$ for some $w \in W$, then $f(w) \in D_w$ for all $w \in W$. The set of all conceptual properties of the structure $\mathfrak{W}$ will be denoted by $C_\mathfrak{W}$. The additional conditions necessary for fully free model structures are:
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\[ \emptyset \neq G \subseteq \bigcap_{w \in W} D_w \quad \text{and} \quad G \in \bigcap_{w \in W} D_w \]  

Informally: The distinguished non-empty set \( G \) is a subset of every domain of the 1\textsuperscript{st} sort, and, respectively,

Informally: \( G \) is a member of every domain of the 2\textsuperscript{nd} sort,

\[ \forall w \in W \; \forall a, b \in G \; \forall X \in D_w (a \in X \iff b \in X) \]  

Informally: No property of any 2\textsuperscript{nd} sort domain separates elements of the set \( G \),

\[ \forall w \in W \; (X \in D_w \implies D_w - X \in D_w) \]  

\( R \) is serial, i.e. \( \forall w \in W \; \exists v \in W \; w R v. \)

By an assignment in a fully free model structure \( \mathcal{W} \) we mean a function \( a \) which maps variables of the 1\textsuperscript{st} sort to members of \( \bigcup_{w \in W} D_w \) and variables of the 2\textsuperscript{nd} sort to conceptual properties of the structure (i.e. members of \( C_\mathcal{W} \)). An assignment \( a \) is extended to all terms \( A \) by putting:

\[ (a(-A))(w) = \begin{cases} 
\bigcup_{w \in W} D_w - (a(A))(w) & \text{if } (a(A))(w) \notin D_w, \\
D_w - (a(A))(w) & \text{if } (a(A))(w) \in D_w,
\end{cases} \]

for every \( w \in W \).

We introduce a special notation for the set of all functions \( f \in C_\mathcal{W} \) such that \( f(w) \in D_w \) for every \( w \in W \), namely, \( C^\Box_\mathcal{W} \).

If \( a \) is an assignment, then the symbol \( a_\xi^o \) denotes the assignment defined by:

\[ a_\xi^o(\zeta) \overset{df}{=} \begin{cases} 
o & \text{if } \zeta = \xi, \\
a(\zeta) & \text{if } \zeta \neq \xi.
\end{cases} \]

Of course, \( o \) is tacitly assumed to be an entity suitable for the variable \( \xi \) depending on its sort and both \( a \) and \( a_\xi^o \) are assumed to be assignments in the same structure. We say that assignments \( a, b \) agree apart from \( \xi \) (symbolically: \( a \equiv^?_\xi b \)) if for some \( o \), \( a_\xi^o = b \). Note that \( \equiv^?_\xi \) is an equivalence relation on the set of all assignments of a strong free model structure. The equivalence class of \( a \) with respect to \( \equiv^?_\xi \) will be further denoted by \( \{a_\xi^o\} \).

And for every \( w \in W \), \( \{a_\xi^o, w\} \) will be the equivalence subclass of \( \{a_\xi^o\} \) defined as follows:
(i) for every 1st sort variable $x$ and every $w \in W$, $\{a^x_{x,w} \} = \{b \mid b \in \{a^x_x\}$ and $b(x) \in D_w\}$.

(ii) for every 2nd sort variable $\alpha$ and every $w \in W$, $\{a^\alpha_{\alpha,w} \} = \{b \mid b \in \{a^\alpha_\alpha\}$ and $b(\alpha) \in C_{\neg \neg W}^\alpha\}.$

A pair of the form $\langle W, a \rangle$ will be called fully free model and the symbol $\models$ will be used for the satisfiability relation – the expression $W, a \models \phi$ reads: the formula $\phi$ is satisfied by the strong free model $\langle W, a \rangle$; and the expression $W, a, w \models \phi$, where $w \in W$ reads: the formula $\phi$ is satisfied in the world $w$ of the strong free model $\langle W, a \rangle$. If no misunderstanding is likely as to the particular structure $W$ in which an assignment $a$ has been chosen, we simplify the notation by writing: $a \models \phi$ and $a, w \models \phi$ instead of $W, a \models \phi$ and $W, a, w \models \phi$, respectively. Given a strong free model $\langle W, a \rangle$, $a, w \models \phi$ is defined as usual, for any possible world $w \in W$ by the following conditions, where $x$ is a variable of the 1st sort, $A$ is a term of the 2nd sort, $\xi$ is a variable of arbitrary sort and $\phi, \psi$ are the formulas:

(i) $a, w \models E(x)$ iff $a(x) \in D_w$;

(ii) $a, w \models E(A)$ iff $a(A) \in C_{\neg \neg W}^\alpha$;

(iii) $a, w \models A(x)$ iff $a(x) \in (a(A))(w)$;

(iv) $a, w \models \phi \land \psi$ iff $a, w \models \phi$ and $a, w \models \psi$;

(v) $a, w \models \neg \phi$ iff not $a, w \models \phi$ (symbolically: $a, w \not\models \phi$);

(vi) $a, w \models \forall \xi \phi$ iff $b, w \models \phi$ for every $b \in \{a^\xi_{\xi,w}\}$;

(vii) $a, w \models L\phi$ iff $a, v \models \phi$ for every $v \in R(w)$;

(viii) $a, w \models P(A)$ iff $a(A) \in C_{\neg \neg W}^\alpha$ and $G \subseteq (a(A))(v)$ for every $v \in W$ such that $wRv$.

In this place, we can clarify the semantic status of the non-empty set $G$ in the definition of a fully free model structure. Although this set was chosen to represent the property of the God-being, the clause: $a, w \models G(x)$ iff $a(x) \in (a(G))(w)(= G)$ was not introduced, which could possibly have been expected. The reason for such logical perspective is that the predicate $G$ is not a primitive term of the language. The significance of the set $G$ manifests itself in the clause (viii) of the definition of satisfiability relation.

It may be instructive to compare how formulas of the kinds: $E(x)$ and $A(x)$ are valued in our model structures, see, the clauses (i) and (iii),
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respectively. And so, a formula $E(x)$ is satisfied under an assignment $a$ at a world $w$ iff the referent $a(x)$ of $x$ is a member of the 1st order domain of $w$. This means that the symbol $E$ determines referents of 1st order variables to be members of appropriate domains (what justifies our name: existence determinator). Now, a formula $A(x)$ is satisfied under an assignment $a$ at a world $w$ iff $a(x)$ belongs to the referent of $A$ with respect to the assignment $a$ at the world $w$. Yet, an enquiring reader might ask why we do not interpret $E$ as a constant predicate, whose referents in particular worlds would be 1st order domains of these worlds. The answer is that the definition of fully free model structures lacks any kind of obligation that the 1st order domains of particular worlds belong to corresponding domains of the 2nd order. And further, a formula $E(A)$ is satisfied under an assignment $a$ at a world $w$ iff the referent $a(A)$ of $A$ is a member of $C^{\exists}_w$. This clause only determines that $a(A)$ is a member of $C^{\exists}_w$, without specifying what kind of element it precisely is. Notice that from $(a(A))(w) \in D_w$ it follows that $a(A) \in C^{\exists}_w$, because all conceptual properties $f \in W \mapsto 2\cup_{w \in W} D_w$ of any fully free model structure satisfy the following condition: if $f(w) \in D_w$ for some $w \in W$, then $f(w) \in D_w$ for all $w \in W$. This condition guarantees that all formulas of the type $ME(A) \rightarrow E(A)$ (see, (2.11)) are satisfied in fully free model structures.$^5$

$^5$An intent reader might ask whether Barcan formulas of the 2nd order and the formulas of the type $ME(A) \rightarrow E(A)$ are equivalent in our Anderson-like theories, because their 1st order counterparts are refereed in many free modal logics as equivalent. We shall show that Barcan formulas of the 2nd order do not entail the formulas of the type $ME(A) \rightarrow E(A)$, and conversely - the formulas of the type $ME(A) \rightarrow E(A)$ do not entail Barcan formulas of the 2nd order, on the basis of remaining axioms of Anderson-like theories.

To prove that Barcan formulas of the 2nd order do not entail the formulas of the type $ME(A) \rightarrow E(A)$, let $\mathfrak{M} = \langle W, R, \mathcal{D}_1, \mathcal{D}_2, G \rangle$ be a model structure such that $W = \{w, v\}$, $R = \{(w, v), (v, v)\}$, $\mathcal{D}_1 = \{D_w, D_v\}$ where $D_w = \{a, g\}$ and $D_v = \{b, g\}$, $\mathcal{D}_2 = \{D_w, D_v\}$ where $D_w = 2^{D_w}$ and $D_v = 2^{D_v}$, and $G = \{g\}$. $C^{\exists}_{\mathfrak{M}}$ is now defined as the set of all functions $f \in W \mapsto 2\cup_{w \in W} D_w$ satisfying the following condition: for every $w, v \in W$, if $f(w) \in D_w$ and $wRv$ then $f(v) \in D_v$. Another change concerns the satisfiability of $E(A)$; namely now, $a, w \models E(A)$ iff $(a(A))(w) \in D_w$. Then, under an assignment $a$ such that $(a(A))(w) = (a(A))(v) = \{b\}$, we obtain that $a, w \models ME(A)$ and $a, w \not\models E(A)$. And one may show that all remaining axioms are true in this model structure and that inference rules are true-preserving.

Conversely, to prove that the formulas of the type $ME(A) \rightarrow E(A)$ do not entail Barcan formulas of the 2nd order, let $\mathfrak{M} = \langle W, R, \mathcal{D}_1, \mathcal{D}_2, G \rangle$ be a model structure such
The set of all formulas satisfied in a world $w$ of a fully free model $\langle \mathcal{W}, a \rangle$ will be denoted by $\text{Sat}(\mathcal{W}, a, w)$ or simply by $\text{Sat}(a, w)$, if the fully free model structure in question is clear from the context.

As customary, we say that a formula $\phi$ is true in a fully free model structure $\mathcal{W}$ (symbolically: $\mathcal{W} \models \phi$) iff $a, w \models \phi$, for every assignment $a$ in $\mathcal{W}$ and every world $w \in W$. The set of all formulas true in $\mathcal{W}$ will be denoted by $\text{Th}(\mathcal{W})$. We also put $\text{Th}(\mathcal{K}) \overset{\text{df}}{=} \bigcap \{ \text{Th}(\mathcal{W}) : \mathcal{W} \in \mathcal{K} \}$, for an arbitrary class of structures $\mathcal{K}$. If $X$ is a set of formulas then we write $\mathcal{W} \models X$, $\mathcal{K} \models X$ if $X \subseteq \text{Th}(\mathcal{W})$, $X \subseteq \text{Th}(\mathcal{K})$ respectively. We write $X \models_\mathcal{K} \phi$ to express that for every assignment $a$ in a structure $\mathcal{W} \in \mathcal{K}$ and for every $w \in W$, if $X \subseteq \text{Sat}(\mathcal{W}, a, w)$ then $\phi \in \text{Sat}(\mathcal{W}, a, w)$.

The following fact is sometimes called substitution lemma. Its proof – a routine induction on the degree of complexity of $\phi$ – will be omitted.

**Proposition 3.1** If $A$ is a term of the same sort as a variable $\xi$ then $a, w \models \phi(\xi/A)$ iff $a^A_\xi, w \models \phi$.

We will need a certain subset $W^{acc} \subseteq W$. Members of $W^{acc}$ are called accessible worlds and $W^{acc}$ is defined as the $R$-image of $W$. We also define that $W = \{ w, v, v_1 \}$, $R = \{ \langle w, v \rangle, \langle w, v_1 \rangle, \langle v, v \rangle, \langle v_1, v \rangle \}$, $\mathcal{D}_1 = \{ D_w, D_v, D_{v_1} \}$ where $D_w = \{ g \}$, $D_v = \{ a, g \}$ and $D_{v_1} = \{ b, g \}$. And $\mathcal{D}_2 = \{ D_w, D_v, D_{v_1} \}$ where $D_w = 2^{D_w}$, $D_v = 2^{D_v}$ and $D_{v_1} = 2^{D_{v_1}}$, and $G = \{ g \}$. $C_M$ is defined as the set of all functions $f \in W \mapsto D$ where $D = \{ a, b, g \}$. By an assignment in the model structure $\mathcal{W}$ we mean a function $a$ which maps variables of the 1st sort to members of $C_M$ and pairs $\langle \alpha, u \rangle$, where $\alpha$ is a variable of the 2nd sort and $u$ is an element of $W$, to members of $2^D$ such that: (i). An assignment $a$ is extended to all terms $A$ by putting:

$$ a(-A, u) = \begin{cases} \bigcup_{u \in W} D_u - a(A, u) & \text{if } a(A, u) \notin D_u, \\ D_u - a(A, u) & \text{if } a(A, u) \in D_u, \end{cases} $$

for every $u \in W$; and (ii). For every $u, u_1 \in W$, if $a(A, u) \notin D_u$ and $uRu_1$, then $a(A, u) \notin D_{u_1}$. It can be easily proved that all formulas of the type $ME(A) \rightarrow E(A)$ are true in the modal structure $\mathcal{W}$. In contrast, the Barcan formula $\forall \alpha \exists y \alpha(y) \rightarrow \alpha(x)$ is false in the model structure $\mathcal{W}$, because for an assignment $a$ such that $a(x) = f$, where $f(u) = g$ for every $u \in W$, $a, w \models \forall \alpha \exists y \alpha(y) \rightarrow \alpha(x)$ and $a, w \not\models \forall \alpha \exists y \alpha(y) \rightarrow \alpha(x)$. And again, one may show that all remaining axioms are true in $\mathcal{W}$ and that inference rules are true-preserving.
inaccessible worlds putting $W^{\text{inacc}} \overset{\text{df}}{=} W - W^{\text{acc}}$. By a special fully free model structure we shall mean a sextuple of the form $\mathfrak{W} = (W, R, D_1, D_2, \mathcal{E}_2, G)$ where $\mathcal{E}_2 = (\mathcal{E}_w)_{w \in W^{\text{inacc}}}$ and $\emptyset \neq \mathcal{E}_w \subseteq D_w$ for every $w \in W^{\text{inacc}}$, and the remaining parameters $W, R, D_1, D_2$ and $G$ are determined in the same way as in ordinary fully free model structures. By conceptual properties of a special fully free model structure we shall mean those functions $f \in W \mapsto \mathcal{P} \bigcup_{w \in W} D_w$ that satisfy the following condition: If $f(w) \in \Delta_w$ for some $w \in W$ then $f(w) \in \Delta_w$ for every $w \in W$, where $\Delta_w = D_w$ if $w \in W^{\text{acc}}$ and $\Delta_w = \mathcal{E}_w$ if $w \in W^{\text{inacc}}$. The set of all conceptual properties of the special fully free model structure $\mathfrak{W}$ will be denoted by $C_{\mathfrak{W}}$.

Given an assignment $a$ in a special fully free model structure $\mathfrak{W}$, its extension to all $2^{nd}$ sort terms is now defined as follows: for every $w \in W$,

$$(a(-A))(w) = \begin{cases} \bigcup_{w \in W} D_w - (a(A))(w) & \text{if } (a(A))(w) \notin \Delta_w \\ D_w - (a(A))(w) & \text{if } w \in W^{\text{acc}} \text{ and } (a(A))(w) \in D_w; \\ \text{an arbitrary element of } \mathcal{E}_w, & \text{if } w \in W^{\text{inacc}} \text{ and } (a(A))(w) \in \mathcal{E}_w. \end{cases}$$

Of course, the symbol $C_{\mathfrak{W}}^{\square}$, where $\mathfrak{W}$ is a special fully free model structure, now denotes the set of all functions $f \in C_{\mathfrak{W}}$ such that for every $w \in W$: $f(w) \in D_w$ if $w \in W^{\text{acc}}$, and $f(w) \in \mathcal{E}_w$ if $w \in W^{\text{inacc}}$.

Now, we will define certain classes of model structures which will play the role of semantical counterparts of fully free Anderson-like theories. Each class will be affixed with the same acronym as its corresponding fully free Anderson-like theory, however, the symbols: $\mathcal{V}^A$, $5$, $b$, $c$, $d$, $n$, $s$ and $\star$ will be interpreted in a different manner according to the following simple rules:

(V) The first symbol of an acronym i.e., $\mathcal{V}^A$ stands for the class of all fully free model structures which subsequently undergo restrictions forced by successive symbols of the acronym;

(5) The symbol $5$ in an acronym indicates that structures in the class are Euclidean i.e., they obey the condition: if $wRv$ and $wRv_1$, then $vRv_1$; for every $w, v, v_1 \in W$.
(b) The symbol \( b \) indicates that structures in the class are symmetric i.e., if \( wRv \), then \( vRw \); for every \( w, v \in W \);

(c) The symbol \( c \) indicates that structures in the class are Euclidean and symmetric;

(d) The symbol \( d \) indicates that structures in the class are Euclidean and transitive i.e., if \( wRv \) and \( vRv_1 \), then \( wRv_1 \); for every \( w, v, v_1 \in W \);

(n) The symbol \( n \) indicates that structures in the class obey the following condition: \( \forall w \in W (D_w \subseteq D) \)
Informally: For every world \( w \), the 1st sort domain of \( w \) is a member of the 2nd sort domain of \( w \);

(p) The symbol \( p \) indicates that structures in the class obey the monotonicity condition: \( \forall w, v \in W (wRv \implies D_w \subseteq D_v) \)
Informally: For any worlds \( w \) and \( v \): if \( w \) has access to \( v \), then the 1st sort domain of \( w \) is a subset of the 1st sort domain of \( v \);

(s) The symbol \( s \) indicates that structures in the class obey the condition: \( \forall w \in W \forall a \in \bigcup_{w \in W} D_w (\bigcap \{ X \mid X \in D_w \text{ and } a \in X \} \in D_w) \)
Informally: For every world \( w \) and every object \( a \): the intersection of all properties of \( w \) containing \( a \) is a member of the 2nd sort domain of \( w \);

(\( \star \)) If an acronym ends with \( \star \) then all structures in the class are required to be special.

In the next sections the result, clarified in the below lemma, will be used.

**Lemma 3.2** For every assignment \( a \) in a fully free model structure of any kind (ordinary or special) and for every world \( w \in W \) (no matter whether accessible or not) the following condition holds: \( a, w \models \mathcal{G}(x) \iff a(x) \in \mathcal{G} \)

**Proof:** See, the proof of Lemma 2.4(i) in [14].

4 Soundness

When we talk about the soundness of a fully free Anderson-like theory with respect to a class of model structures, we mean that all provable formulas in that theory are true in every model structure of this corresponding class of model structures. Thus, it suffices to prove that all axioms of that theory are true in every model structure of this corresponding class and that its inference rules are true-preserving.
(i) Soundness of $\mathcal{V}^{A5}$:

We perform only the problematic steps. Let $\mathfrak{W} = \langle W, R, D_1, D_2, \mathcal{G} \rangle$ be an arbitrary $\mathcal{V}^{A5}$ fully free modal structure, $w \in W$ and let $a$ be some assignment in $\mathfrak{W}$.

1. Suppose that $a, w \not\models \forall x \phi \land E(y) \rightarrow \phi(x/y)$. Then, $a, w \models E(y)$ and $a, w \not\models \phi(x/y)$. This implies that $\forall b \in \{a^y_w\}(b, w \models \phi)$, i.e., for every $b \in \{a^y_w\}$ such that $b(x) \in D_w$, $b, w \models \phi$; and $a(y) \in D_w$. Consequently, for the assignment $b \in \{a^y_w\}$ such that $b(x) = a(y)$, $b, w \models \phi$, and hence $a, w \models \phi(x/y)$ - a contradiction.

2. Suppose that $a, w \not\models \forall \alpha \phi \land E(A) \rightarrow \phi(\alpha/A)$. Thus, $a, w \models \forall \alpha \phi$, $a, w \models E(A)$ and $a, w \not\models \phi(\alpha/A)$. From this it follows that $b, w \models \phi$ for every assignment $b \in \{a^y_w\}$, and $(\alpha(A))(w) \in D_w$. This means - for every $b \in \{a^y_w\}$ such that $b(\alpha) \in C_{\mathfrak{W}}$, $b, w \models \phi$, and $\alpha(A) \in C_{\mathfrak{W}}$, from which it follows that $a, w \models \phi(\alpha/A)$ - a contradiction.

3. Suppose that $a, w \not\models \forall \alpha(\alpha(x) \rightarrow \exists y \alpha(y))$. Then, there exists an assignment $b \in \{a^y_w\}$ such that $b, w \not\models \alpha(x) \rightarrow \exists y \alpha(y)$. From this it follows, in particular, that $b, w \models \alpha(x)$ and $b, w \not\models \exists y \alpha(y)$. The former case means that $b(x) \in (b(\alpha))(w)$. And the latter case means that for every assignment $c \in \{b^y_{w'}\}$, $c, w \not\models \alpha(y)$, i.e., $c(y) \not\models (b(\alpha))(w)$, and consequently, $b(x) \not\models (b(\alpha))(w)$ - a contradiction.\(^6\)

\(^6\)The interesting thing from our point of view is that the formulas: $\alpha(x) \rightarrow \exists y \alpha(y)$, $\exists x \mathcal{L} \alpha(x) \rightarrow \mathcal{L} \exists x \alpha(x)$, $\mathcal{L} \exists x \alpha(x) \rightarrow \exists x \mathcal{L} \alpha(x)$ and $\forall \alpha(\exists x \alpha(x) \rightarrow \exists x \mathcal{L} \alpha(x))$ are not true in the class of $\mathcal{V}^{A5}$ fully free modal structures.

And so, let $\mathfrak{W} = \langle W, R, D_1, D_2, \mathcal{G} \rangle$ be a $\mathcal{V}^{A5}$ fully free modal structure such that $W = \{w, v\}$, $R = W \times W$, $D_1 = \{D_w, D_v\}$ where $D_w = \{a, g\}$ and $D_v = \{b, g\}$, $D_2 = \{D_w, D_v\}$ where $D_w = 2^{D_w}$ and $D_v = 2^{D_v}$, and $\mathcal{G} = \{g\}$.

Taking an assignment $a$ in $\mathfrak{W}$ such that $(a(\alpha))(w) = (a(\alpha))(v) = \{b\}$ and $a(x) = b$, we have $a, w \models \alpha(x)$ and $a, w \not\models \exists y \alpha(y)$. Thus, $a, w \not\models \alpha(x) \rightarrow \exists y \alpha(y)$.

Let now $a$ be an assignment in $\mathfrak{W}$ such that $a(\alpha))(w) = (a(\alpha))(v) = \{a\}$. Clearly, $\mathfrak{W}, a, w \models \exists x \mathcal{L} \alpha(x)$ and $\mathfrak{W}, a, w \not\models \mathcal{L} \exists x \alpha(x)$. Therefore, $\exists x \mathcal{L} \alpha(x) \rightarrow \mathcal{L} \exists x \alpha(x) \not\in \text{Th}(\mathfrak{W})$.

The reader can easily check that under an assignment $a$ in $\mathfrak{W}$ such that $(a(\alpha))(w) = \{a\}$ and $(a(\alpha))(v) = \{b\}$, $\mathcal{L} \exists x \alpha(x) \rightarrow \exists x \mathcal{L} \alpha(x) \not\in \text{Th}(\mathfrak{W})$ and also $\forall \alpha(\mathcal{L} \exists x \alpha(x) \rightarrow \exists x \mathcal{L} \alpha(x)) \not\in \text{Th}(\mathfrak{W})$.

This example of a $\mathcal{V}^{A5}$ fully free modal structure can be also used to show that these above three formulas are not true in remaining classes of fully free modal structures.
(4) To prove that \( a, w \models (x \approx y) \rightarrow (\phi(z/x) \leftrightarrow \phi(z/y)) \), where \( z \) does not occur within the scope of a modal operator, and \( x, y \) are free for \( z \) in \( \phi \), let us assume that \( a, w \models (x \approx y) \), i.e., \( a, w \models \forall \alpha(\alpha(x) \leftrightarrow \alpha(y)) \). Hence, it follows that for every assignment \( b \in \{ a_\alpha, w \} \), \( b, w \models \alpha(x) \leftrightarrow \alpha(y) \). Now, the proof that \( a, w \models \phi(z/x) \leftrightarrow \phi(z/y) \) proceeds by induction on the length of \( \phi \). Obviously, this relation holds if \( \phi \) is an atomic formula \( \alpha(z) \). So, let us consider \( \phi = \psi \land \psi_1 \), where on the strength of the induction hypothesis, \( a, w \models \psi(z/x) \leftrightarrow \psi(z/y) \) and \( a, w \models \psi_1(z/x) \leftrightarrow \psi_1(z/y) \). Therefore, \( a, w \models \psi(z/x) \land \psi_1(z/x) \) iff \( a, w \models \psi(z/x) \) and \( a, w \models \psi_1(z/x) \) iff \( \psi(z/y) \land \psi_1(z/y) \). And clearly, \( a, w \models \phi(z/x) \leftrightarrow \phi(z/y) \). In the similar manner one can prove the latest relation if \( \phi = \neg \psi \) and, on the strength of the induction hypothesis, \( a, w \models \psi(z/x) \leftrightarrow \psi(z/y) \). Further, the requirement of freeness of \( x \) and \( y \) for \( z \) in \( \phi \) is essential, because supposing that \( \phi = \forall t \psi \) we obtain the following equivalences:

\[ a, w \models \forall t \psi(z/x) \text{ iff for every } b \in \{ a^2_t, w \}, b, w \models \psi(z/x) \]  
(by the induction hypothesis) for every \( b \in \{ a^2_t, w \}, b, w \models \psi(z/y) \) iff \( a, w \models \forall t \psi(z/y) \). Therefore, \( a, w \models \forall t \psi(z/x) \leftrightarrow \forall t \psi(z/y) \). Finally, supposing that \( \phi = \forall \alpha \psi \) we have two cases to consider.

The first one is that the formula \( \psi \) has no subformula of the form \( \alpha(z) \). Then clearly, \( a, w \models \forall \alpha \psi(z/x) \) iff for every \( b \in \{ a^2_\alpha, w \}, b, w \models \psi(z/x) \) iff (by the induction hypothesis) for every \( b \in \{ a^2_\alpha, w \}, b, w \models \psi(z/y) \) iff \( a, w \models \forall \alpha \psi(z/y) \). Therefore, \( a, w \models \forall \alpha \psi(z/x) \leftrightarrow \forall \alpha \psi(z/y) \) And the second one is that the formula \( \psi \) has subformulas of the form \( \alpha(z) \). Then, \( a, w \models \forall \alpha \psi(z/x) \) iff for every \( b \in \{ a^2_\alpha, w \}, b, w \models \psi(z/x) \) iff (because \( a, w \models (x \approx y) \), and hence, \( a, w \models \alpha(z/x) \leftrightarrow \alpha(z/y) \)) for every \( b \in \{ a^2_\alpha, w \} \) for every \( b \in \{ a^2_\alpha, w \}, b, w \models \psi(z/y) \) iff \( a, w \models \forall \alpha \psi(z/y) \). Therefore, \( a, w \models \forall \alpha \psi(z/x) \leftrightarrow \forall \alpha \psi(z/y) \).

(5) To prove that \( a, w \models \forall \alpha, \beta(\forall x(\alpha(x) \leftrightarrow \beta(x)) \rightarrow (\phi(\gamma/\alpha) \leftrightarrow \phi(\gamma/\beta))) \), where \( \gamma \) does not occur within the scope of a modal operator, and \( \alpha, \beta \) are free for \( \gamma \) in \( \phi \), let us assume that \( b \in \{ a^2_\alpha, w \} \) and \( c \in \{ b^2_\beta, w \} \) and \( c, w \models \forall x(\alpha(x) \leftrightarrow \beta(x)) \). Hence, for every \( d \in \{ c^2_x, w \}, d, w \models \alpha(x) \leftrightarrow \beta(x) \). Now, proceeding by induction on the length of \( \phi \) we establish that \( c, w \models \phi(\gamma/\alpha) \leftrightarrow \phi(\gamma/\beta) \). Trivially, this relation holds, if \( \phi = \gamma(z) \). The cases \( \phi = \psi \land \psi_1 \) and
\( \phi = \neg \psi \) are considered analogously as in the point (4). Supposing that \( \phi = \forall x \psi \) we have two cases to consider. The first one is that the formula \( \psi \) has no subformula of the form \( \gamma(x) \). Then, \( c, w \models \forall x \psi(\gamma(\alpha)(x)) \) iff for every \( \delta \in \{a^2_{x,w}\}, \delta, w \models \psi(\gamma(\alpha)(x)) \) iff (by the induction hypothesis) for every \( \delta \in \{a^2_{x,w}\}, \delta, w \models \psi(\gamma(\beta)(x)) \) iff \( c, w \models \forall x \psi(\gamma(\beta)(x)) \). Therefore, \( c, w \models \forall x \psi(\gamma(\alpha)(x)) \iff \forall x \psi(\gamma(\beta)(x)) \). And the second one is that the formula \( \psi \) has subformulas of the form \( \gamma(x) \). Then, \( c, w \models \forall x \psi(\gamma(\alpha)(x)) \) iff for every \( \delta \in \{a^2_{x,w}\}, \delta, w \models \psi(\gamma(\alpha)(x)) \) iff (because \( c, w \models \forall x(\alpha(x) \leftarrow \beta(x)) \)) and hence, for every \( \delta \in \{a^2_{x,w}\}, \delta, w \models (\alpha(x) \leftarrow \beta(x)) \) for every \( \delta \in \{a^2_{x,w}\} \), \( \delta, w \models \psi(\gamma(\beta)(x)) \) iff \( c, w \models \forall x \psi(\gamma(\alpha)(x)) \). Therefore, \( c, w \models \forall x \psi(\gamma(\alpha)(x)) \iff \forall x \psi(\gamma(\beta)(x)) \). Finally, supposing that \( \phi = \forall \rho \psi \) we obtain the following equivalences (here, the requirement of freeness of \( \alpha, \beta \) for \( \gamma \) in \( \phi \) is essential) : \( c, w \models \forall \rho \psi(\gamma(\alpha)) \) iff for every \( \delta \in \{a^2_{\rho,w}\}, \delta, w \models \psi(\gamma(\alpha)) \) iff (by the induction hypothesis) for every \( \delta \in \{a^2_{\rho,w}\}, \delta, w \models \psi(\gamma(\beta)) \) iff \( c, w \models \forall \rho \psi(\gamma(\beta)) \).

(6) To prove that \( a, w \models \forall \alpha \forall \beta(\mathcal{P}(\alpha) \land \exists x(\alpha(x) \rightarrow \beta(x)) \rightarrow \mathcal{P}(\beta)) \), let us assume that there exist assignments \( b \in \{a^2_{\alpha,w}\} \) and \( c \in \{b^2_{\beta,w}\} \) such that \( c, w \models \mathcal{P}(\alpha) \), \( c, w \models \exists x(\alpha(x) \rightarrow \beta(x)) \) and \( c, w \models \neg \mathcal{P}(\beta) \). From the first relation, we obtain that \( c(\alpha) \in C^{\mathcal{P}} \) and \( \mathcal{G} \subseteq (\mathcal{a}(\alpha))(v) \) for every \( v \in W \) such that \( wRv \). And from the second relation, we obtain that \( (c(\alpha))(v) \subseteq (c(\beta))(v) \) for every \( x \in D_v \) and every \( v \in W \) such that \( wRv \). Hence, \( c(\beta) \in C^{\mathcal{P}} \) and \( \mathcal{G} \subseteq (\mathcal{a}(\beta))(v) \) for every \( v \in W \) such that \( wRv \), i.e., \( c, w \models \neg \mathcal{P}(\beta) \) - a contradiction.

(7) To prove that \( a, w \models G(x) \rightarrow (L(x \uparrow y) \rightarrow (\phi(\gamma(x) \leftrightarrow \phi(\gamma(y)))) \), where \( x \) and \( y \) are free for \( z \) in \( \phi \), let us assume that \( a, w \models G(x) \) and \( a, w \models L(x \uparrow y) \). From the former relation, by Lemma 3.2, we obtain that \( a(x) \in \mathcal{G} \). And from the second relation, by (3.1), we have that \( \{a(x), a(y)\} \subseteq \mathcal{G} \), and hence, \( \{a(x), a(y)\} \subseteq D_w \) for every \( w \in W \). And therefore, by (3.2), for every \( b \in \{a^2_{\alpha,w}\} \), \( a(x) \in (b(\alpha))(w) \) iff \( a(y) \in (b(\alpha))(w) \). Now, by induction on the construction of \( \phi \) we establish that \( a, w \models \phi(\gamma(x) \leftrightarrow \phi(\gamma(y)) \) for every \( w \in W \). We consider only the case: \( \phi = L\psi \), because in the remaining cases the proofs are the same as in the point (4). And so, supposing that for any \( v \in W \), \( a, v \models \phi(\gamma(x) \leftrightarrow \phi(\gamma(y)) \), we obtain the following equivalences: \( a, w \models L\psi(z/x) \iff a, v \models L\psi(z/x) \) 

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for every \( v \in W \) such that \( wRv \) iff (from the induction hypothesis) \( a, v \models L\psi(z/y) \) for every \( v \in W \) such that \( wRv \) iff \( a, w \models L\psi(z/y) \). Therefore, \( a, w \models L\psi(z/x) \leftrightarrow L\psi(z/y) \) for every \( w \in W \).

(8) To prove that \( a, w \models \exists \beta(\mathbf{P}(\beta) \land (\beta \approx G)) \), where \( \approx \) is defined by (2.30), let \( b \in \{a_{\beta,w}^2\} \) be an assignment such that \( (b(\beta))(w) = G \) for every \( w \in W \). Then, we obtain that \( b, w \models \mathbf{P}(\beta) \) and, by Lemma 3.2, \( b, w \models \forall x(\beta(x) \leftrightarrow G(x)) \), which means that \( b, w \models \mathbf{P}(\beta) \land \forall x(\beta(x) \leftrightarrow G(x)) \). And therefore, \( a, w \models \exists \beta(\mathbf{P}(\beta) \land (\beta \approx G)) \).

(9) Suppose that \( a, w \not\models \forall \alpha \forall x (\neg \alpha(x) \rightarrow (\neg \alpha(x))) \). Hence, we have that there exist \( b \in \{a_{\alpha,w}^\beta\} \) with \( (b(\alpha))(w) \in D_w \) and \( c \in \{b_{\alpha,w}^x\} \) with \( c(x) \in D_w \) such that \( c, w \models \neg \alpha(x) \) and \( c, w \not\models (\neg \alpha(x)) \), which means that \( c(x) \notin (b(\alpha))(w) \) and \( c(x) \notin (b(\neg \alpha))(w) \). Trivially, from the latter, we obtain \( c(x) \notin D_w \) or \( c(x) \in (b(\alpha))(w) \) - a contradiction. 7

(10) Suppose that \( \phi \rightarrow L\chi \in \text{Th}(\mathfrak{W}) \) and \( \phi \rightarrow L\forall x\chi \notin \text{Th}(\mathfrak{W}) \), where - what is important - \( x \) is not free in \( \phi \). This means, in the first case, that \( a, w \models \phi \rightarrow L\chi \) for every assignment \( a \) in \( \mathfrak{W} \) and every \( w \in W \). As to the second case, we have that there exists an assignment \( a \) in \( \mathfrak{W} \) and \( w \in W \) such that \( a, w \not\models \phi \rightarrow L\forall x\chi \), which implies \( a, w \models \phi \) and \( a, w \not\models L\forall x\chi \). And from the latter, it follows that for some \( v \in R(w) \) and \( b \in \{a_{\alpha,w}^\beta\} \), \( b, v \not\models \chi \). Hence, \( b, w \not\models L\chi \), and consequently, \( b, w \not\models \phi \rightarrow L\chi \), i.e., \( \phi \rightarrow L\chi \notin \text{Th}(\mathfrak{W}) \) - a contradiction.

(11) For the proof that every inference rule \( R_{4n}, n > 0 \), is truth-preserving, let us suppose that for a given \( n, n > 0 \), \( \phi \rightarrow .\psi_1 <

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7The reader may be naturally interested whether the formulas \( \forall x(\neg \alpha(x) \rightarrow (\neg \alpha(x))) \) and \( \forall \alpha (\neg \alpha(x) \rightarrow (\neg \alpha(x))) \) are true in the class of \( \mathbf{V}^A \) fully free model structures. For negative answers, let us consider the \( \mathbf{V}^A \) fully free model structure \( \mathfrak{W} = (W, R, \mathcal{D}_1, \mathcal{D}_2, G) \) such that \( W = \{w, v\} \), \( R = W \times W \), \( \mathcal{D}_1 = \{D_w, D_v\} \) where \( D_w = \{a, g\} \) and \( D_v = \{b, g\} \), \( \mathcal{D}_2 = \{D_w, D_v\} \) where \( D_w = 2^{D_w} \) and \( D_v = 2^{D_v} \), and \( G = \{g\} \). Let now \( a \) be an assignment in \( \mathfrak{W} \) such that \( a(x) = a \) and \( (a(\alpha))(w) = (a(\alpha))(v) = \{b\} \). Then, obviously, \( a, w \not\models \forall x(\neg \alpha(x) \rightarrow (\neg \alpha(x))) \).

It is also not difficult to see that under assignments \( a \) and \( b \in \{a_{\alpha,w}^\beta\} \) such that \( a(x) = b \) and \( (b(\alpha))(w) = \{a\} \), \( a, w \not\models \forall \alpha (\neg \alpha(x) \rightarrow (\neg \alpha(x))) \).

This example of a \( \mathbf{V}^A \) fully free model structure can be also used to show that both these formulas are not true in remaining classes of fully free model structures.
... < \psi_n < L_\chi \in \text{Th}(\mathcal{W})$ and $\phi \rightarrow \psi_1 < ... < \psi_n < L\forall x \chi \notin \text{Th}(\mathcal{W})$, where $x$ is not free in $\phi$, $\psi_1,...$, or $\psi_n$. This means that $a, w \models \phi \rightarrow \psi_1 < ... < \psi_n < L_\chi$ for every assignment $a$ in $\mathcal{W}$ and every $w \in W$, and there exists an assignment $a$ in $\mathcal{W}$ and $w \in W$ such that $a, w \not\models \phi \rightarrow \psi_1 < ... < \psi_n < L\forall x \chi$. The latter implies that $a, w \models \phi$ and there exist $v_1, ..., v_{n+1}$ with $wRv_1$ and $v_iRv_{i+1}$ for every $i, 1 \leq i \leq n$, such that $a, v_i \models \psi_i$ and $a, v_{n+1} \not\models \forall x \chi$. And further, there exists an assignment $b \in \{a_{x,v_{n+1}}\}$ such that $b, v_n \not\models \chi$. From this it follows that $b, v_n \not\models L_\chi$, $b, v_n \not\models \psi_n \not\models \psi_n < L_\chi$, $b, v_n \not\models \psi_1 < ... < \psi_n < L_\chi$, and finally, $b, w \not\models \phi \rightarrow \psi_1 < ... < \psi_n < L_\chi$ - a contradiction.

(12) Suppose that $\phi \rightarrow (x \not\approx y) \in \text{Th}(\mathcal{W})$ and $\neg \phi \notin \text{Th}(\mathcal{W})$, where $x$ is not free in $\phi$. So, we have that $a, w \models \phi \rightarrow (x \not\approx y)$ for every assignment $a$ in $\mathcal{W}$ and every $w \in W$, and there exists an assignment $a$ in $\mathcal{W}$ and $w \in W$ such that $a, w \models \phi$. As a consequence of the first case, we have that for an assignment $b$ such that $b(x) = a(y)$, $b, w \not\models (x \not\approx y)$. And since $x$ is not free in $\phi$, it follows that $b, w \not\models \phi$, i.e., $b, w \models \neg \phi$, for every assignment $a$ in $\mathcal{W}$ and every $w \in W$ - a contradiction.

(13) For the proof that every inference rule $R5_n, n > 0$, is true-preserving proceeds, let us suppose that for given $n, n > 0$, $\phi \rightarrow \psi_1 < ... < \psi_n < (x \not\approx y) \in \text{Th}(\mathcal{W})$ and $\phi \rightarrow \psi_1 < ... < \psi_n < L\neg \psi_n \notin \text{Th}(\mathcal{W})$, where $x$ is not free in $\phi$, $\psi_1,...$, or $\psi_n, n > 0$. This means that $a, w \models \phi \rightarrow \psi_1 < ... < \psi_n < (x \not\approx y)$ for every assignment $a$ in $\mathcal{W}$ and every $w \in W$, and there exists an assignment $a$ in $\mathcal{W}$ and $w \in W$ such that $a, w \not\models \phi \rightarrow \psi_1 < ... < \psi_n < L\neg \psi_n$. As a consequence of the first case, we have that for every assignment $a$ in $\mathcal{W}$ and every $w \in W$, $a, w \models \phi \rightarrow \forall v \in R(w)(a, v \models \psi_1 \rightarrow \forall v_1 \in R(v)(a, v_1 \models \psi_2 \rightarrow ... \forall v_{n-1} \in R(v_{n-2})(a, v_{n-1} \models \psi_n \rightarrow a, v_n \models (x \not\approx y))...)$. And clearly, for an assignment $b$ such that $b(x) = a(y)$, $b, v_n \not\models (x \not\approx y)$. Finally, since $x$ is not free in $\phi$, $\psi_1,...$, or $\psi_n, n > 0$, then $\forall v_n \in R(v_{n-2})(a, v_{n-1} \models \neg \psi_n)$, and consequently, $a, w \models \phi \rightarrow \psi_1 < ... < \psi_n < L\neg \psi_n$ for every assignment $a$ in $\mathcal{W}$ and every $w \in W$ - a contradiction.

(ii) Soundness of $\mathcal{V}^{A} 5n$

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By (i), we show here only that (2.39), i.e., $\exists \alpha (\alpha \approx \text{NE})$, is true in any $\mathcal{V}^A5n$ model structure, where $\approx$ is defined by (2.30). So, suppose that $a, w \not\vDash \exists \alpha (\alpha \approx \text{NE}(x))$. From this it follows that $\forall b \in \{a^2_{\alpha, w}\} \exists c \in \{b^2_{x, w}\}(c, w \not\vDash \alpha(x) \leftrightarrow \text{NE}(x))$. Let $b \in \{a^2_{\alpha, w}\}$ be such that $\forall w \in W((b(\alpha))(w) = D_w)$. Consequently, $\exists c \in \{b^2_{x, w}\}[c(x) \notin D_w$ or $c, w \not\vDash \text{NE}(x)]$. Clearly, $\forall c \in \{b^2_{x, w}\}(c(x) \in D_w)$. Hence, it follows that $\exists c \in \{b^2_{x, w}\}(c, w \not\vDash \text{NE}(x))$. But, in the exactly same way as the proof of Lemma 2.4(iii) in [14] it can be proved that $\forall c \in \{b^2_{x, w}\}(c, w \vDash \text{NE}(x))$, which gives a contradiction.

(iii) Soundness of $\mathcal{V}^A5s$ :

By (ii), we show here only that (2.40), i.e., $\exists \alpha (\alpha \approx 1_x)$, is true in any $\mathcal{V}^A5s$ model structure. And so, let $b \in \{a^2_{\alpha, w}\}$ be an assignment such that $b(x) \in D_w$ and let $c \in \{b^2_{x, w}\}$ be an assignment such that for every $w \in W$: $(c(\alpha))(w) = \bigcap\{X \mid X \in D_w \text{ and } b(x) \in X\}$ if $b(x) \in D_w$. By the semantical condition $s$, the definition of this assignment $c \in \{b^2_{x, w}\}$ is correct. Now, according to our choice of the assignment $c$, it should be easy to establish that $c, w \vDash \forall y (\alpha(y) \leftrightarrow (x \approx y))$, and consequently, $a, w \vDash \exists \alpha \forall y (\alpha(y) \leftrightarrow (x \approx y))$.

We leave to the reader the proofs of the soundness of the remaining fully free Anderson-like ontological proofs with respect to appropriate model structures.

5 A Preliminary Machinery to Strong Completeness

Before we pass on to the proofs of strong completeness theorems in the strict sense, we will present some preparatory technical results. Similarly as in [15], we will adopt here the Thomason’s [16] method proving the completeness theorem for the $1^{st}$ order free modal logic $S4$ to our Anderson-like theories. Certainly the reader will understandably accept our decisions about leaving out proofs of Lemmas; which would only be repetitions or small modifications of that from [15].

We shall assume that all the formulas have been arranged in some denumerable sequence: $\phi_1, \phi_2, \ldots, \phi_i, \ldots$. We shall also suppose that some
particular enumerations are fixed so that we may speak of the $1^{st}$, $2^{nd}$, ..., $i^{th}$, ... variable of the $1^{st}$ or $2^{nd}$ sort, respectively.

Given a set $X$ of the formulas of $\mathcal{L}$, we say that $X$ is **Th-consistent** if there exists no formula $\phi$ of $\mathcal{L}$ such that both $X \vdash_{\text{Th}} \phi$ and $X \vdash_{\text{Th}} \neg \phi$; and **Th-inconsistent**, otherwise. $X$ is maximally **Th-consistent** if it is **Th-consistent** and for any formula $\phi$ of $\mathcal{L}$ that does not belong to $X$, $X \cup \{\phi\}$ is **Th-inconsistent**.

**Lemma 5.1** Let $X$ be a **Th-consistent** set of formulas of $\mathcal{L}$ and $M(\phi_1 \wedge \ldots \wedge \phi_n) \in X$, $n \geq 1$. Then, $\{\phi_1, \ldots, \phi_n\}$ is also **Th-consistent**.

**Proof**: By an easy verification. 

**Lemma 5.2** Let $X$ be a maximally **Th-consistent** set of formulas of $\mathcal{L}$. Then, $X \vdash_{\text{Th}} \phi$ iff $\phi$ belongs to $X$.

**Proof**: By an easy verification.

**Lemma 5.3** Let $X$ be a maximally **Th-consistent** set of formulas of $\mathcal{L}$ and $M(\phi_1 \wedge \ldots \wedge \phi_n) \in X$, $n \geq 1$. Then, for any formula $\psi$ of $\mathcal{L}$: $M(\phi_1 \wedge \ldots \wedge \phi_n \wedge \psi) \in X$ or $M(\phi_1 \wedge \ldots \wedge \phi_n \wedge \neg \psi) \in X$.

Let $f_0, f_1, \ldots$ and $h_0, h_1, \ldots$ be two sequences of functions defined as follows:

(i) $f_0(\exists \xi \phi, \zeta) = M\exists \xi \phi \rightarrow M(E(\zeta) \wedge \phi(\xi/\zeta))$;

(ii) $f_1(\psi_1, \exists \xi \phi, \zeta) = M\psi_1 \rightarrow M(\psi_1 \wedge f_0(\exists \xi \phi, \zeta))$;

(iii) $f_{i+1}(\psi_1, \ldots, \psi_{i+1}, \exists \xi \phi, \zeta) = M\psi_{i+1} \rightarrow M(\psi_{i+1} \wedge f_i(\psi_1, \ldots, \psi_i, \exists \xi \phi, \zeta))$ and

(iv) $h_1(\psi_1, x, y) = M\psi_1 \rightarrow M(\psi_1 \wedge (x \equiv y))$;

(v) $h_{i+1}(\psi_1, \ldots, \psi_{i+1}, x, y) = M\psi_{i+1} \rightarrow M(\psi_{i+1} \wedge h_i(\psi_1, \ldots, \psi_i, x, y))$.

**Lemma 5.4** For all $i > 0$, if $X \vdash_{\text{Th}} \neg f_i(\psi_1, \ldots, \psi_i, \exists \chi \phi, \zeta)$ where $\zeta$ does not occur free in $\psi_1, \ldots, \psi_i, \exists \xi \phi$ or any member of $X$, then $X$ is **Th-inconsistent**.

**Lemma 5.5** For all $i > 0$, if $X \vdash_{\text{Th}} \neg h_i(\psi_1, \ldots, \psi_i, x, y)$ where $y$ is different from $x$ and $y$ does not occur free in $\psi_1, \ldots, \psi_i$ or any member of $X$, then $X$ is **Th-inconsistent**.
Let $X$ be a set of formulas of $\mathcal{L}$ and $\text{Th}$ be of one of fully free Anderson-like theories in $\mathcal{L}$. We shall say that $X$ is $\text{Th}$-saturated in $\mathcal{L}$ if it meets the following conditions:

(i) $X$ is maximally $\text{Th}$-consistent;

(ii) For every formula $\phi$ and variable $\xi$ of $\mathcal{L}$, $\forall \xi \phi \in X$ if $\phi(\xi/\zeta) \in X$ for all variables $\zeta$ of $\mathcal{L}$;

(iii) For every 1st sort variable $y$ there is a 1st sort variable $x$ such that $(x \approx y) \in X$;

(iv) For every formula $\phi$ and variable $\xi$ of $\mathcal{L}$ there is a variable $\zeta$ of $\mathcal{L}$ such that $f_0(\exists \xi \phi, \zeta) \in X$;

(v) For all $n > 0$, for all sets $\{\psi_1, \ldots, \psi_n, \exists \xi \phi\}$ of formulas of $\mathcal{L}$ there is a variable $\zeta$ of $\mathcal{L}$ such that $f_n(\psi_1, \ldots, \psi_n, \exists \xi \phi, \zeta) \in X$;

(vi) For all $n > 0$, for every 1st sort variable $y$ and all sets $\{\psi_1, \ldots, \psi_n\}$ of formulas of $\mathcal{L}$ there is a 1st sort variable $x$ such that $h_n(\psi_1, \ldots, \psi_n, (x \approx y)) \in X$.

Let $X$ be a $\text{Th}$-consistent set of formulas of $\mathcal{L}$, on account of fully free Anderson-like theories $\text{Th}$. Let $\mathcal{L}'$ be a language obtained from $\mathcal{L}$ by adding an infinite number of new 1st order variables $X' = \{x'_1, x'_2, \ldots\}$ and an infinite number of new 2nd order variables $Y' = \{\alpha'_1, \alpha'_2, \ldots\}$. Moreover, suppose that the set of nonnegative integers was partitioned into denumerably many denumerable sets $S_0, S_1, S_2, \ldots$. We define the infinite sequence of Thomason’s sets (in short, $t$-sets) $X_0, X_1, X_2, \ldots$ of formulas of $\mathcal{L}'$ in this way that $X_0 = X$ and if $X_i$ was already introduced then according to the following cases:

(0) $i \in S_0$. Let $\exists \xi \phi$ be the alphabetically first formula of $\mathcal{L}'$ of the kind $\exists \zeta \delta$ such that for all $\zeta$ of $X'$ (or, $Y'$), $(\exists \xi \phi \rightarrow \phi(\xi/\zeta)) \land \mathcal{E}(\zeta) \notin X_i$. Then, we put $X_{i+1} = X_i \cup \{\exists \xi \phi \rightarrow \phi(\xi/\tau) \land \mathcal{E}(\tau)\}$ where $\tau$ is the first member of $X'$ (or, $Y'$) not occurring in any member of $X_i$ or $\exists \xi \phi$;

(1) $i \in S_1$. Let $\exists \xi \phi$ be the alphabetically first formula of $\mathcal{L}'$ of the kind $\exists \zeta \delta$ such that for all $\zeta$ of $X'$ (or, $Y'$), $f_0(\exists \xi \phi, \zeta) \notin X_i$. Then, we put $X_{i+1} = X_i \cup \{f_0(\exists \xi \phi, \tau)\}$ where $\tau$ is the first member of $X'$ (or, $Y'$) not occurring in any member of $X_i$ or $\exists \xi \phi$;

(2) $i \in S_2$. Let $y$ be the alphabetically first 1st sort variable of $\mathcal{L}'$ such that for all $x' \in X'$, $(x' \approx y) \notin X_i$. Then, we put $X_{i+1} = X_i \cup \{z' \approx y\}$.
where $z'$ is the first member of $X'$ not occurring in any member of $X_i$ and $z'$ is different from $y$;

(3) $i \in S_{2n+1}$, where $n > 0$. Let $\psi_1 \lor \ldots \lor \psi_n \lor \exists \xi \phi$ be the alphabetically first formula of $L'$ of the kind $\delta_1 \lor \ldots \lor \delta_n \lor \exists \xi \delta$ such that for all $\zeta$ of $X'$ (or, $Y'$), $f_i(\psi_1, \ldots, \psi_n, \exists \xi \phi, \zeta) \notin X_i$. Then, we put $X_{i+1} = X_i \cup \{f_n(\psi_1, \ldots, \psi_i, \exists \xi \phi, \tau)\}$ where $\tau$ is the first member of $X'$ (or, $Y'$) not occurring in any member of $X_i$ or in $\psi_1 \lor \ldots \psi_n \lor \exists \xi \phi$;

(4) $i \in S_{2n+2}$, where $n > 0$. Let $\psi_1 \lor \ldots \lor \psi_n \lor (x \approx y)$ be the alphabetically first formula of $L'$ of the kind $\delta_1 \lor \ldots \lor \delta_n \lor (a \approx b)$ such that for all $x' \in X'$, $h_i(\psi_1, \ldots, \psi_n, x', y) \notin X_i$. Then, we put $X_{i+1} = X_i \cup \{h_n(\psi_1, \ldots, \psi_n, z', y)\}$ where $z'$ is the first member of $X'$ not occurring in any member of $X_i$ or $\psi_1 \lor \ldots \psi_n$, and $z'$ is different from $y$.

**Lemma 5.6** Let $X_0, X_1, X_2, \ldots$ be a sequence of t-sets of formulas of $L'$. Then, the union $X_\infty = \bigcup_{i \geq 0} X_i$ is Th-consistent set of formulas of $L'$.

Let $X$ be a Th-consistent set of formulas of $L$. Let $X_0, X_1, X_2, \ldots$ be a sequence of t-sets of formulas of $L'$. By a normal Thomason’s Th-extension (in short, normal t-Th-extension) of $X$ in $L'$ we shall understand an extension of $X_\infty = \bigcup_{i \geq 0} X_i$ to a maximal Th-consistent set of formulas of $L'$.

**Lemma 5.7** Let $X$ be a Th-consistent set of formulas of $L$. Then, a normal t-Th-extension of $X$ in $L'$ is a Th-saturated set in $L'$.

Let $X$ be a Th-saturated set, on account of fully free Anderson-like theories Th, in $L$ and $\mathcal{M} \psi \in X$. Moreover, suppose that the set of non-negative integers was partitioned into denumerably many denumerable sets $S_0, S_1, S_2, \ldots$. By a special Thomason’s Th-extension (in short, a special t-Th-extension) of $\psi$ in $L$ we shall understand the union $X_\infty = \bigcup_{i \geq 0} X_i$, where $X_0 = \{\psi_0\} = \{\psi\}$ and if $X_i = \{\psi_0, \psi_1, \ldots, \psi_i\}$ then $X_{i+1}$ is given according to the following cases:

(0) $i \in S_0$. Let $\chi$ be the alphabetically first formula of $L$ such that $\chi \notin X_i$ and $\neg \chi \notin X_i$. Then, we put $X_{i+1} = X_i \cup \{\psi_{i+1}\}$, where $\psi_{i+1}$ is $\chi$ if $\mathcal{M}(\psi_0 \land \ldots \land \psi_i \land \chi) \in X$, and $\psi_{i+1}$ is $\neg \chi$ otherwise.

[According to Lemma 5.3, $\psi_{i+1}$ is defined and $\mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i \land \psi_{i+1}) \in X$];
(1) \( i \in S_1 \). If there is no formula of the kind \( \exists \varsigma \delta \) such that \( \exists \varsigma \delta \in X_i \), we put \( \psi_{i+1} \) to be \( \psi_i \), i.e., \( X_{i+1} = X_i \). If there is a formula of the kind \( \exists \varsigma \delta \) such that \( \exists \varsigma \delta \in X_i \), then we choose the alphabetically first formula \( \exists \tau \chi \in X_i \) and the first variable \( \varsigma \) of \( \mathcal{L} \) such that \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i) \rightarrow \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i \land E(\varsigma) \land \chi(\tau/\varsigma)) \in X \) holds, and put \( X_{i+1} = X_i \cup \{ \psi_{i+1} \} \), where \( \psi_{i+1} \) is \( E(\varsigma) \land \chi(\tau/\varsigma) \).

[Suppose that for some \( k \leq i \), \( \psi_k \in \exists \tau \chi \). Let \( \varsigma \) be the alphabetically first variable of \( \mathcal{L} \), not occurring in any formula of \( \{ \psi_0, \psi_1, \ldots, \psi_i \} \) and of the same order as \( \tau \). Because \( \vdash \text{Th } \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_k \land \exists \tau \chi \land \psi_k \land \ldots \land \psi_i) \rightarrow \mathcal{M}(\exists u(\psi_0 \land \psi_1 \land \ldots \land \psi_{k-1} \land \exists \tau \chi \land \psi_k \land \ldots \land \psi_i) \land \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_{k-1} \land \exists \tau \chi \land \psi_{k+1} \land \ldots \land \psi_i) \in X \), then \( \mathcal{M}(\exists u(\psi_0 \land \psi_1 \land \ldots \land \psi_{k-1} \land \chi(\tau/\varsigma) \land \psi_k \land \ldots \land \psi_i) \) \( \in X \). Hence, by the \( \text{Th} \)-saturation of \( X \) in \( \mathcal{L} \), \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_k \land \chi(\tau/\varsigma) \land \psi_k \land \ldots \land \psi_i) \rightarrow \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_k \land \chi(\tau/\varsigma) \land \psi_{k+1} \land \ldots \land \psi_i) \in X \) for some \( \varsigma \) of \( \mathcal{L} \). Finally, in view of the Proposition 2.1(vi), \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i \land E(\varsigma) \land \chi(\tau/\varsigma)) \in X \), i.e., \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_{i+1}) \in X \).]

(2) \( i \in S_2 \). Let \( y \) be the alphabetically first \( 1^{st} \) sort variable of \( \mathcal{L} \) such that for all \( 1^{st} \) sort variables \( u \) of \( \mathcal{L} \), \( (u \uparrow y) \notin X_i \). Let \( z \) be the alphabetically first \( 1^{st} \) sort variable of \( \mathcal{L} \) such that \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i) \rightarrow \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i \land (z \uparrow y)) \in X \). Then, we put \( X_{i+1} = X_i \cup \{ \psi_{i+1} \} \), where \( \psi_{i+1} \) is \( (z \uparrow y) \).

[Because \( X \) is \( \text{Th} \)-saturated in \( \mathcal{L} \), therefore there exists a \( 1^{st} \) sort variable \( z \) of \( \mathcal{L} \) such that \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i) \rightarrow \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i \land (z \uparrow y)) \in X \). But, by induction hypothesis, \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i) \in X \). Thus, \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i \land (z \uparrow y)) \in X \), i.e., \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_{i+1}) \in X \).]

(3) \( i \in S_3 \). If there is no formula of the kind \( \mathcal{M} \exists \varsigma \delta \) such that \( \mathcal{M} \exists \varsigma \delta \in X_i \), we put \( \psi_{i+1} \) to be \( \psi_i \), i.e., \( X_{i+1} = X_i \). If there is a formula of the kind \( \mathcal{M} \exists \varsigma \delta \) such that \( \mathcal{M} \exists \varsigma \delta \in X_i \), then we choose the alphabetically first formula \( \mathcal{M} \exists \tau \chi \in X_i \) and the first variable \( \tau' \) of \( \mathcal{L} \) such that \( \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i) \rightarrow \mathcal{M}(\psi_0 \land \psi_1 \land \ldots \land \psi_i \land f_0(\exists \tau \chi, \tau')) \in X \) holds, and put \( X_{i+1} = X_i \cup \{ \psi_{i+1} \} \), where \( \psi_{i+1} \) is \( f_0(\exists \tau \chi, \tau') \).

[Suppose that for some \( k \leq i \), \( \psi_k \in \mathcal{M} \exists \tau \chi \). Let \( \tau' \) be the alphabetically first variable of \( \mathcal{L} \) not occurring in any formula of \( \{ \psi_0, \psi_1, \ldots, \psi_i \} \) and of the same order as \( \tau \). Because \( (\psi_0 \land \psi_1 \land \ldots \land \psi_k \land \mathcal{M} \exists \tau \chi \land \psi_{k+1} \land \ldots \land \psi_i) \in X \) and \( \mathcal{M} \exists \tau \chi \rightarrow \mathcal{M}(\mathcal{E}(\tau') \land \phi(\tau/\tau')) \in X \), then \( (\psi_0 \land \psi_1 \land \ldots \land \psi_k \land \mathcal{M} \exists \tau \chi \land \psi_{k+1} \land \ldots \land \psi_i) \).]
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ψ_{k-1} \land M[E(τ') \land φ(τ/τ')] \land ψ_{k+1} \land ... \land ψ_i \in X. Therefore, in view of the Proposition 2.1(vi), M(ψ_0 \land ψ_1 \land ... \land ψ_i \land M[E(τ') \land φ(τ/τ')]) \in X, and consequently, M(ψ_0 \land ψ_1 \land ... \land ψ_i \land f_0(∃τχ, τ')) \in X i.e., M(ψ_0 \land ψ_1 \land ... \land ψ_{i+1}) \in X.

(4) \quad i \in S_{2n+2}, where n > 0. Let χ_1 \lor ... \lor χ_n \lor (x^1 \approx y) be the alphabetically first formula of L of the kind δ_1 \lor ... \lor δ_n \lor (a \approx b) such that for all 1st sort variables u of L, h_n(χ_1, ..., χ_n, u, y) \notin X_i. Let z be the first 1st sort variable of L such that M(ψ_0 \land ψ_1 \land ... \land ψ_i) \rightarrow M(ψ_0 \land ψ_1 \land ... \land ψ_i \land h_n(χ_1, ..., χ_n, z, y)) \in X. Then, we put X_{i+1} = X_i \cup \{ψ_{i+1}\} where ψ_{i+1} is h_n(χ_1, ..., χ_n, z, y).

[Because X is Th-saturated in L, therefore there exists a 1st sort variable z of L such that M(ψ_0 \land ψ_1 \land ... \land ψ_i) \rightarrow M(ψ_0 \land ψ_1 \land ... \land ψ_i \land h_n(χ_1, ..., χ_n, z, y)) \in X. But, by induction hypothesis, M(ψ_0 \land ψ_1 \land ... \land ψ_i) \in X. Thus, M(ψ_0 \land ψ_1 \land ... \land ψ_i \land h_n(χ_1, ..., χ_n, z, y)) \in X, i.e., M(ψ_0 \land ψ_1 \land ... \land ψ_{i+1}) \in X.]

(5) \quad i \in S_{2n+3}, where n > 0. Let χ_1 \lor ... \lor χ_i \lor ∃ξχ be the alphabetically first formula of L of the kind δ_1 \lor ... \lor δ_i \lor ∃ξδ such that for all variables ζ of L f_n(χ_1, ..., χ_n, ∃ξχ, ζ) \notin X_i. Then, we put X_{i+1} = X_i \cup \{ψ_{i+1}\} where ψ_{i+1} is f_n(χ_1, ..., χ_n, ∃ξχ, τ) and τ is the alphabetically first variable such that M(ψ_0 \land ψ_1 \land ... \land ψ_i) \rightarrow M(ψ_0 \land ψ_1 \land ... \land ψ_i \land f_n(χ_1, ..., χ_n, ∃ξχ, τ)) \in X.

[Because X is Th-saturated in L, therefore there exists variable τ of L such that M(ψ_0 \land ψ_1 \land ... \land ψ_i) \rightarrow M(ψ_0 \land ψ_1 \land ... \land ψ_i \land f_n(χ_1, ..., χ_n, ∃ξχ, τ)) \in X. But, by induction hypothesis, M(ψ_0 \land ψ_1 \land ... \land ψ_i) \in X. Thus, M(ψ_0 \land ψ_1 \land ... \land ψ_i \land f_n(χ_1, ..., χ_n, ∃ξχ, τ)) \in X, i.e., M(ψ_0 \land ψ_1 \land ... \land ψ_{i+1}) \in X.]

Lemma 5.8 Let X be a Th-saturated set in L and Mψ \in X. Then, the special t-Th-extension X_∞ of {ψ} in L is Th-saturated in L. Moreover, \{φ \mid Lφ \in X\} \subseteq X_∞.

6 Strong Completeness

We are now in a position to prove

Theorem 6.1 (Strong completeness) Let X be a set of formulas. Then, for every \Re ∈ \{V^A 5, V^A b, V^A c, V^A 5n, V^A bn, V^A cn, V^A 5s, V^A bs, V^A cs, V^A 5ns, V^A bns, V^A cns\}, X \models_\Re φ implies X \models_\Re φ.
Proof: We confine ourselves only to the situation when \( \mathcal{R} \in \{ \mathcal{V}^A5, \mathcal{V}^A5n, \mathcal{V}^A5s \} \), because banking on these proofs, it is not hard to prove the remaining cases.

Proof of: \( X \models_{\mathcal{V}^A5} \phi \) implies \( X \models_{\mathcal{V}^A5} \phi \)

We consider only the non-trivial case, when \( X \not\models_{\mathcal{V}^A5} \phi \). Hence, \( X \cup \{-\phi\} \) is \( \mathcal{V}^A5 \)-consistent. As usual, we are going to construct a model \( \langle \mathcal{W}, a \rangle \), where \( \mathcal{W} \in \mathcal{V}^A5 \), such that for every \( \psi \in X \cup \{-\phi\} \) and for each \( w \in \mathcal{W} \), \( \mathcal{W}, a, w \models \psi \).

First we form a frame \( \langle W, R \rangle \) by taking \( W \) to be the family consisting of the normal \( \mathcal{V}^A5 \)-extension \( w_1 \) in \( L' \) of the set \( X \cup \{-\phi\} \), the special \( t-\mathcal{V}^A5 \)-extensions in \( L' \) of formulas \( \psi \) such that \( M\psi \in w_1 \), the special \( t-\mathcal{V}^A5 \)-extensions in \( L' \) of formulas \( \psi \) such that \( M\psi \) is a member of a special \( t-\mathcal{V}^A5 \)-extension in \( L' \) of formulas \( \psi \) such that \( M\psi \in w_1 \), etc.

The members of \( W \) will be ordered in the following four steps:

Step 1 We assign a rank to each \( w \in W \) (rank \( (w) \), for short). And so, \( \text{rank} \ (w_1) = 1 \); and \( \text{rank} \ (v) = \text{rank} \ (w) + 1 \) if \( v \) is a special \( t-\mathcal{V}^A5 \)-extension in \( L' \) of a formula \( \psi \) such that \( M\psi \in w \) and \( v \) has not yet got a rank \( (v) \leq \text{rank} \ (w) \).

Step 2 For every \( w \in W \), we order various special \( t-\mathcal{V}^A5 \)-extensions in \( L' \) of formulas \( \phi \) and \( \psi \) such that \( \{M\phi, M\psi\} \subseteq w \). So, suppose \( w' \) and \( w'' \) are distinct special \( t-\mathcal{V}^A5 \)-extensions in \( L' \) of formulas \( \phi \) and \( \psi \), respectively, such that \( \{M\phi, M\psi\} \subseteq w \). Then \( w' \) is to precede or follow \( w'' \) according to whether \( \phi \) precedes or follows \( \psi \).

Step 3 We partition \( W \) into cells \( W^1, W^2, \ldots, W^r, \ldots \), consisting for each \( r, r \geq 1 \), of the members of \( W \) of rank \( r \), and next we order the members of each cell. If \( W^r \) has exactly one member, we declare it the first member of \( W^r \). Otherwise, we employ the following inductive procedure:

Case 1 \( r = 2 \). Then the members of \( W^2 = \{w \mid w \text{ is a special } t-\mathcal{V}^A5 \text{ extension in } L' \text{ of a formula } \psi \text{ such that } M\psi \in w_1 \} \) already come in an order of their own (see, Step 2);

Case 2 \( r > 2 \). Given any two members of \( W^r \), one - let us call it \( w' \), - is sure to be for some \( j' \) and \( k' \), the \( k' \)th special \( t-\mathcal{V}^A5 \)-extension in \( L' \) of some formula \( \psi' \) such that \( M\psi' \) belongs to the \( j' \)th member.
of $W^{r-1}$, and the other - let us call it $w''$ - is sure to be for some $j''$ and $k''$, the $k''^\text{th}$ special $t$-$\mathcal{V}$-extension in $\mathcal{L}'$ of some formula $\psi''$ such that $M\psi''$ belongs to the $j''^\text{th}$ member of $W^{r-1}$. Then $w'$ will precede $w''$ in $W^r$ if $j' + k' < j'' + k''$ or, when $j' + k' = j'' + k''$ and $j' < j''$; otherwise, $w'$ will follow $w''$ in $W^r$.

**Step 4** We now order the members of $W$ in a single run:

(i) $w_1$, the one member of $W^1$, is to precede all other members of $W$;

(ii) $w'$ being the $j'^\text{th}$ member of $W^{r'}$ (  $r' > 1$ ) , and $w''$ the $j''^\text{th}$ member of $W^{r''}$ (  $r'' > 1$ ) , $w'$ is to precede $w''$ if $j' + r' < j'' + r''$ or, when $j' + r' = j'' + r''$ and $r' < r''$; otherwise, $w'$ follows $w''$.

Now, let us suppose for induction that the set $w_n$, $n > 1$, is already defined. Thus, there exist parameters $j \geq 1$ and $r \geq 2$ such that $w_n$ is the $j^\text{th}$ member of $W^r$. For each $i$, $2 \leq i < r + j$, we next put

$$V^i = W^i - \{v \mid v \in W^i \text{ and } v \text{ precedes or equals } w_n\},$$

and

$$V = \{v \mid v \text{ is the first member of some } V^i, 2 \leq i < r + j\}.$$

In the case of $V = \emptyset$, $w_n$ is the last member of $W$. Supposing then that $V \neq \emptyset$, we define $w_{n+1}$ as the first member of $V$. It is easily shown, when $w_n$ is not the last member of $W$, that there is no member of $W$ which follows $w_n$ and precedes $w_{n+1}$ does not exist.

And next, for any $w, v \in W$, we define the accessibility relation $R$ as follows:

(R) $wRv$ if and only if $\{\phi \mid L\phi \in w\} \subseteq v$.

We are now in a position to prove that

(* ) For every formula $\phi$ and all $w \in W$, $L\phi \in w$ if and only if $\phi \in v$ for each $v \in W$ such that $wRv$.

Let $\phi$ be any formula and $w$ any member of $W$. We leave it to the reader to verify that for every axiom $\phi$ of $\mathcal{V}$-$\mathcal{A}$-$5$, $\vdash_{\mathcal{V}} M\phi$. Hence, trivially, $w$ has members of the sort $M\phi$. And therefore, if $L\phi \in w$, then by Lemma 5.8 and (R), $\phi \in v$ for each $v \in W$ such that $wRv$. Suppose, on the other hand, that $\phi \in v$ for each $v \in W$ such that $wRv$, and let $L\phi \notin w$. Because $w$ is $\mathcal{V}$-$\mathcal{A}$-$5$-saturated, then with respect to Lemma 5.2, $M\neg\phi \in w$. Hence, by
the construction of members of \( W \), there exists \( w \in W \) such that \( \neg \phi \in w \), which contradicts the assumption that \( \phi \in v \) and \( v \) is \( \mathcal{V}^{A_5} \)-consistent.

The proofs that \( R \) is serial and Euclidean can be found in [15].

Next three facts perform an auxiliary role:

\( (\bullet \bullet) \) If \( G(x) \in v \) for some \( v \in W \), then \( G(x) \in w \) for each \( w \in W \).

\( (\bullet \bullet \bullet) \) If \( G(x) \in v \) for some \( v \in W \), then \( E(x) \in w \) for each \( w \in W \).

\( (\bullet \bullet \bullet \bullet) \) If \( E(A) \in v \) for some \( v \in W \), then \( E(A) \in w \) for each \( w \in W \).

The proofs of \( (\bullet \bullet) \) and \( (\bullet \bullet \bullet) \) are the same as their proofs in [15]. And the proof of the fact \( (\bullet \bullet \bullet \bullet) \) goes as follows:

First, we show that if \( E(A) \not\in \{w_1\} \) and \( E(A) \not\in v \) for every \( w \in W \). So, suppose that \( E(A) \not\in \{w_1\} \) and \( E(A) \in v \) for some \( v \in W \). Then, there exists a finite sequence \( w_1, w_2, \ldots, w_k = v \) such that for each \( i \), \( 1 < i \leq k \), \( w_i \) is a special \( t \)-\( \mathcal{V}^{A_5} \)-extension of some formula \( \psi_{i-1} \) such that \( M \psi_{i-1} \in w_i-1 \). And, by the maximal consistency of \( w_1 \), \( \neg E(A) \in w_1 \).

But, since \( \vdash_{\mathcal{V}^{A_5}} \neg E(A) \rightarrow L \neg E(A) \), then by Lemma 5.2 and Proposition 2.1(ii), \( L \neg E(A) \in w_1 \). Now, by applying the definition \( (R) \) to the latter, we obtain that \( \neg E(A) \in w_2 \), i.e., \( E(A) \not\in w_2 \). Now, for the induction step, let us assume that we have already proved that \( E(A) \not\in w_i \) for some \( i \), \( 2 \leq i < k \). So, by the maximal consistency of \( w_i \), \( \neg E(A) \in w_i \), and by the same argument as before, we obtain that \( L \neg E(A) \in w_i \). Consequently, \( \neg E(A) \in w_{i+1} \), i.e., \( E(A) \not\in w_{i+1} \), and hence, \( E(A) \not\in v \) - a contradiction. From this result it immediately follows that if \( E(A) \in v \) for some \( v \in W \), then \( E(A) \in w_1 \). Next, suppose that \( E(A) \in v \) and \( E(A) \not\in v_1 \) for some \( v, v_1 \in W \). Again, there exists a finite sequence \( w_1, w_2, \ldots, w_k = v_1 \) such that for each \( i \), \( 1 < i \leq k \), \( w_i \) is a special \( t \)-\( \mathcal{V}^{A_5} \)-extension of some formula \( \psi_{i-1} \) such that \( M \psi_{i-1} \in w_i-1 \), and \( E(A) \in w_1 \). But, since \( \vdash_{\mathcal{V}^{A_5}} E(A) \rightarrow L \neg E(A) \) (see, T11 in Appendix), then by Lemma 5.2 and Proposition 2.1(ii), \( L \neg E(A) \in w_1 \). Hence, by applying the definition \( (R) \) to the latter, we obtain that \( E(A) \in w_2 \). Routine editing of the induction argument will show that \( E(A) \in v_1 \) - a contradiction. From this result it immediately follows that if \( E(A) \in v \) for some \( v \in W \), then \( E(A) \in w \) for every \( w \in W \) - which finishes the proof of the fact \( (\bullet \bullet \bullet \bullet) \).
Fully Free Semantics ...

Given some (any chosen) member \( w \) of \( W \), we define
\[
\mathcal{G} = \{ x | G(x) \in w \},
\]
By (\( \bullet \bullet \)), it can easily be seen that this definition is correct.

Now with each \( w \in W \) we associate the 1\(^{st}\) order domain
\[
D_w = \{ x | x \text{ is a 1}\^{st}\text{ order variable of } \mathcal{L}' \text{ and } E(x) \in w \},
\]
and we put
\[
\mathcal{D}_1 = (D_w)_{w \in W}.
\]

Clearly, by (\( \bullet \bullet \bullet \)), \( \mathcal{G} \subseteq \bigcap_{w \in W} D_w \).

Next, we associate the following sets with each 2\(^{nd}\) order term \( A \) and \( w \in W \):
\[
F(A, w) = \{ a \in D_w | \{ E(A), A(a) \} \subseteq w \}
\]
(According to (2.9), Proposition 2.1(i) and Lemma 5.2, \( E(A) \rightarrow (A(a) \rightarrow \ E(a)) \in w \). Hence, \( \{ E(A), A(a) \} \subseteq w \) implies that \( E(a) \in w \), and therefore, \( F(A, w) \in 2^{D_w} \) for every \( w \in W \),
\[
G(A, w) = \{ a \in \bigcup_{w \in W} D_w | A(a) \in w \},
\]
and we put
\[
\text{for every } w \in W, D_w \text{ to be the family of all sets } F(A, w) \in 2^{D_w}, \]
\[
\mathcal{D}_2 = (D_w)_{w \in W},
\]

\( C_{\mathcal{M}} \) is the set of all functions \( f \in W \mapsto \bigcup_{w \in W} D_w \) satisfying the following condition: if \( f(w) \in D_w \) for some \( w \in W \), then \( f(w) \in D_w \) for all \( w \in W \);
and

\( \mathcal{C}_{\mathcal{G}} \) is the set of all functions \( f \in C_{\mathcal{M}} \) such that \( f(w) \in D_w \) for every \( w \in W \).

To prove that \( \mathcal{G} \in \bigcap_{w \in W} D_w \) let us notice that, by (\( \bullet \bullet \)), \( F(\mathcal{G}, w) = F(\mathcal{G}, w_1) \) for every \( w, w_1 \in W \). Hence, trivially, \( \mathcal{G} = F(\mathcal{G}, w) \in \bigcap_{w \in W} D_w \).
To prove that for every \( w \in W \), \(-X \in D_w \) if \( X \in D_w \), let us assume that \( F(A, w) \in D_w \). Then, for every \( a \in F(A, w) \), \(-A(a) \notin w \). It follows that for every \( x \in D_w - F(A, w) \), \(-A(x) \in w \). Thus, \( F(-A, w) \in D_w \). But, \( F(-A, w) = -F(A, w) \), and therefore \(-F(A, w) \in D_w \).

In this way we have finished our construction of the model structure \( W = \langle W, R, D_1, D_2, G \rangle \), called a canonical \( V^A5 \)- fully free model structure.

The assignment \( a \) in the canonical \( V^A5 \)- fully free model structure such that for any \( 1^{st} \) order variable \( x \) of \( L' \), \( a(x) = x \), and for any \( 2^{nd} \) sort term \( A \) of \( L' \) and each \( w \in W \):

\[
(a(A))(w) = \begin{cases} 
F(A, w) & \text{if } E(A) \in w, \\
G(A, w) & \text{if } E(A) \notin w, 
\end{cases}
\]

will be called a canonical assignment.

One can show that

\[\text{(TL) Given the canonical } V^A5 \text{- fully free model structure } W = \langle W, R, D_1, D_2, G \rangle \text{ and the canonical assignment } a \text{ in it: for any formula } \phi \text{ of } L' \text{ and each } w \in W, \ a, w \models \phi \text{ if and only if } \phi \in w.\]

The proof of (TL) proceeds by simultaneous induction on the complexity of \( \phi \). We only focus on the problematic steps.

\( \phi \) is of the form \( E(x) \):

Then, \( a, w \models E(x) \iff a(x) \in D_w \iff x \in D_w \), by the definition of \( D_w \), this last iff \( E(x) \in w \).

\( \phi \) is of the form \( E(A) \):

Then, \( a, w \models E(A) \iff a(A) \in C^a \iff a(A)(w) \in D_w \) for every \( w \in W \) iff \( F(A, w) \in D_w \) for every \( w \in W \). The last implies that \( E(A) \in w \). Conversely, suppose that \( E(A) \in w \). Then, according to (\( \bullet \bullet \bullet \bullet \)), \( E(A) \in w \) for every \( w \in W \). Because \( \vdash_{V^A5} \forall \alpha \forall x (\alpha(x) \lor -\alpha(x)) \), then using Lemma 5.2, we obtain that \( \forall \alpha \forall x (\alpha(x) \lor -\alpha(x)) \in w \) for every \( w \in W \). Hence, by (2.7), Proposition 2.1(i) and Lemma 5.2, \( E(A) \rightarrow \forall x (A(x) \lor -A(x)) \in w \) for every \( w \in W \). Consequently, for every \( w \in W \) and every \( x \in D_w \), \( A(x) \lor -A(x) \in w \). Thus, for every \( w \in W \) and every \( x \in D_w \), \( \{E(A), A(x)\} \subseteq w \) or \( \{E(A), -A(x)\} \subseteq w \), which implies that \( F(A, w) = \{a \in D_w | \{E(A), A(a)\} \subseteq w \} \in D_w \) for every \( w \in W \). And finally, we have that \( a, w \models E(A) \).
Fully Free Semantics …

$\phi$ is of the form $A(x)$:
We consider two possible cases. Case 1: $E(A) \in w$. Then, $a, w \models A(x)$ iff $a(x) \in a(A)(w)$ iff $x \in F(A, w)$, by the definition of $F(A, w)$, the last iff $\{E(A), A(x)\} \subseteq w$. And the last implies that $A(x) \in w$. But, by the assumption, $E(A) \in w$, it should be then clear that $\{E(A), A(x)\} \subseteq w$ iff $A(x) \in w$. Case 2: $E(A) \notin w$. Then, $a, w \models A(x)$ iff $a(x) \in a(A)(w)$ iff $x \in G(A, w)$, by the definition of $G(A, w)$, this last iff $A(x) \in w$.

$\phi$ is of the form $\forall \xi \psi$:
Then, $a, w \models \forall \xi \psi$ iff $b, w \models \psi$ for every $b \in \{a_{\xi,w}^2\}$, and further on the strength of Proposition 3.1, this last iff $b, w \models \psi(\xi/b(\xi))$ for every $b \in \{a_{\xi,w}^2\}$, and by the inductive hypothesis, iff $\psi(\xi/b(\xi)) \in w$ for every assignment $b \in \{a_{\xi,w}^2\}$. This last, because $w$ is $\mathcal{V}_A^5$-saturated, implies that $\forall \xi \psi \in w$. Suppose now that $\forall \xi \psi \in w$. Then, by applying (2.7), and Lemma 5.2, it follows that $(E(\xi) \rightarrow \psi(\xi/\xi)) \in w$ for every $1^{st}$ or $2^{nd}$ order term $\xi$. This means that, for every assignment $b \in \{a_{\xi,w}^2\}$, $\psi(\xi/b(\xi)) \in w$.

$\phi$ is of the form $P(A)$:
Then, $a, w \models P(A)$ iff $a(A) \in C_{\mathcal{M}}$ and $G \subseteq a(A)(v)$ for every $v \in R(w)$ iff $a(A)(w) \in D_w$ for every $w \in W$ and $x \in a(A)(v)$ for every $x \in G$ and every $v \in R(w)$. This last implies that $E(A) \in w$ and $A(x) \in v$ for every $x \in G$ and every $v \in R(w)$ iff, on the strength of the condition (●), $E(A) \in w$ and $LA(x) \in w$ for every $x \in G$. We have already demonstrated that $G(x) \in w$ for every $x \in G$. Hence, by definition (2.1), $\forall \alpha[P(\alpha) \leftrightarrow LA(\alpha)] \in w$ for every $x \in G$. But then, by (2.7) and Lemma 5.2, $E(A) \land LA(x) \rightarrow P(A) \in w$ for every $x \in G$. Therefore, $P(A) \in w$. Conversely, suppose that $P(A) \in w$. Hence, by (2.26) and Lemma 5.2, $E(A) \in w$. Further, by $T15$ (see, Appendix) and Lemma 5.2, $\forall \alpha[P(\alpha) \leftrightarrow LA(\alpha)] \in w$ and consequently, on the strength of the axiom (2.7) and Lemma 5.2, $E(A) \land P(A) \rightarrow LA(x)(G(x) \rightarrow A(x)) \in w$. But then, $a(A) \in C_{\mathcal{M}}$ and $G \subseteq a(A)(v)$ for every $v \in R(w)$. This means that $a, w \models P(A)$.

This concludes our proof of (TL).

referring back to the assumption $X \nvdash_{\mathcal{V}_A^5} \phi$ we apply now the semantic instrument, which we have here introduced. So, let $\mathfrak{W} = \langle W, R, D_1, D_2, G \rangle$ be the canonical $\mathcal{V}_A^5$- fully free model structure and let $a$ be the canonical assignment in it. Because $X \cup \{\neg \phi\} \subseteq w_1$, then, for all $\psi \in X \cup \{\neg \phi\}$,
\( \mathcal{M}, a, w_1 \models \psi \). Therefore, \( X \not\models_{\mathcal{A}_5} \phi \), which completes the proof of: \( X \models_{\mathcal{A}_5} \phi \) implies \( X \not\models_{\mathcal{A}_5} \phi \).

**Proof of:** \( X \models_{\mathcal{A}_5} \phi \) implies \( X \not\models_{\mathcal{A}_5} \phi \)

Banking on the result: \( X \models_{\mathcal{A}_5} \phi \) implies \( X \not\models_{\mathcal{A}_5} \phi \), it suffices only to prove that \( D_w \in \mathcal{D}_w \) for every \( w \in \mathcal{W} \). And so, by T14 (see, Appendix), \( \models_{\mathcal{A}_5} \forall x \text{NE}(x) \) for every \( w \in \mathcal{W} \). But, by (2.39), \( \text{NE} \) is a predicate, so we can put \( D_w = F(\text{NE}, w) \in \mathcal{D}_w \) for every \( w \in \mathcal{W} \), which completes the proof of: \( X \models_{\mathcal{A}_5} \phi \) implies \( X \not\models_{\mathcal{A}_5} \phi \).

**Proof of:** \( X \models_{\mathcal{A}_5} \phi \) implies \( X \not\models_{\mathcal{A}_5} \phi \)

Relying on the result: \( X \models_{\mathcal{A}_5} \phi \) implies \( X \not\models_{\mathcal{A}_5} \phi \), it suffices only to show that \( \bigcap \{X \mid X \in \mathcal{D}_w \text{ and } x \in X\} \subseteq \mathcal{D}_w \) for every \( w \in \mathcal{W} \). And so, according to (2.40) with the definition (2.30), for each 1st sort variable \( x \), \( l_x \) is a term of the 2nd order. Thus, \( F(l_x, w) \in \mathcal{D}_w \) for every \( x \in \bigcup_{w \in \mathcal{W}} \mathcal{D}_w \). It remains to show that \( F(l_x, w) = \bigcap \{X \mid X \in \mathcal{D}_w \text{ and } x \in X\} \) for every \( x \in \bigcup_{w \in \mathcal{W}} \mathcal{D}_w \). First, we prove that \( \bigcap \{X \mid X \in \mathcal{D}_w \text{ and } x \in X\} \subseteq F(l_x, w) \) for every \( x \in \bigcup_{w \in \mathcal{W}} \mathcal{D}_w \) and every \( w \in \mathcal{W} \). Clearly, \( \bigcap \{X \mid X \in \mathcal{D}_w \text{ and } x \in X\} = \emptyset \) if \( x \notin \mathcal{D}_w \), and consequently, the required inclusion holds. If \( x \in \mathcal{D}_w \), then \( x \in F(l_x, w) \), and of course the required inclusion also holds. Now, we prove that \( F(l_x, w) \subseteq \bigcap \{X \mid X \in \mathcal{D}_w \text{ and } x \in X\} \) for every \( x \in \bigcup_{w \in \mathcal{W}} \mathcal{D}_w \) and every \( w \in \mathcal{W} \). It should be clear that the required inclusion holds if \( x \notin \mathcal{D}_w \). Finally, suppose that for some \( w \in \mathcal{W} \) and for some \( x \in \mathcal{D}_w \): 
\( a \in F(l_x, w) \) and \( a \notin \bigcap \{X \mid X \in \mathcal{D}_w \text{ and } x \in X\} \). Since the following equivalences hold: 
\( a \in F(l_x, w) \iff a \in \{y \in \mathcal{D}_w \mid l_x(y) \in w\} \) iff \( a \in \mathcal{D}_w \) and \( \forall \alpha(\alpha(x) \leftrightarrow \alpha(a)) \in w \), in the former case, we consequently have: 
\( a \in \mathcal{D}_w \) and \( \forall \alpha(\alpha(x) \leftrightarrow \alpha(a)) \in w \). In the latter one, the assumption: \( a \notin \bigcap \{X \mid X \in \mathcal{D}_w \text{ and } x \in X\} \) means that there exists \( X \in \mathcal{D}_w \) such that \( a \notin X \), \( x \in X \) and \( X = F(A, w) \) for some 2nd sort term \( A \). Therefore, \( a \notin \{y \in \mathcal{D}_w \mid A(y) \in w\} \), i.e., \( a \notin \mathcal{D}_w \) or \( A(a) \notin w \), hence \( a \notin \mathcal{D}_w \) or \( \forall \alpha(\alpha(x) \leftrightarrow \alpha(a)) \notin w \) - a contradiction, which means that the required inclusion holds.

**Theorem 6.2 (Strong completeness)** Let \( X \) be a set of formulas. Then, for every \( \mathcal{R} \in \{\mathcal{V}_4^{5p}, \mathcal{V}_4^{6p}, \mathcal{V}_4^{6c}, \mathcal{V}_4^{5ps}, \mathcal{V}_4^{6ps}, \mathcal{V}_4^{5c}, \mathcal{V}_4^{6c}, \mathcal{V}_4^{5np}, \mathcal{V}_4^{6np}, \mathcal{V}_4^{5bn}, \mathcal{V}_4^{6bn}, \mathcal{V}_4^{5cnp}, \mathcal{V}_4^{6cnp}, \mathcal{V}_4^{5bnps}, \mathcal{V}_4^{6bnps}, \mathcal{V}_4^{5cnps}, \mathcal{V}_4^{6cnps}\} \), \( X \models_{\mathcal{R}} \phi \) implies \( X \models_{\mathcal{R}} \phi \).
Proof: According to the proof of Theorem 6.1, we need only to show that for every \( w, v \in W \), \( D_w \subseteq D_v \) if \( wRv \). So, using (2.41), (2.7) and Lemma 5.2, we obtain that for every \( w \in W \) and every 1\(^{st}\) sort variable \( x \) of \( L' \), \( E(x) \rightarrow LE(x) \in w \). And supposing that \( x \in D_w \), by the definition of \( D_w \) we have that \( x \) is a 1\(^{st}\) sort variable of \( L' \) and \( E(x) \in w \), and hence, \( LE(x) \in w \). Consequently, \( E(x) \in v \), i.e., \( x \in D_v \), for every \( v \in W \) such that \( wRv \). This means that \( D_w \subseteq D_v \) for every \( v \in W \) such that \( wRv \).

Theorem 6.3 (Strong completeness) Let \( X \) be a set of formulas.
Then, for every \( \mathcal{R} \in \{ \mathcal{V}^A5\star, \mathcal{V}^A5s\star, \mathcal{V}^A5n\star, \mathcal{V}^A5ns\star, \mathcal{V}^A5nps\star, \mathcal{V}^Adn\star, \mathcal{V}^Adns\star \} \), \( X \models_\mathcal{R} \phi \) implies \( X \vdash_\mathcal{R} \phi \).

Proof: Like that of Theorem 5.3 (i)-(viii) in [15], respectively.

Theorem 6.4 (Strong completeness) Let \( X \) be a set of formulas.
Then, for every \( \mathcal{R} \in \{ \mathcal{V}^A5p\star, \mathcal{V}^A5ps\star, \mathcal{V}^Adp\star, \mathcal{V}^A5np\star, \mathcal{V}^Adps\star, \mathcal{V}^A5nps\star, \mathcal{V}^Adnp\star, \mathcal{V}^Adnp\star \} \), \( X \models_\mathcal{R} \phi \) implies \( X \vdash_\mathcal{R} \phi \).

Proof: Putting together the proofs of Theorem 6.2 and of Theorem 6.3, respectively.

7 Appendix

Particularly important are the following theorems:

T1: For every \( \mathcal{Th} \in \{ \mathcal{V}^A5, \mathcal{V}^A5b, \mathcal{V}^A5\star \} \), \( \vdash_{\mathcal{Th}} \exists \xi E(\xi) \)

Proof: Cf. the proof of T1 in [15].

T2: For every \( \mathcal{Th} \in \{ \mathcal{V}^A5, \mathcal{V}^A5b, \mathcal{V}^A5\star \} \), \( \vdash_{\mathcal{Th}} \exists \xi [(\exists \xi \phi \rightarrow \phi) \land E(\xi)] \)

Proof: Cf. the proof of T2 in [15].

T3: For every \( \mathcal{Th} \in \{ \mathcal{V}^A5, \mathcal{V}^A5b, \mathcal{V}^A5\star \} \), \( \vdash_{\mathcal{Th}} E(\zeta) \land \phi(\zeta/\zeta) \rightarrow \exists \xi \phi \)

Proof: Trivially, by (2.7).

T4: For every \( \mathcal{Th} \in \{ \mathcal{V}^A5, \mathcal{V}^A5b, \mathcal{V}^A5\star \} \), \( \vdash_{\mathcal{Th}} \exists \alpha (\exists \alpha \phi \rightarrow \phi) \)

Proof: See, the proof of T4 in [15].

T5: For every \( \mathcal{Th} \in \{ \mathcal{V}^A5, \mathcal{V}^A5b, \mathcal{V}^A5\star \} \), \( \vdash_{\mathcal{Th}} \exists \alpha G(y) \)

Proof: See, the proof of T5 in [15].
T6: For every \( \text{Th} \in \{ \forall A^5, \forall^A b, \forall^A 5^* \} \), \( \vdash_{\text{Th}} \exists x G(x) \rightarrow \exists y G(x) \)

Proof: See, the proof of T6 in [15].

T7: For every \( \text{Th} \in \{ \forall A^5, \forall^A b, \forall^A 5^* \} \), \( \vdash_{\text{Th}} \exists x G(x) \)

Proof: See, the proofs of T7 and T8 in [15].

T8: For every \( \text{Th} \in \{ \forall A^5, \forall^A b, \forall^A 5^* \} \), \( \vdash_{\text{Th}} \exists x G(x) \rightarrow \exists y G(x) \)

Proof: See, the proof of T9 in [15].

T9: For every \( \text{Th} \in \{ \forall A^5, \forall^A b, \forall^A 5^* \} \), \( \vdash_{\text{Th}} \exists (G(x) \land E(x)) \)

Proof: See, the proof of T10 in [15].

T10: For every \( \text{Th} \in \{ \forall A^5, \forall^A b, \forall^A 5^* \} \), \( \vdash_{\text{Th}} G(x) \land G(y) \rightarrow (x \frac{1}{2} y) \)

Proof: See, the proof of T12 in [15].

T11: For every \( \text{Th} \in \{ \forall A^5, \forall^A b, \forall^A 5^* \} \), \( \vdash_{\text{Th}} E(A) \rightarrow LE(A) \)

Proof:

1. \( \forall \alpha \text{LE}(\alpha) \land E(A) \rightarrow \text{LE}(A) \) (2.7)
2. \( \forall \forall \text{LE}(\alpha) \) (2.8), R2
3. \( \forall \forall \text{LE}(\alpha) \) (2.15), 2, R1
4. \( E(A) \rightarrow \text{LE}(A) \) 1, 3, R1

T12: For every \( \text{Th} \in \{ \forall A^5, \forall^A b, \forall^A 5^* \} \), \( \vdash_{\text{Th}} \forall \alpha, \beta ( \alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow \text{L}(\alpha \frac{2}{2} \beta )) \)

Proof:

1. \( \alpha \text{ Ess } x \rightarrow \forall \beta ( \text{L}(\beta(x) \rightarrow \forall y(\alpha(y) \rightarrow \beta(y))) \) (2.2)
2. \( \forall \beta ( \text{L}(\beta(x) \rightarrow \forall y(\alpha(y) \rightarrow \beta(y))) \rightarrow (E(\alpha) \rightarrow (\forall y(\alpha(y) \rightarrow \alpha(y))) \rightarrow \text{L}(\alpha(x))) \) (2.4), (2.7), R1
3. \( \alpha \text{ Ess } x \rightarrow (E(\alpha) \rightarrow \text{L}(\alpha(x))) \) (2.4), R3, R2, 1, 2, R1
4. \( \beta \text{ Ess } x \rightarrow \forall \alpha ( \text{L}(\alpha(x) \rightarrow \forall y(\beta(y) \rightarrow \alpha(y))) \) (2.2)
5. \( \forall \alpha ( \text{L}(\alpha(x) \rightarrow \forall y(\beta(y) \rightarrow \alpha(y))) \rightarrow (E(\beta) \rightarrow (\forall y(\beta(y) \rightarrow \beta(y))) \rightarrow \text{L}(\beta(x))) \) (2.4), (2.7), R1
6. \( \beta \text{ Ess } x \rightarrow (E(\beta) \rightarrow \text{L}(\beta(x))) \) (2.4), R3, R2, 4, 5, R1
7. \( \alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow (E(\beta) \rightarrow (\text{L}(\beta(x) \leftrightarrow \forall y(\alpha(y) \rightarrow \beta(y))) \land \text{L}(\beta(x))) \) (2.4), (2.2), (2.7), (2.5), 6, R1

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8. \( \alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow (E(\beta) \rightarrow L \forall y(\alpha(y) \rightarrow \beta(y))) \)

9. \( \alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow (E(\alpha) \rightarrow (L \alpha(x) \leftrightarrow L \forall y(\beta(y) \rightarrow \alpha(y))) \land L \alpha(x)) \)

10. \( \alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow (E(\alpha) \rightarrow L \forall y(\beta(y) \rightarrow \alpha(y))) \)

11. \( E(\alpha) \land E(\beta) \rightarrow (\alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow L \forall y(\alpha(y) \leftrightarrow \beta(y))) \)

12. \( \forall \alpha, \beta(\alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow L(\alpha \approx \beta)) \)

\( \text{T13: } \vdash_{\mathcal{V}^{A5}} \forall \alpha, \beta(\alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow (\alpha \approx \beta)) \)

**Proof:** The steps 1 – 11 are the same as for T12

12. \( \forall \alpha, \beta(\alpha \text{ Ess } x \land \beta \text{ Ess } x \rightarrow (\alpha \approx \beta)) \)

\( \text{T14: } \) For every \( \text{Th} \in \{\mathcal{V}^{A5}, \mathcal{V}^{A5b}, \mathcal{V}^{A5*}\} \), \( \vdash_{\text{Th}} \forall x \text{NE}(x) \)

**Proof:**

1. \( \forall \alpha(\alpha(x) \rightarrow \exists y \alpha(y)) \)

2. \( \forall x L \forall \alpha(\alpha(x) \rightarrow \exists y \alpha(y)) \)

3. \( \forall x \forall \alpha L(\alpha(x) \rightarrow \exists y \alpha(y)) \)

4. \( \forall x \forall \alpha (L \alpha(x) \rightarrow L \exists y \alpha(y)) \)

5. \( \forall \beta(L \beta(x) \leftrightarrow L \forall y(\alpha(y) \rightarrow \beta(y))) \land E(\alpha) \rightarrow (L \alpha(x) \leftrightarrow L \forall y(\alpha(y) \rightarrow \alpha(y))) \)

6. \( \forall \beta(L \beta(x) \leftrightarrow L \forall y(\alpha(y) \rightarrow \beta(y))) \land E(\alpha) \rightarrow L \alpha(x) \)

7. \( \forall \alpha E(\alpha) \rightarrow \forall x \forall \alpha(\forall \beta(L \beta(x) \leftrightarrow L \forall y(\alpha(y) \rightarrow \beta(y))) \rightarrow L \alpha(x)) \)

8. \( \forall x \forall \alpha \forall \beta(L \beta(x) \leftrightarrow L \forall y(\alpha(y) \rightarrow \beta(y))) \rightarrow L \exists y \alpha(y) \)

9. \( \forall x \forall \alpha(\alpha \text{ Ess } x \rightarrow L \exists y \alpha(y)) \)

10. \( \forall x \text{NE}(x) \)

\( \text{T15: } \) For every \( \text{Th} \in \{\mathcal{V}^{A5}, \mathcal{V}^{A5b}, \mathcal{V}^{A5*}\} \), \( \vdash_{\text{Th}} \forall \alpha(\mathcal{P}(\alpha) \rightarrow L \forall x(G(x) \rightarrow \alpha(x))) \)

**Proof:**

1. \( M(G(x) \land \neg \alpha(x)) \rightarrow MG(x) \)

2. \( M(G(x) \land \neg \alpha(x)) \rightarrow LG(x) \)

3. \( M(G(x) \land \neg \alpha(x)) \rightarrow LG(x) \land M(G(x) \land \neg \alpha(x)) \)

4. \( \neg LG(x) \lor \neg M(G(x) \land \neg \alpha(x)) \rightarrow \neg M(G(x) \land \neg \alpha(x)) \)

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5. \((LG(x) \rightarrow L(G(x) \rightarrow \alpha(x))) \rightarrow L(G(x) \rightarrow \alpha(x))\) \((2.4), 4, R1\)
6. \(\forall xL(G(x) \rightarrow \alpha(x)) \land E(x) \rightarrow L(G(x) \rightarrow \alpha(x))\) \((2.7)\)
7. \(\forall xL(G(x) \rightarrow \alpha(x)) \land G(x) \rightarrow L(G(x) \rightarrow \alpha(x))\) \((2.4), (2.9), (2.7), (2.28)\), 6, R1
8. \(\forall xL(G(x) \rightarrow \alpha(x)) \land LG(x) \rightarrow L(G(x) \rightarrow \alpha(x))\) \((2.4), (2.27), 7, R1\)
9. \(\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow (LG(x) \rightarrow L(G(x) \rightarrow \alpha(x)))\) \((2.4), 8, R1\)
10. \(\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow L(G(x) \rightarrow \alpha(x))\) \((2.4), 5, 9, R1\)
11. \(\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow \forall x(G(x) \rightarrow \alpha(x))\) \(10, R4_0\)
12. \(G(x) \rightarrow \forall \alpha(\neg \alpha(x) \rightarrow \neg P(\alpha))\) \((2.4), (2.1), R1\)
13. \(E(\alpha) \rightarrow (G(x) \land M \land \neg \alpha(x) \rightarrow \neg P(\alpha))\) \((2.4), 12, (2.7), R1\)
14. \(E(\alpha) \rightarrow (MG(x) \land M \land \neg \alpha(x) \rightarrow \neg P(\alpha))\) \((2.4), (2.1), R2, (2.12), (b)(or, (5), (2.27)), 13, R1\)
15. \(E(\alpha) \rightarrow (M(G(x) \land \neg \alpha(x)) \rightarrow \neg P(\alpha))\) \((2.4), R2, (2.12), 14, R1\)
16. \(E(\alpha) \rightarrow (\exists xM(G(x) \land \neg \alpha(x)) \rightarrow \neg P(\alpha))\) \((2.4), 15, R3, (2.5), (2.6), R1\)
17. \(E(\alpha) \rightarrow (P(\alpha) \rightarrow \forall xL(G(x) \rightarrow \alpha(x)))\) \((2.4), 16, R1\)
18. \(E(\alpha) \rightarrow (P(\alpha) \rightarrow L\forall x(G(x) \rightarrow \alpha(x)))\) \((2.4), 11, 17, R1\)
19. \(\forall \alpha(P(\alpha) \rightarrow L\forall x(G(x) \rightarrow \alpha(x)))\) \(18, R3, (2.5), (2.8), R1\)
20. \(\forall \alpha(L\forall x(G(x) \rightarrow \alpha(x)) \rightarrow P(\alpha))\) \((2.25), (2.7), (2.28), R1\)
21. \(\forall \alpha(P(\alpha) \leftrightarrow L\forall x(G(x) \rightarrow \alpha(x)))\) \((2.4), 18, 19, R1\)

\textbf{T16:} For every \(Th \in \{\forall^4, 5, \forall^4 b, \forall^4 5\ast\}, \vdash_{Th} \forall \alpha(\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow L\forall x(G(x) \rightarrow \alpha(x)))\)

Proof:

1. \(E(\alpha) \rightarrow (G(x) \rightarrow (L\alpha(x) \rightarrow P(\alpha)))\) \((2.7), (2.1), R1\)
2. \(E(\alpha) \rightarrow ((G(x) \rightarrow L\alpha(x)) \rightarrow (G(x) \rightarrow P(\alpha)))\) \((2.4), 1, R1\)
3. \(E(\alpha) \rightarrow ((LG(x) \rightarrow L\alpha(x)) \rightarrow (G(x) \rightarrow P(\alpha)))\) \((2.4), (2.1), (2.28), (2.29), 2, R1\)
4. \(E(\alpha) \rightarrow (L(G(x) \rightarrow \alpha(x)) \rightarrow (G(x) \rightarrow P(\alpha)))\) \((2.4), (2.12), 3, R1\)
5. \(E(\alpha) \rightarrow (\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow \forall x(G(x) \rightarrow P(\alpha)))\) \(4, R3, (2.5), R1\)
6. \(E(\alpha) \rightarrow (\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow (\exists xG(x) \rightarrow P(\alpha)))\) \((2.4), (2.5), 5, R1\)
7. \(E(\alpha) \rightarrow (\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow (\exists xLG(x) \rightarrow P(\alpha)))\) \((2.4), (2.27), R3, (2.5), 6, R1\)
8. \(E(\alpha) \rightarrow (\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow (L \exists xG(x) \rightarrow P(\alpha)))\) \((2.4), (2.1), (2.5), (2.29), T8, 7, R1\)
9. \(E(\alpha) \rightarrow (\forall xL(G(x) \rightarrow \alpha(x)) \rightarrow P(\alpha))\) \((2.4), 8, T7, R1\)
10. \(P(\alpha) \rightarrow (E(\alpha) \rightarrow L\forall x(G(x) \rightarrow \alpha(x)))\) \((2.4), 9, T15, R1\)
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11. \( E(\alpha) \rightarrow (\forall x L(G(x) \rightarrow \alpha(x)) \rightarrow \forall x (G(x) \rightarrow \alpha(x))) \) \hspace{1cm} (2.4), 9, 10, R1

12. \( \forall \alpha (\forall x L(G(x) \rightarrow \alpha(x)) \rightarrow \forall x (G(x) \rightarrow \alpha(x))) \) \hspace{1cm} 11, R3, (2.5), (2.8), R1

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**T17:** For every \( Th \in \{ \mathcal{V}^{45}, \mathcal{V}^{4b}, \mathcal{V}^{45\ast} \} \), \( \vdash_{Th} \forall x (G(x) \rightarrow \alpha(x)) \rightarrow \forall x L(G(x) \rightarrow \alpha(x)) \)

Proof:

1. \( \forall \alpha (\alpha(x) \rightarrow E(x)) \rightarrow (E(G) \rightarrow (G(x) \rightarrow E(x))) \) \hspace{1cm} (2.7)

2. \( G(x) \rightarrow E(x) \) \hspace{1cm} 1, (2.9), (2.28), R1

3. \( (E(x) \rightarrow (G(x) \rightarrow \alpha(x))) \rightarrow (G(x) \rightarrow (G(x) \rightarrow \alpha(x))) \) \hspace{1cm} (2.4), 2, R1

4. \( (E(x) \rightarrow (G(x) \rightarrow \alpha(x))) \rightarrow (G(x) \rightarrow \alpha(x)) \) \hspace{1cm} (2.4), 3, R1

5. \( \forall x (G(x) \rightarrow \alpha(x)) \rightarrow (G(x) \rightarrow \alpha(x)) \) \hspace{1cm} (2.4), (2.7), 4, R1

6. \( \forall x (G(x) \rightarrow \alpha(x)) \rightarrow L(G(x) \rightarrow \alpha(x)) \) \hspace{1cm} 5, R2, (2.12), R1

7. \( \forall x (G(x) \rightarrow \alpha(x)) \rightarrow \forall x L(G(x) \rightarrow \alpha(x)) \) \hspace{1cm} 6, R3, (2.5), (2.6), R1

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**T18:** For every \( Th \in \{ \mathcal{V}^{45n}, \mathcal{V}^{4bn}, \mathcal{V}^{45n\ast} \} \), \( \vdash_{Th} P(NE) \)

Proof:

1. \( \forall \alpha L(\alpha(x) \rightarrow \exists y \alpha(y)) \) \hspace{1cm} (2.4), (2.18), T3, R3, (2.5), R2, (2.12), (2.15), R2

2. \( \forall \alpha (La(x) \rightarrow L\exists y \alpha(y)) \) \hspace{1cm} (2.12), R3, (2.5), 1, R1

3. \( \forall \alpha (\forall \beta (L\beta(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow \beta(y)))) \wedge E(\alpha) \rightarrow (La(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow \alpha(y)))) \) \hspace{1cm} (2.7), R3

4. \( \forall \alpha (\forall \beta (L\beta(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow \beta(y)))) \rightarrow L(\alpha(x)) \) \hspace{1cm} (2.4), R3, (2.5), (2.6), R2, (2.12), 3, (2.8), R1

5. \( \forall \alpha (\forall \beta (L\beta(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow \beta(y)))) \rightarrow L\exists y \alpha(y)) \)

6. \( \forall \alpha (L\exists y \alpha(y) \rightarrow ((P(\alpha) \leftrightarrow La(x)) \rightarrow L\exists y \alpha(y))) \) \hspace{1cm} (2.4), R3

7. \( \forall \alpha (\forall \beta (L\beta(x) \leftrightarrow L\forall y[\alpha(y) \rightarrow \beta(y)])) \rightarrow ((P(\alpha) \leftrightarrow La(x)) \rightarrow L\exists y \alpha(y))) \)

8. \( \forall \alpha (\alpha \text{ Ess } x \rightarrow ((P(\alpha) \leftrightarrow La(x)) \rightarrow L\exists y \alpha(y))) \) \hspace{1cm} (2.4), R3, (2.5), (2.2), 7, R1

9. \( \forall \alpha (P(\alpha) \leftrightarrow La(x)) \rightarrow \forall \alpha (\alpha \text{ Ess } x \leftrightarrow L\exists y \alpha(y)) \) \hspace{1cm} (2.4), R3, (2.5), 8, R1

10. \( (G(x) \rightarrow \forall \alpha (P(\alpha) \leftrightarrow La(x))) \rightarrow (G(x) \rightarrow \forall \alpha (\alpha \text{ Ess } x \rightarrow L\exists y \alpha(y))) \) \hspace{1cm} (2.4), 9, R1

11. \( G(x) \rightarrow \forall \alpha (\alpha \text{ Ess } x \rightarrow L\exists y \alpha(y)) \)

12. \( \forall \alpha (\alpha \text{ Ess } x \rightarrow L\exists y \alpha(y)) \rightarrow NE(x) \rightarrow (G(x) \rightarrow NE(x)) \) \hspace{1cm} (2.4), 11, R1

13. \( L\forall x (G(x) \rightarrow NE(x)) \) \hspace{1cm} 12, (2.3), R3, R2

14. \( P(G) \wedge L\forall x (G(x) \rightarrow NE(x)) \rightarrow P(NE) \) \hspace{1cm} (2.25), (2.39)

15. \( P(NE) \)

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T19: For any $\text{Th} \in \{\forall^A5s, \forall^A\text{bps}, \forall^A5\text{ps}\}$, $\vdash_{\text{Th}} \forall\alpha(\alpha \text{ Ess } x \land \alpha \text{ Ess } y \rightarrow L(E(x) \lor E(y) \rightarrow (x \approx y)))$

Proof:

1. $\forall\beta(L\beta(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow \beta(y))) \rightarrow (E(l_x) \rightarrow (Ll_x(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow l_x(y))))$ (2.7), (2.40)
2. $Ll_x(x)$ (2.38), R2
3. $\alpha \text{ Ess } x \rightarrow L\forall y(\alpha(y) \rightarrow (x \approx y))$ (2.4), (2.2), 1, (2.40), 2, R1
4. $\alpha \text{ Ess } x \rightarrow L(E(y) \rightarrow (\alpha(y) \rightarrow (x \approx y)))$ (2.4), (2.7), R2, (2.12), 3, R1
5. $\forall\beta(L\beta(y) \leftrightarrow L\forall x(\alpha(x) \rightarrow \beta(x))) \rightarrow (E(\alpha) \rightarrow (L\alpha(y) \leftrightarrow L\forall x(\alpha(x) \rightarrow \alpha(x))))$ (2.7)
6. $\alpha \text{ Ess } x \rightarrow (E(\alpha) \rightarrow L\alpha(y))$ (2.4), R3, R2, 5, R1
7. $\alpha \text{ Ess } x \rightarrow (E(\alpha) \rightarrow L(E(y) \rightarrow (x \approx y)))$ (2.4), R1, (2.12), 4, 6, R1
8. $\forall\beta(L\beta(y) \leftrightarrow L\forall x(\alpha(x) \rightarrow \beta(x))) \rightarrow (E(l_y) \rightarrow (Ll_y(y) \leftrightarrow L\forall x(\alpha(x) \rightarrow l_y(x))))$ (2.7), (2.40)
9. $Ll_y(y)$ (2.38), R2
10. $\alpha \text{ Ess } y \rightarrow L\forall x(\alpha(x) \rightarrow (y \approx x))$ (2.4), (2.2), 8, (2.40), 9, R1
11. $\alpha \text{ Ess } y \rightarrow L(E(x) \rightarrow (\alpha(x) \rightarrow (y \approx x)))$ (2.4), (2.7), R2, (2.12), 10, R1
12. $\forall\beta(L\beta(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow \beta(y))) \rightarrow (E(\alpha) \rightarrow (L\alpha(x) \leftrightarrow L\forall y(\alpha(y) \rightarrow \alpha(y))))$ (2.7)
13. $\alpha \text{ Ess } y \rightarrow (E(\alpha) \rightarrow L\alpha(x))$ (2.4), R3, R2, 12, R1
14. $\alpha \text{ Ess } y \rightarrow (E(\alpha) \rightarrow L(E(x) \rightarrow (y \approx x)))$ (2.4), R1, (2.12), 11, 13, R1
15. $\alpha \text{ Ess } x \land \alpha \text{ Ess } y \rightarrow (E(\alpha) \rightarrow L(E(x) \lor E(y) \rightarrow (x \approx y)))$ (2.4), R2, (2.12), 7, 14, R1
16. $\forall\alpha(\alpha \text{ Ess } x \land \alpha \text{ Ess } y \rightarrow L(E(x) \lor E(y) \rightarrow (x \approx y)))$ 11, R3, (2.5), (2.8), R1

T20: For any $\text{Th} \in \{\forall^A5\text{ps}, \forall^A\text{bps}, \forall^A5\text{ps}\}$, $\vdash_{\text{Th}} \forall x(l_x \text{ Ess } x)$

Proof:

1. $\alpha(x) \rightarrow ((x \approx y) \rightarrow \alpha(y))$ (2.4), (2.20), R1
2. $\alpha(x) \rightarrow (l_x(y) \rightarrow \alpha(y))$ (2.4), (2.38), 1, R1
3. $\alpha(x) \rightarrow \forall y(l_x(y) \rightarrow \alpha(y))$ 2, R3, (2.5), (2.6), R1
4. $L\alpha(x) \rightarrow L\forall y(l_x(y) \rightarrow \alpha(y))$ 3, R2, (2.12), R1
5. $\forall x(L\alpha(x) \rightarrow L\forall y(l_x(y) \rightarrow \alpha(y)))$ 4, R3
6. \((x \approx x) \to \alpha(x)\) \quad (2.20)

7. \(\forall y((x \approx y) \to \alpha(y)) \to \alpha(x)\) \quad (2.4), (2.7), 6, R1

8. \(\forall y(I_x(y) \to \alpha(y)) \to \alpha(x)\) \quad (2.4), (2.38), 7, R1

9. \(\forall y(I_x(y) \to \alpha(y)) \to \alpha(x)\) \quad 8, R2, (2.12), R1

10. \(\forall x LE(x) \to \forall x(L\forall y(I_x(y) \to \alpha(y)) \to \alpha(x))\) \quad (2.4), 9, R3, (2.5), R1

11. \(\forall x(L\forall y(I_x(y) \to \alpha(y)) \to L\alpha(x))\) \quad 10, (2.41), R1

12. \(\forall x(I_x \text{ Ess } x)\) \quad (2.4), R3, (2.5), 5, 11, (2.3), R1

Bibliography


Part V

Ontological Proofs and Kinds of Necessity
Conceptual Modality and the Ontological Argument

Anthony C. Anderson

1 The Modal Ontological Argument

Modern versions of the ontological argument involve the use of modal notions such as necessity and possibility. These arguments are at least implicit in St. Anselm’s writings and reach their most sophisticated forms in the works of Alvin Plantinga [11] and Kurt Gödel [6]. My immediate purpose is to secure certain modal principles used in such versions of the ontological argument.

Robert Merrihew Adams [1] observed that a certain version of the argument is valid in the relatively weak modal logic $\mathcal{B}$. All the modal formulations depend on (at least) this not-strikely-self-evident modal principle:

$$(B) \quad \Diamond \Box P \rightarrow P$$

“If it is possible that it’s necessary that $P$, then $P$ is true.”

Nathan Salmon [12] complains that this principle does not seem to be “logically necessary”. Perhaps his point is that $(B)$ is just not sufficiently evident to merit its inclusion as part of logic. Whether or not we call it a principle of logic is of no real consequence for the present purpose, so this part of Salmon’s critique can safely be ignored. However, Salmon produces a rather convincing refutation of the characteristic principle of $(S4)$, viz.

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\((S4)\) \(\square P \rightarrow \square \square P\)

This bears on the present project as follows. The intuitive arguments for \((B)\), such as they are, are just as good (or just as bad) when they are used in support of the stronger modal logic \(S5\), characterized especially by the principle:

\((S5)\) \(\Diamond P \rightarrow \square \Diamond P\)

Now \((S5)\), in the presence of uncontroversial modal logical principles, implies both \((B)\) and \((S4)\). If Salmon’s criticisms succeed in defeating \((S4)\), then they also refute \((S5)\). This doesn’t directly impugn \((B)\), but it reflects very badly on the arguments to which one might appeal in support of it, proceeding, as they often do, by way of first trying to justify something in the neighborhood of \((S5)\). So whether or not principle \((B)\) is properly said to be a principle of modal logic, we must consider Salmon’s argument.

There are two more direct attacks on \((B)\) in the recent philosophical literature by Michael Dummett [5] and Yannis Stephanou [13]. A much older and almost entirely neglected criticism of \((S5)\) may be extracted from some observations of Alonzo Church [4] in his first paper on the Frege-Church logic of sense and denotation.

To deal with the arguments of Salmon, Dummett and Stephanou, I delineate a notion of modality appropriate for the purpose. When this is done, the arguments will be found to be simply irrelevant to the modal logic thus construed. Church’s argument turns on an interesting deficiency in the now standard formulations of modal logic and the response points to a needed addition to the basic notions Church would utilize.

The upshot for the modal ontological argument will be that everything worthy of debate depends on the claim that it is possible (in the appropriate sense) that God exists. Along the way I will argue that this proposition is not to be regarded as obvious, easily established by reflection or thought-experiment, or otherwise to be taken for granted. Leibniz and Gödel both attempted proofs of the possibility premise, but it is, or should be, agreed that their arguments do not succeed. The best version known to me just uses the possibility premise more or less as it stands.

Here’s the general form of the modal ontological argument. Let \(G\) abbreviate “God exists”.

\((1)\) \(\Diamond G\) \hspace{1cm} \text{Premise}

\((2)\) \(\Box (G \rightarrow \Box G)\) \hspace{1cm} \text{Premise}

\((3)\) \(\Box (G \rightarrow \Box G) \rightarrow (\Diamond G \rightarrow \Diamond \Box G)\) \hspace{1cm} \text{Modal logic } K^*(\text{Premise})
I have labeled even uncontroversial principles of modal logic as premises. The important thing is not that they are “logic” but that we can be justified in taking them to be true. Premise (3) is not quite an instance of the modal principle usually called \((K)\), but follows from it fairly easily, given some other elementary principles. It will suffice for my purposes that the notions of necessity and possibility used allow us to see that if it is necessarily the case that if \(P\) then \(Q\), and if \(P\) is possible, then \(Q\) is possible.

Premise (2) is not uncontroversial. Plantinga and Gödel just define the property of being God or, in Plantinga’s case, of being unsurpassibly great, so that the analogue (2) is provable. Kant criticized the ontological argument and concluded that all that it proves is that if God exists, then He necessarily exists. Well, that’s what (2) says, or rather, it claims that this is itself a necessary truth. Perhaps the most natural way of characterizing the Christian God does not guarantee the truth of (2). Nevertheless, it seems clear that some concepts in the near neighborhood do guarantee the truth of (2). What’s wrong with the argument if we take it to concern the existence of such a God?

Premise (6) is, as already observed, initially the most suspicious. It is a direct instance of \((B)\) and whether or not this principle is correct depends of course on what concept of modality is being used. I turn first to the problem of specifying the appropriate notion.

2 Conceptual Modality

Here is an edited remark by Kurt Gödel:

In a second sense a proposition is called [necessary] if it [must hold] “owing to the meaning of the concepts occurring in it”, where this meaning may
perhaps be undefinable (i.e., irreducible to anything more fundamental) ([7], p. 151)

He immediately adds this elaboration [also edited]:

[I]t is perfectly possible that ... a proposition [necessary in this sense] might be undecidable (or decidable only with [a certain] probability). For our knowledge of the world of concepts may be as limited and incomplete as that of the world of things.²

The editing I have done replaces “analytic” with “necessary” and “analytic in this sense” with “necessary in this sense”. Also I have put “must hold” for “holds”. With these emendations, I think that Gödel may be taken to be pointing toward a natural, useful, and important conception of modality. Because the necessity involved holds due to the meaning or better, the “nature”, of the concepts involved, I will call it “conceptual necessity” and the general modality “conceptual modality”. This is not an entirely happy choice, but I can think of none better. Unfortunately, this term has been used in the literature for a variety of quite different kinds of modality.

I am going to take for granted here the Platonism, or at least objectivism, about concepts that Gödel is presupposing. Concepts, in this sense, necessarily exist and they have certain necessary relations to one another. We may not be able to discover these relations by a priori reflection or proof, but they may hold all the same. I distinguish three kinds of concepts: propositions, attributes, and individual concepts. This is not Gödel’s terminology, although I think he would have little reason to quarrel with it. Following Frege, I will take propositions to be the kinds of abstract entities appropriate to be the meanings of sentences when such meanings are context independent. If there are Russellian propositions that have contingent things as constituents (and I think that there are), then they must be distinguished from these purely conceptual entities. Propositions (of this Fregean sort) are taken to be concepts of truth-values, attributes – the sorts of things that can be attributed to something or other – are concepts of sets (or better: characteristic functions of sets) and individual concepts are concepts of, well, individuals of various kinds. It should be emphasized that the use of the term ‘concept’ is not meant to imply any subjective or human psychological element in the notion. An attribute is

²I cannot now find the exact source of this quote. I am morally certain that it is due to Gödel.
not something that depends for its existence on the existence of any human mind.

Among these, certain relations must hold, not simply because some of them contain others as a parts or constituents, but just because of what the concepts are. Being red excludes being blue, but not because one is included in the other. These are what Leibniz called “disparities” and their necessary relations are to be included in the theory of modality that I contemplate. Notice that these are attributes and so there is nothing especially psychological\textsuperscript{3} about the claim that it is conceptually necessary that being red excludes being blue. Formally, I would take incompatibility or disparity as a primitive relation between concepts. It is not too far off to think of incompatibility as inconsistency – although there is no intended reference to formal provability, or indeed to any kind of provability. There is no implication that the claim that two concepts are incompatible or inconsistent with one another implies a formal contradiction in such a way that it can be deduced using some standard rules of logic. I suppose “Blue” and “not being a color” are inconsistent with one another. And “being an everywhere differentiable real function” and “being a nowhere continuous real function” are not incompatible, although it takes some ingenuity to show this.

Well, what is this modality? I think that it may be “metaphysical, or absolute, modality as determined by the essential properties and relations of concepts”. I do not see that it contains an epistemic component, although I am not completely confident that it doesn’t coincide in extension with “ideal knowability a priori when there are no limits of any kind placed on the knower”.

To see that it is not the same as a priori knowability by us – or by finite idealizations of us – consider this example. A perfect number is a number that is the sum of its aliquot parts, that is, its smaller factors. Thus $6 = 1 + 2 + 3$ and $28 = 1 + 2 + 4 + 7 + 14$ are perfect numbers. Every known perfect number is even, but there is so far no proof of the general claim. Suppose that there really are no odd perfect numbers. Then for any odd number the calculation can be carried out showing that it is not the sum of its smaller factors. And all of these calculations can be given as proofs in Peano Arithmetic. So in principle each such proof would be available

\textsuperscript{3} Timothy Williamson pointed out that it is quite odd to say that something is a concept if it is not graspmable by any thinking being. I am willing to accept this – if the thinking being is unlimited. All attributes are graspable in principle, but perhaps not by us or by any possible human being with finite capacities.
to us, although some would be too long to be carried out by any actual human being. Even so it might be that there is no humanly usable proof of the general claim that no odd number is a perfect number. But it would be true, and, I say, conceptually necessary. Each of the non-identities is conceptually necessary. For example, “9 ≠ 1 + 3” and “1 and 3 are the only smaller factors of 9” are both conceptually necessary, and the same for all the other cases. The quantifier in “It is not the case that there exists an x, such that x is odd and x is a perfect number” ranges over the natural numbers, the domain being determined by that very concept. So it would have to be true and true because of natures of the concepts involved.

It is thus not quite clear what is the relation between conceptual necessity and “ideal a priori knowability”. The example given indicates that if finiteness of proof is required, then conceptual necessity will not obviously coincide with provability. But there is another problem. What axioms are to be allowed for an ideal a priori knower? Here I am at a loss. I have no doubt that something is conceptually necessary if and only if God knows it simply from the consideration of concepts. But I suppose that He just sees that it is so and necessarily so.

I have avoided the terms “logical necessity” and “analytic necessity” as inappropriate for the present notion. Ideally there should here follow a long discussion as to why these terms are unsuitable, together with examples from the literature. For brevity, let me just say that conceptual modality does not depend on form, generality, some specified list of “logical notions”, conceivable, semantical rules, definition, stipulation, or epistemic notions such as provability or deducibility.

Let us suppose that we have a grasp, perhaps somewhat halting and unsure, of the idea of conceptual necessity. What of principle (B)? There is a fairly convincing argument that this principle is correct for conceptual

4 Saul Kripke [10] uses a similar example for a similar purpose.

5 There are some oddities when we consider propositions containing names of abstract entities. “John’s favorite concept is a concept of a number” is not conceptually necessary. Suppose we fix the reference of the name “Bob” to be John’s favorite concept and it is in fact: the even prime. Is the Russellian proposition “Bob is a concept of a number” (if it is one) conceptually necessary? Well, all of its constituents are concepts. And it is necessary simply in virtue of their essential relations. Here we seem to have the “two ways” something can be a constituent of a proposition postulated by Nathan Salmon to deal with a problem that led Russell to his Theory of Descriptions.
modality. Perhaps it should be said instead, that if we elaborate the concepts involved, it will just be seen that \((B)\) is necessarily true. It may be that the assumptions are so close to the principle itself, that this shouldn’t be counted as an argument from independently knowable premises. Suppose that a proposition is true. Remember that propositions (in the present relevant sense) are purely conceptual entities. If a proposition is true, then the concepts that make it up couldn’t be incompatible with one another. Of course, if the proposition is true, those concepts aren’t incompatible with one another. But more, that very proposition, made up of those very concepts, could not be impossible. Those constituent concepts are essential to its very identity and their relations of compatibility or incompatibility are necessary or essential to them. So if a proposition is true, then it is conceptually impossible that it is inconsistent.

It is perhaps slightly more natural, and more convincing, to argue as follows. If a proposition is true, then it is conceptually possible. But conceptual possibility, and indeed related modal properties, are essential properties of the propositions that have them. Hence, if a proposition is true, then it is necessarily possible. The modally sophisticated reader will see that this is just the usual derivation of (an equivalent of) \((B)\) from \((S5)\).

Perhaps some will say that this is just a begging of the question. Anyone likely to doubt \((B)\) or \((S5)\) might be just as suspicious of the reasoning using the principles just proclaimed. Maybe so. But then, maybe not. Thus understood, as concerning the necessary relations of concepts, the principles seem (to me) to be impeccable.

Using a black diamond for conceptual possibility or consistency, we seem to have:

\[(B')\quad P \rightarrow \neg \Diamond \neg \Diamond P\]

The reasoning is completely general, so we can choose \(P\) to be \(\neg Q\):

\[\begin{align*}
(1) \quad & \neg Q \rightarrow \neg \Diamond \neg \Diamond \neg Q \\
(2) \quad & \Diamond \neg \Diamond \neg Q \rightarrow Q \\
(3) \quad & \Diamond \Box Q \rightarrow Q
\end{align*}\]

This is the desired form of principle \((B)\). Of course, the black box here stands for conceptual necessity.

This is the best case I know of for this principle. There is another sort of consideration that may carry some weight. As already noted, the \((B)\)-principle is a theorem of the stronger modal logic \(S5\). From the logician’s
point of view S5 is a very attractive logic. Iterated modalities all reduce pleasantly to single modal connectives, the usual “possible worlds” semantics for S5 has no need of “accessibility relations” between worlds, and decision procedures using semantic tableaux and the like are beautifully simple for this case (i.e., for the modal propositional calculus). That’s all very nice, but does conceptual modality obey these pleasant principles? If we were inventing the logic, rather than discovering it, we might allow aesthetic considerations to hold sway. Actually, they might carry some weight in any case, but certainly not enough for the present purpose.

Sometimes it is suggested that there is a simple way to show that S5 governs what is called “metaphysical modality”. If metaphysical modality obeys S5 and what I have been calling “conceptual modality” is just an application to the world of concepts, then if the general argument is correct, then all is well. The argument is this. Metaphysical modality is by definition the strongest kind of modality. S5 is the strongest natural modal logic, including as it does K, T, B, and S4. Hence metaphysical modality will obey S5, at least.

There is a problem here with the idea of strength. It isn’t clear that the idea applies generally to modality. In the usual logical terminology, we would say that one kind of necessity, ‘black box’ say, is stronger than another, ‘white box’ if and only if:

\[
\Box P \Rightarrow \square P
\]

is valid (in some appropriate sense), but its converse is not. In this sense, I suppose that conceptual necessity is stronger than metaphysical necessity because “Water is a compound” is metaphysically but not conceptually necessary. If so, then metaphysical necessity is not the strongest kind of necessity. Perhaps metaphysical possibility is the strongest kind of possibility, that is, for any kind of possibility, ‘black diamond’, if white diamond is metaphysical possibility then:

\[
P \Rightarrow \Diamond P
\]

but not conversely. My problem here is that for any kind of possibility, there is a restriction of it which is stronger than it. Say, for example, that a proposition is epistemically possible for a given knower if no propositions are known (perhaps by him) which together necessarily imply that it is false. If a proposition is epistemically possible in this sense, then it is metaphysically possible. Another example: Let us say that a proposition P is ‘possible for me’ if some action which I am capable of performing
will bring it about that $P$. Then possibility for me is stronger than metaphysical possibility. I suppose that some types of possibility are more natural than others, but from a purely logical point of view there can be no strongest.

We can of course speak of one logical system as being stronger than another if and only if the theorems of the latter are theorems of the former, but not vice versa. We could pursue this and other versions of the argument nearby, but I think there is little hope of showing that $\mathbf{S}_5$ is to be preferred to $\mathbf{T}$, $\mathbf{K}$, $\mathbf{B}$, $\mathbf{S}_4$, $\mathbf{S}.2$, and the like because it is a stronger logic in this sense.

3 Arguments Against $\mathbf{B}$ (or $\mathbf{S}_4$)

Nathan Salmon’s argument against ($\mathbf{S}_4$), construed as concerning metaphysical modality, seems (to some us) to be compelling. In the present context we need only note that it uses premises about possibility such as:

\[ S \quad \text{It is not possible that Woody (a certain table) should have originated from mass m.} \]

This is not a conceptual necessity, whether or not we construe proper names as “directly denoting”. Actually, it would be natural to just exclude such propositions from the domain of conceptual necessity.

Michael Dummett and Yannis Stephanou have produced arguments directly against ($\mathbf{B}$) Actually what they have argued (as does Salmon) is that metaphysical modality does not obey $\mathbf{B}$. We can deal with them quickly by citing crucial premises in their reasoning.

Dummett’s argument appears in his paper “Could There Be Unicorns?” He supposes that the unicorn myth might have been appropriately connected with an actual sort animal of a certain species – even though the myth itself does not give enough information to determine such a species. If so, then unicorns would have been necessarily of that species. So, says Dummett, it might have been true that:

\[ D \quad (\exists x \text{Unicorn}(x) \land \Box \forall x (\text{Unicorn}(x) \rightarrow \text{Species}(x, A))). \]

His argument goes on from here, but we can see that the inner modality, if construed as conceptual necessity, has no plausibility at all. We just should not admit that it is possible that such a conceptual connection should have held.

Yannis Stephanou [13] argues that the following, together with another premise, lead to an unacceptable conclusion – if we accept ($\mathbf{B}$).
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(S') If Aristotle did not exist, then it is impossible that someone should be the actual Aristotle.

This is taken to be a necessary truth. We need only note that even if Aristotle does not exist, there is no conceptual impossibility in someone’s being the actual Aristotle, granting, what is doubtful, that we can make sense of the claim.

Alonzo Church [4] gave an argument that seems to tell against S4, but an analogous case can be made against S5 – and hence it bears on our project of justifying (B).

Here is the argument in full:

[T]hat it is necessary that everything has some property or other is no doubt itself necessary; but that the proposition mentioned on lines 27-28 of page 272 of Lewis and Langford’s Symbolic Logic is necessary is true but not necessary. For although the proposition mentioned on lines 27-28 of page 272 of Lewis and Langford’s Symbolic Logic in fact is that everything has some property or other, this identity is presumably not a necessary one. That is, in “It is necessary that p is necessary”, we take the type of p to be $O_2$; the quoted expression may have the value truth for one propositional concept as value of p and have the value falsehood for another propositional concept, although the two propositional concepts are concepts of the same proposition and although the proposition in question is necessary. ([4], note 22, p. 22)

Well, what shall we say? Has Church shown that the fundamental principle (S4) is not logically correct? And if not, why not?

Most formulations of modal logic take necessity, possibility, and the like, to be properly represented as sentential connectives. Church, on the other hand, takes the fundamental conception of a modality to be as a property of propositions. Now his argument turns on the fact that propositions (like anything else) can be denoted by expressions that contingently pick them out. Standard formulations of modal logic do not allow such things – a proposition is always represented by a sentence. In Church’s way of doing things this would correspond to limiting names of propositions to be of the following form: “the proposition that S”. Such an expression necessarily (and “standardly”) denotes the proposition that is the meaning of the sentence. If we restrict the expressive resources of modal logic in this way, principles such as (S4) and (S5) are unexceptionable. To give this reply in detail would involve an excursion into the logic of sense and denotation and extensions thereof. We avoid this long digression and hope
that interested readers will be able to see that the reply does fully meet Church’s complaints about \((S4)\).

Summing up: there is a reasonably compelling case for the principle \(B\) as applied to conceptual modality. The only arguments that have been offered against are not cogent. So the use of the principle in the modal ontological argument is to that extent justified.

Premise (2) deserves reconsideration in the light of conceptual modality. If God is characterized as a perfect being or as a being having all perfections, it is not immediately obvious that \textit{being such that one’s existence is conceptually necessary} is a perfection or an attribute of a perfect being. But here our terminology actually misleads. Recall that we include \textit{attributes} in the category of concepts. Suppose that a being’s \textit{essential attributes} guarantee by their nature that the being exists. Then the existence of that being would be conceptually necessary. The ‘beings’ existence is guaranteed by its essential nature. This might well be taken to be a perfection.

Sometimes this premise is secured by definition, e.g. in Plantinga’s version. This seems unwise. If one gives a truly stipulative definition, then the cogency of the argument does not depend on its use. If they are claimed to be necessary equivalences between concepts, then they should just be stated as such. Typically the defined term, say “unsurpassable greatness” has an independent meaning and defining it as, say, “maximal excellence in every possible world” produces a proposition that is supposed to be true by stipulation, but really might be thought to concern distinct concepts. Critics have been known to say that the ontological arguer puts the rabbit into the hat with a definition and then produces it from the hat with great flourish. In general it is best to avoid “gerrymandered” or technical concepts, especially if one makes a claim about the possible exemplification of such a concept.

Is the argument then a \textit{proof} of the existence of (a certain kind of) God? If the premises of a proof are required to be absolutely self-evident, then it is not a proof. But then many proofs in mathematics are not proofs in that sense either. I don’t suppose that proofs in Set Theory that use the Axiom of Choice are absolutely self-evident. And the Power Set Axiom is probably less evident to most mathematicians than, say, the Pairing Axiom. The question is: Is it reasonable (for so-and-so, say you or me) to believe the premises of the argument?

What about premise (1)? Is it possible that God exists? If this is conceptual possibility, then this is \textit{not} a triviality. We cannot establish
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it with any show of reasonability by exercises in conceiving or imagining. Recall that conceptual possibilities depend on all the necessary relations between this concept and others. These need not reveal themselves in our contemplation of the concept of God, or of Godlikeness.

I do not believe in “stand-alone” arguments in philosophy, or at least, I doubt that there are any concerning any key issues. In judging reasonable belief, the Principle of Total Evidence (stated and elaborated by Bernoulli, Keynes, and Carnap) requires us to consider all known relevant evidence when settling on the probability of a proposition – in the sense in which probability measures degree of reasonable belief. So even the existence of the amount and variety of evil may bear on the rationality of believing the possibility premise of the modal ontological argument. But then so too, perhaps, will the fine-tuning of the constants in the laws of nature. We should know by now that empirical evidence can bear on modal propositions.

Locally, one can accept the possibility of God’s existence as an hypothesis. If someone deduces a contradiction from the concept of Godlikeness, then the hypothesis is to be rejected.

In defense of the premise, the best approach seems to be that suggested by Robert M. Adams [2]. If it can be argued that the hypothesis that it is possible that there be an infinite mind makes sense of mathematics (especially set theory), then this would offer some independent support for the premise.

Bibliography


Does the Kind of Necessity which Is Represented by S5 Capture a Theologically Defensible Notion of a Necessary Being?

Stamatios Gerogiorgakis

1 Introduction

S5 is the modal system which provides the semantics of Gödel’s [2] ontological proof. It also provides the semantics of the emendations of this proof which were elaborated by Anderson [1] and, lately, Szatkowski [7]. But does the kind of necessity which is represented by S5 meet the traditional philosophical objections against ontological proofs? I will try to answer this question. Especially, I will focus on Kantian objections to the demonstrations of the existence of a necessary being. The topic “Kant versus S5” is not a very intensively discussed one. To my knowledge, only Szatkowski ([7], pp. 142 - 143) addresses it, arguing that ontological proofs modelled in S5 may survive Kant’s objection that existence is not a real predicate. My arguments here are for the opposite.

1 A first draft of this paper was presented in the congress Ontological Proofs Today /Dowody ontologiczne dziś, which took place in Bydgoszcz/Poland on 6-8 September 2011. The present version has benefited much from criticism and commentaries by Prof. Dr. Uwe Meixner and Prof. Dr. Kordula Świetorzecka. I inessentially changed the original title (“Does the Kind of Necessity which Is Represented by S5 Capture the Notion of a Necessary Being?”) for the sake of precision.
In the *Critique of Pure Reason*, Immanuel Kant ([3], pp. A 452 - 455/B 480 - 483) introduced what he thought to be an antinomy concerning God’s existence. This antinomy, usually referred to as the fourth antinomy, consists of two arguments: one for the existence of an absolutely necessary being and one against it. In spite of the contradictory conclusions of these arguments, Kant thought that both are sound! Kant’s ([3], pp. A 598/B 626) well known way out of the riddle, which his arguments presented, is expressed in a nutshell by the aphorism that existence is not a real predicate. Although far from being formally justified, Kant’s aphorism is being accepted by many philosophers. For the sake of simplicity I will call them collectively “Kantians” but this tag is to be attached also to intuitionists and constructivists who are not consciously Kantians. By saying that existence is not a real predicate, Kantians mean that claims on behalf of the actual existence of some entity have to be substantiated in a certain way. In other words, they mean that the truthful ascription of some predicates to a term does not imply that the term refers to an actually existing entity. In order for an entity to be accepted as actually existing, a special acquaintance with it by way of what Kantians call experience in time and space must be given.

I will argue that the Kantian claim that existence is not a real predicate, is anything but well founded. However there is no effective argument against this claim from the ontological-argument-proponents’ side. Challenged by Kantians, proponents of ontological arguments modelled in S5 would see themselves involved in positions which are less and less defensible.

What perhaps could make my argument particularly unpleasant to a proponent of ontological arguments, is the fact that Kantians are not the most dangerous opponents of theism. In fact, Kantians are agnostics rather than atheists. They would refrain from deciding whether God exists but they would not think this impossible. *Eo ipso*, Kantians would not think impossible that some sentences by which God is said to be ascribed certain properties, are true.

If one looks for acrimonious enemies of traditional theism, then these would not be Kantians, but rather Russellians. Russell ([6], pp. 23 - 24) thought that God, conceived (as is the case in the Christian, Jewish and Muslim traditions) as an individual and tagged with a proper name, should be a datum rather than a universal. In the same traditions, however, God is not understood as a datum perceivable by someone. Therefore the entity which the aforementioned traditions coin as ‘God’ cannot exist, since it is
conceived as a datum and as a non-datum at the same time and this is inconsistent. Russelians would analyse, in effect, the sentence:

(1)  \textit{God is the ultimately perfect being}

as: ‘There exists an entity which is the ultimately perfect being and this entity is God’. The last sentence, however, is, in the Russelians’ view, false, since the concept of (the Christian, Jewish and Muslim) God is inconsistent. Consequently, the Christian, Jewish and Muslim God cannot exist. This is why a Russelitian would hold (1) and, in fact, every ascription of any property to the Christian, Jewish and Muslim God to be false. In the Russelians’ view this would make the case of the God of the theistic tradition analogous to the case of Sherlock Holmes: every ascription of a predicate to Sherlock Holmes is, according to a Russelitian, also false. Consider that the sentence:

(2)  \textit{Sherlock Holmes is a detective}

is analysed à la Russell as: ‘There exists an entity which is a detective and this entity is Sherlock Holmes’ – but we know that Sherlock Holmes does not exist. (2) is false since it affirms Holmes’s existence just like (1) is false since it affirms God’s existence.

Kantians would also assume some analogy between God and Sherlock Holmes but for different reasons. Kantians see in the God of the theistic tradition a representation, and, since they take representations to be what are the data to the Russelians (Kantians believe that we form judgments not on things in themselves but only on their empirical representations), Kantians would assume that God does have some properties which may be \textit{truthfully} and actually ascribed to Him; at least the property of being a character in the Old and the New Testament. And since Sherlock Holmes is also a representation which is introduced in detective stories, Kantians would ascribe to Sherlock Holmes at least the property of being a character in detective stories. In fact, most Kantians would not hesitate to ascribe to God more properties. They would associate the representation of God with the representations of ultimate perfection and greatness, without caring whether God exists.

Kantian representationalism has some merits: Sherlock Holmes’s being (represented as) a fictional character in the actual world allows for the actual existence of a story in the actual world, in which the term ‘Sherlock Holmes’ refers. If \textit{no actual properties} at all were ascribed to Sherlock Holmes, there could have been no account for the difference between terms
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which refer to nonexistentss like Sherlock Holmes (= the chief detective of the Conan Doyle novels) and terms which do not refer at all, like ‘the chief detective in Alice in Wonderland’ – the examples are credited to Terence Parsons ([5], pp. 366 - 367). And God’s being (represented as) an object of faith in the actual world, perhaps also as a perfect being, accounts for the actual existence of religion.

A Kantian is, as one sees, a tame opponent of theism: Kantians admit that the term ‘God’ refers, and make lengthy discussions on God. Kantian religious talk has many reasons, but the most important one is that Kantians, albeit usually agnostics, take the existence of a perfect being to be a very useful and logically harmless hypothesis; a useful one for reasons which pertain to morality and are not my subject here, and a logically harmless one because God’s property to be perfect does not imply, in the Kantian’s view, the existence of the holder of the property. The representation of ultimate perfection which the Kantian associates with the representation of God, implies, in the Kantian’s view, the actual existence of God not more than Sherlock Holmes’s property to be a character of a book implies the existence of Sherlock Holmes. Actual existence is not a real predicate to be inferred from other properties.

In the pages to follow, I argue that a proponent of Anderson-like proofs has no chances of victory against an opponent of theism as tame as the Kantian.

2 Contexts for which S5 is Adequate

As the case with Sherlock Holmes and, generally, with fictional entities shows, there is some plausibility in the Kantian claim that existence is no real predicate. But there is no formal proof of this claim. How could there be one? After all there are entities, whose existence in the actual world necessarily follows from some property, which we ascribe to them. Take, for example, the second largest prime number which is calculated until now. It is the number $2^{42,643,801} - 1$. It was discovered in 2009, about one year after those prime numbers were discovered, which are today known as the first and the third largest. Why was it reasonable to search for prime numbers into the interval instead of searching for a prime number which is larger than the largest known to date? Because we knew by Bertrand’s theorem, that there is a host of prime numbers in this interval, with which we had no acquaintance. The reality did not disappoint the expectations which Bertrand’s theorem nourished. Bertrand’s theorem
“predicted” prime numbers to be necessarily there, and in fact we started to discover them. As one sees, Kant’s claim is obviously not correct of objects, whose existence necessarily follows from some property, which they have. The property of an interval between natural numbers to be large enough that at least one prime number is hosted in it according to Bertrand’s theorem necessarily implies the existence of at least one prime number, without the special acquaintance with this number, which a Kantian would demand.

Bearing this in mind, proponents of some version of an ontological argument could overcome Kantian arguments by maintaining that ultimate perfection and greatness are not like the other properties of God. In the tradition of the ontological argument, exactly these properties are held to imply God’s existence.

But is God an entity whose existence follows necessarily from some of His properties? Anderson-like proofs provide strong evidence to affirm this question. They very roughly assert God’s existence as a consequence of His ultimate perfection – a property which we ascribe to God by definition. Anderson-like proofs are modelled in $S5$, a modal system which is commonly associated with logical necessity. Entities which are modelled in terms of logical necessity, are eo ipso actual: they are there in every possible world. Therefore God exists in every possible world and, consequently, in the actual world. The Kantian dictum “Existence is not a real predicate” does hold for entities like Sherlock Holmes, it does not hold however for entities whose existence is logically necessary. And God is by logical necessity such an entity, if “God exists” is a valid conclusion of an Anderson-like proof – the underlying assumption here being that $S5$ is an adequate framework for logical necessity.

3 Contexts for which $S5$ is Inadequate

It is very improbable, however, that this argument will work. You can perhaps persuade Kantians that there are things, which exist by logical necessity in virtue of some property they have, a property other than existence that is. You can also persuade them that prime numbers are good candidates for such things. But this is not to say that God is a candidate for such a thing. Kantians would counter-argue that having been able to calculate the second prime number known to date, we were in effect able to construct the number. But God cannot be said to be constructed. Kantians would warn that any unexplored territory of experience apart of logic
and mathematics might turn out to defy logical necessity. Necessity talk involves possible worlds. And Kantians would warn that if you enhance your possible worlds with entities which are not numbers, then you will not be able to model them in a frame, in which the possible worlds would have an accessibility relation which is reflexive, symmetrical and transitive. In other words, you would not be able to model possible worlds populated with entities which are not numbers in an $S5$-frame.

Take, for example, Sherlock Holmes. Assume some possible world $w_1$ which is inhabited by the detective Sherlock Holmes. This world sees and is seen by another world, $w_2$, in which there is not a detective Sherlock Holmes. Let the frame be reflexive, symmetrical and transitive. We assume:

- The detective Sherlock Holmes exists in $w_1$.
- The detective Sherlock Holmes does not exist in $w_2$.

We get further due to reflexivity:

- Possibly, the detective Sherlock Holmes exists in $w_1$.
- Possibly, the detective Sherlock Holmes does not exist in $w_2$.

But we also get due to symmetry:

- Possibly, the detective Sherlock Holmes does not exist in $w_1$.
- Possibly, the detective Sherlock Holmes exists in $w_2$.

As one sees in both possible worlds the sentences are true:

- Possibly, the detective Sherlock Holmes does not exist.

and

- Possibly, the detective Sherlock Holmes exists.

Consequently:

- It is necessary that it is possible that the detective Sherlock Holmes does not exist.

and

- It is necessary that it is possible that the detective Sherlock Holmes exists.

Then we get in $S5$ for each of these sentences:
(1) □◊¬ex : SH
(2) ◊◊¬ex : SH
(3) ◊¬□¬¬ex : SH
(4) ◊¬□ex : SH

and

(1') ◊□ex : SH
(2') ¬◊¬□¬¬ex : SH
(3') ¬◊□¬ex : SH
(4') ¬◊¬□ex : SH

And so, (4) and (4') contradict.

So, since we stipulated Holmes and accepted a frame which is reflexive, symmetrical and transitive, we have to admit either that Sherlock Holmes necessarily exists or that Holmes necessarily fails to exist. The lesson to draw from this, is, of course, that we may not stipulate the existence of Sherlock Holmes in any world in such a frame. Kantians would argue that this may hold also for God and this is why they would continue to be suspicious against a priori proofs of the necessary existence of God.

Now, if you continue for some reason to be very dedicated to the idea that the existence of God is logically necessary, you can try to provide an a posteriori proof of the existence of a necessary being by way of some version of the design argument, in a way much analogous to the way in which children “prove” a posteriori the truth of ‘5+3=8’ by using their fingers, and then to leave at least the option open that the necessary existence of this being is modelled in S5. In the best possible case, however, the result of such an attempt would be the existence of a being out of natural necessity. But natural necessity involves asymmetry, whereas S5 - frames, in which logical necessity is represented, involve symmetry. Here is a short argument for this:

Symmetry suggests that for any possible worlds x and y, x sees y iff y sees x. Now, imagine next to our actual world another world which almost matches ours, with the only difference that at this world the average height of humans is only one meter forty. We can imagine of course the logical possibility of an average height of 1 m 40, which means that this other world is logically possible. Vice versa, the inhabitants of the other world can imagine the possibility of an average height
Does the Kind of Necessity... of 1 m 75. But logical possibility is here not my issue. My issue is, what is naturally possible – i.e., possible under the proviso of natural laws. In our actual world the natural possibility of an average height of 1 m 40 is given: Let a disease wipe out everyone on the planet except of the pygmies and you can imagine this without having to ignore any natural law. But at the other world they cannot imagine a height average of 1 m 75 if they have to preserve natural laws in imagining so, and this despite their ability to imagine taller humans in logical terms. They could not possibly affect the body-growth of their population to enable the existence of Carl Lewises and Mike Powells, athletes, that is, who could make long-jumps of 8,70 m.

Now, we can “see” the other world in terms of long-jump efforts, since we know what it would mean for the discipline if only pygmies were participating in long-jump sport events. But in a world inhabited only by pygmies, long-jumps of 8,70 m are not possible under the proviso of natural laws. They are naturally impossible. We can “see” the other world in terms of efforts in track and field sport events, but we cannot be “seen” from the other world in the same terms, since at the other world the natural possibility of athletes with our standards is not given. Consequently, natural possibility and necessity are modelled in an asymmetrical frame. Therefore they should not be represented in $\mathcal{S}_5$. Therefore an a posteriori proof of the truth of: ‘There is a necessary being’ would not present an indication that the necessity involved might be modelled in $\mathcal{S}_5$.

This conclusion has brought us a bit further in our considerations: Whether an alleged proof of the existence of God, which is supposed to be modelled in $\mathcal{S}_5$, is sound or not, depends on logic alone. Consequently, if our ontological proofs turned out to be false or inadequate and we had to content ourselves with some version of the design argument, then it would not be very clever to attempt to model the necessity of God’s existence in $\mathcal{S}_5$. The design argument would provide, if at all, some natural necessity and, unless we could “mend” our ontological proofs, this would be natural necessity alone. But natural necessity alone involves asymmetrical frames.

4 Necessary Properties of Nonexistent

As a reaction to this, the proponent of Anderson-like proofs can remind that fictional entities have some properties in the actual world, this being a
belief also of Kantians as I had the opportunity to show in my introduction. There are two major philosophical parties, one of which the theist can join: according to the fictional realist party, whose most emblematic piece of work is done by Peter van Inwagen [8], Sherlock Holmes has the property of being a character of a book. And God, even if assumed a nonexistent, would have at least the property of being a character of the Old and the New Testament. The ascription of a predicate like ‘being a character of the Old and the New Testament’ to God will seem harmless to the Kantian, since it does not imply God’s existence. As I have already remarked, the Kantian thinks that existence is not implied by any property.

According to the other party, which is shaped by the Meinongian theory of objects, fiction engages in two different concepts of existence – cf. van Inwagen ([8], pp. 299 - 300) and Parsons ([5], pp. 365 - 366). Meinongians maintain Sherlock Holmes’s existence as the entity which has the properties which Arthur Conan Doyle stipulated (living in Baker Street and being a grandnephew of the French painter Horace Vernet among others), not however his actual existence as a person, with whom actually existing persons may be acquainted. Also this would seem harmless to the Kantian.

Each of both theories has its problems. Unlike others, however, they manage to cope with the same challenge: they cope with Sherlock Holmes’s not existing in the actual world, although Sherlock Holmes is assigned properties in the actual world like being described by sir Arthur Conan Doyle. Following van Inwagen ([8], p. 307 - 308), the fictional realist would say that Sherlock Holmes is actually a character, Excalibur is actually a mythical sword, and the purgatory is actually a mythical place – and this would not have been so if no detective stories, no Celtic myths, no Christian theology ever had existed. Meinongians would go as far as to say that Sherlock Holmes is also actually an abstract (but non-existing) resident of Baker Street and an abstract (but non-existing) grandnephew of Horace Vernet, that Excalibur is actually an abstract (but non-existing) sharp artefact and that in purgatory there is actually an abstract (but non-existing) high temperature. In viewing things so, the fictional realist and the Meinongian distinguish between singular terms which refer to the detective who is a resident of Baker Street 221b in a sense of “being”

---

2Walton’s make-believe theory of fictionality and David Lewis’s analysis of truth in fiction do not account for Sherlock Holmes’s property to be a fictional character in the actual world. Lewis ([4], p. 38) considers it as probably an advantage of the Meinongian account. Walton ([9], p. 67) believes that Sherlock Holmes’s being fictional is a property of the fictional, not of the actual world.
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which is validated in Arthur Conan Doyle’s stories, and other singular terms, which have not ever referred to anything – in this context I already mentioned Parson’s example ‘The chief detective in Alice in Wonderland’.

So, whether based on fictional realism or Meinongianism, proponents of Anderson-like proofs can take numbers and God to be in the worst case fictional entities which do not exist in the actual world, some amount of predications thereof however, to be actually true. Kantians would easily grant this move, since they think that they have a strong backup against it. If, as Kantians insist, no property implies existence (except of existence itself, if you count existence among properties), fictional realists and Meinongians may ascribe to Sherlock Holmes whatever property they wish to, without this implying that Sherlock Holmes exists. The various ontological arguments are based on the assumption that perfection (or greatness) implies existence, but since Kantians reject this assumption, they would not be concerned by God’s being attributed perfection, greatness, or any other property which the premises of ontological arguments usually ascribe to Him. To the Kantian’s view, no such premise would allow for an inference to God’s existence.

However, the theistic argument is not over. Since the Kantian would agree that certain properties are truthfully ascribed to Sherlock Holmes, the theist may assume that the sentence:

*Sherlock Holmes is a detective*

is actually true since Arthur Conan Doyle stipulated so. In virtue of his being a detective, Holmes has also some properties, which are not explicitly stipulated by Arthur Conan Doyle. One of them is being a human, without Doyle having to explicitly stipulate so.

Now, there has existed something in history, which was named ‘Holmes’, made of steel and no human: the frigate ‘Holmes’ of the Royal Navy during the second world war. But since Doyle wrote stories of a detective Holmes, not of the ship named ‘Holmes’, it would be senseless to let Sherlock Holmes to be no human or consist of steel. In a sense, it was necessary for Doyle to write about the human Holmes, given that he wrote about a detective Holmes. Likewise, it was dependent on the warfare necessities of the given time to construct HMS Holmes out of steel. The property of Sherlock Holmes to be human and the property of the WWII-frigate Holmes to have been constructed out of steel, although not directly stipulated, are necessarily implied, since, by definition, detectives are humans and WWII-battleships are made of steel.
Therefore, in a sense, it is necessary that if Sherlock Holmes is a detective, then Sherlock Holmes is a human. Now, there are possible worlds, which do not contain the actual one, in which Sherlock Holmes exists. In all these worlds, which are inhabited by what we hold to be “our” fictional Sherlock Holmes, Sherlock Holmes is a detective by stipulation and a human by the definition of ‘detective’. Therefore, in all these worlds the sentence:

*Sherlock Holmes is necessarily a human*

is true. In all the worlds accessible from these worlds, in which the following sentence holds:

(3)  *Holmes is not necessarily a human*

(notice that I dropped ‘Sherlock’ in (3)), ‘Holmes’ is ambiguous. That is, these worlds are inhabited by other Holmes’s, the frigate or some other, of which (3) holds. But Sherlock Holmes is necessarily a human in all the possible worlds, which are inhabited by Sherlock Holmes.\(^3\) Now, clearly, the actual world is accessible from these worlds. Therefore the fictional Sherlock Holmes is a human in the actual world, i.e., the sentence ‘Sherlock Holmes is a human’ is actually true. In the case of Sherlock Holmes, this does not amount to an ontological argument, since the property of being a human does not entail existence. But in the case of God, even if one accepts that God is, in the worst case, a fiction but perfect by stipulation, God must be perfect in the actual world. But perfection does imply existence, which means that the worst case does not apply and God is not a fictional entity but as real as anything else. The argument goes as follows:

Assume that God, an entity stipulated to be the ultimately perfect, greatest, wisest etc. is a fictional entity. Then:

(4)  *Necessarily God has every perfection*

is true not only in each world which is inhabited by God, but also in each world, in which God is a fiction – the assumption here being an ontological position like fictional realism or Meinongianism. Notice that the sentence ‘God is existing’ is not assumed to be true. Since God is perfect by stipulation, in all the worlds, in which the negation of (4) holds, ‘God’ is ambiguous. That is, these worlds are inhabited by other gods, of

\(^3\) Obviously it would not present a problem to this account to have something called ‘Sherlock Holmes’ which does not refer to our Sherlock Holmes. In this case we could introduce a new proper name either for this other thing or for our Sherlock Holmes.
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which the negation of (4), properly interpreted, holds. But in every world, in which ‘God’ has only the meaning which we usually give the word in English, (4) is true by stipulation. Consequently, God is necessarily perfect not only in all the possible worlds which are inhabited by God, but also in the ones, in which God is a fiction. Now, the actual world is obviously accessible from the worlds in which God is a fiction. Therefore the (let us assume fictional) God is perfect in the actual world, i.e., the sentence ‘God is perfect’ is actually true. By the definition of existence (existence is a perfection) this amounts to the following sentence’s truth in the actual world:

God is existing.

This is an heroic argument since the proponent of Anderson-like proofs denies to give up his main position that God’s existence is logically necessary and modelled in $S5$, at the price of maintaining a position, fictional realism or Meinongianism, which is metaphysically debatable. The Kantian drove the theistic claims into this position in order to give them a coup de grâce.

5 The Kantian Overcomes, and a Conclusion

Imagine a reflexive and transitive frame with an actual world, in which Holmes is the fictional detective described by the author sir Arthur Conan Doyle. And then imagine another possible world in this frame, in which Holmes is just the same, but sir Arthur Conan Doyle was never a real author. In this world Doyle is also a fiction, a character of an author in a book written by, say, Evelyn Waugh. In Waugh’s book (which is also fictional, since Waugh never wrote such a book), the character Doyle is supposed to write a novel about his character Holmes – and you can even read a sketch of Doyle’s alleged novel on Holmes in Waugh’s book. According to this scenario, Holmes is necessarily a human in the universe of his adventures. His adventures are more actual when they are only described by Doyle, less actual, when they are sketched by Evelyn Waugh in his attempt to describe his hero Doyle, who describes his hero Holmes, but never get really actual. Now, the Kantian may ask: who assures us that nothing like the sketch happens in the case of God, our ontological proofs notwithstanding? Godly existence can get, the Kantian argument goes, as actual as it can get and, therefore, be a realised perfection, nevertheless never gets really actual, say for the reason that the part of theology which
concerns perfection is an inconsistent sketch. In such a case the proponent of Anderson-like proofs will not be able to appeal to a symmetrical frame to guarantee that God is existing actually – not even under the assumption of fictional realism or Meinongianism.

With this argument the Kantian shows that the heroic position which the proponent of Anderson-like proofs defends, does not necessarily imply God’s existence. This position cannot take the sharpness from the Kantian claim that existence is not a real predicate.

However, the theist did not lose everything. Unlike Anderson-like proofs and, possibly, unlike every ontological proof, the research in natural theology and the formulation of arguments for the existence of God which involve asymmetrical frames, are tasks still promising.

Bibliography


Modal Collapse in Gödel’s Ontological Proof

Srećko Kovač

1 Introduction

As is well known, the modal second- (or higher-) order system in which Gödel 1970 sketched his ontological proof of God’s existence ([19], pp. 403 - 404) has proposition $\phi \rightarrow \Box \phi$ as a theorem. This was proven by J. H. Sobel in [34] (see also [35] and [36]). Gödel’s system has the $S5$ propositional base. Since $\Box \phi \rightarrow \phi$ is a theorem of $S5$ (by $\Box$ Elimination), modal collapse, $\Box \phi \leftrightarrow \phi$, is provable. Modal collapse is also provable with the $KB$ propositional base (see Theorem 1.8).

Let us briefly describe Gödel’s ontological system in a variation that we shall call $GO$. The language of $GO$ is a second-order modal language with first-order self-identity ($t = t$), $\lambda$-abstracts, and a third-order term of positivity ($P$). In the linear natural deduction format (originating from Jaśkowski), the rules of $GO$ are the following: Assumption, Reiteration (derivable by $\forall2I$ and $\forall2E$, see below), $S5$ modal rules, free first-order quantification rules ($\forall1I$, $\forall1E$, $\exists1I$, $\exists1E$), $\exists xEx$ (axiom of actual existence, see [15]), classical second-order quantification rules ($\forall2I$, $\forall2E$, $\exists2I$, $\exists2E$), $=I$ (axiom scheme $c = c$) and $\lambda$-abstraction rules ($\lambda I$, $\lambda E$). Here are the rules for first-order quantification and $\lambda$-abstraction ($c \notin \mathcal{C}(\Delta)$ is a

\[1\]In analogy with $PA$ for ‘Peano arithmetic’, and following Hájek’s [21] and [23].
constant not occurring in the members of \( \Delta \):

\[
\frac{\Gamma \vdash Ec \rightarrow \phi}{\Gamma \vdash \forall x \phi(x)}
\]

\[
\Gamma \vdash Ec \rightarrow \phi(c)
\]

\[
\Gamma \vdash \exists x \phi(x)
\]

\[
\Gamma, Ec \land \phi(c) \vdash q
\]

For simplicity, we do not include second-order identity (\( X = Y \), nor first-order identity except for self-identity. They do not occur in Gödel’s ontological proof from 1970, although they are useful for proving some further theorems.

We use Gödel’s following abbreviations:

**God, God-like:** \( G(x) =_{def} \forall X (P(X) \rightarrow X(x)) \),

**Essence:** \( E(X,x) =_{def} X(x) \land \forall Y (Y(x) \rightarrow \Box \forall y(Y(y) \rightarrow Y(y))) \),

**Necessary existence:** \( N(x) =_{def} \forall Y (E(Y,x) \rightarrow \Box \exists x Y(x)) \).

The following axioms describe the concept of positivity:

\((GA1)\) \( \forall X (P(\neg X) \rightarrow \neg P(X)) \),

\((GA2)\) \( \forall X \forall Y (P(X) \land \Box \forall x (X(x) \rightarrow Y(x)) \rightarrow P(Y)) \),

\((GA3)\) \( P(G) \),

\((GA4)\) \( \forall X (P(X) \rightarrow \Box P(X)) \),

\((GA5)\) \( P(N) \).

We list the propositions proved within the ontological argument.\(^4\)

\(^2\)In 1970 *definiens* lacks the left conjunct, which is present in an earlier note by Gödel ([19], p. 431) and is required by D. Scott ([35], p. 146).

\(^3\)It is Scott’s version of Gödel’s axiom ([35], p. 145). Fitting’s version (see [13], p. 148) is expressed in higher-order logic, and formalizes Gödel’s formulation ([19], p. 403) that, if \( \phi \) and \( \psi \) are positive, so is their conjunction, “and for any number of summands” (infinitely many of them, too). Fitting then has \( P(G) \) as a provable proposition.

\(^4\)The soundness and completeness proofs from [29] and [http://filist.ifzg.hr/~skovac/WeakenedCorrections.pdf](http://filist.ifzg.hr/~skovac/WeakenedCorrections.pdf) can be adapted to apply to GO.
Proposition 1.1 \( \mathcal{P}(\lambda x.x = x) \).

Theorem 1.1 \( \forall X \left( \mathcal{P}(X) \rightarrow \diamond \exists x X(x) \right) \).\(^5\)

Corollary 1.1 \( \diamond \exists x \mathcal{G}(x) \).

Proposition 1.2 \( \forall x (\mathcal{G}(x) \rightarrow \forall X (X(x) \rightarrow \mathcal{P}(X))) \).

Theorem 1.2 \( \forall x (\mathcal{G}(x) \rightarrow \mathcal{E}(G, x)) \).

Theorem 1.3 \( \exists x \mathcal{G}(x) \rightarrow \Box \exists x \mathcal{G}(x) \).

Theorem 1.4 \( \Box \exists x \mathcal{G}(x) \).

For proofs, see in Gödel [19], Sobel [34] and [35], Fitting [13], Czermak [11], Hájek [23]. Sobel proved the following theorem, too:

Theorem 1.5 (Modal collapse) \( \forall X \forall x (X(x) \leftrightarrow \Box X(x)) \).

Proof: See [35] or [13]. (a) From left to right. Roughly, from the assumption \( P(c) \) (and \( E(c) \)) derive \( (\lambda x. P(c))(d) \) and from the further assumption \( G(d) \) and \( \mathcal{E}(G, d) \) derive \( \Box \forall y (G(y) \rightarrow (\lambda x. P(c))(y)) \). From there and from Theorem 1.4 derive \( \Box P(c) \). Hence, \( P(c) \rightarrow \Box P(c) \) and \( \forall X \forall x (X(x) \rightarrow \Box X(x)) \) follow (by \( \rightarrow I \) and \( \forall I \)). (b) From right to left. Apply \( \Box \) Elimination.

Let us add some propositions that are closely related to modal collapse. The first collapses positivity to being, or, equivalently, “raises” being to positivity.

Theorem 1.6 (Positivity as being) \( \forall X \forall x (X(x) \leftrightarrow \mathcal{P}(\lambda y. X(x))) \).

Proof: (a) From left to right (positivity of being). Assume \( P(c) \) and \( E(c) \) and from there and from Theorem 1.4, assuming \( G(d) \), derive \( \Box \forall x (G(x) \rightarrow (\lambda x. P(c))(x)) \), as in the proof of the modal collapse above. Then, from \( (GA2) \) derive \( (\mathcal{P}(G) \land \Box \forall x (G(x) \rightarrow (\lambda x. P(c))(x))) \rightarrow \mathcal{P}(\lambda x. P(c)) \). Since \( \mathcal{P}(G) \) is an axiom, \( \mathcal{P}(\lambda x. P(c)) \) follows by \( \rightarrow E \). (b) From right to left (being of positivity). This follows from Theorem 1.4 and Theorem 1.2.

Corollary 1.2 (Positivity as necessity) \( \forall X \forall x (\Box X(x) \leftrightarrow \mathcal{P}(\lambda y. X(x))) \).

Proof: From theorems 1.5 and 1.6.

The following theorem proved by Hájek ([23], p. 311) makes explicit the equivalencies between being God-like, positiveness, and necessity:

\(^5\)Gödel proves Theorem 1.1 from Proposition 1.1. There is a shorter way, independent of any non-empty property being provably positive [35], p. 120, and [23].
Modal Collapse in Gödel’s Ontological Proof

Theorem 1.7 \( \forall x (G(x) \leftrightarrow \forall Y (\mathcal{P}(Y) \leftrightarrow \Box Y(x))) \).

If we replace S5 propositional base in GO with KB, we obtain GOKB.

Theorem 1.8 (Modal Collapse in GOKB) \( \text{GOKB} \vdash \forall X \forall x (X(x) \leftrightarrow \Box X(x)) \).

Proof: (a) From left to right. The same as for Theorem 1.5 above, except that, instead of by Theorem 1.4, the justification is by \( \exists x G(x) \), which is a theorem of GOKB (see [29]). (b) From right to left. Assume \( \Box P(c) \) (and \( E(c) \)); then, from \( \neg P(c) \) and Corollary 1.1 a contradiction is derivable in the following way: in a \( \Box \) subproof from assumption \( \exists x G(x) \) we derive \( \Diamond \neg P(c) \) (by B reiteration). Then derive, in the same \( \Box \) subproof, \( \Box P(c) \). \( \Box P(c) \) is derivable in the same way as in the proof from left to right: by means of Theorem 1.2 (which holds in GOKB, too) and \( \Box \forall x (G(x) \rightarrow (\lambda x. P(c))(x)) \).

Remark 1.1 We note that, as a consequence of Proposition 1.1, there is no being with only negative properties, since each being is self-identical.

Since \( \Box \exists x Gx \) is provable in GO, quantification is never vacuous (semantically: each world has a non-empty domain) and the rule of actual existence \( E \) is derivable. Moreover, since modal collapse is provable in GO, the Barcan formula and the converse Barcan formula are also provable (semantically: we have a constant domain across worlds).

Had we extended the system to a full logic with identity, e.g., with the interchangeability of identicals in atomic formulas, modal collapse would make the full substitution rule derivable and all first-order terms rigid:

\[
\frac{t_1 = t_2, \phi(t_2)}{\phi(t_1/t_2)}
\]

It would also make the necessity of identity and non-identity derivable:

\[
\frac{t_1 = t_2}{\Box t_1 = t_2} \quad \frac{\neg t_1 = t_2}{\Box \neg t_1 = t_2}
\]

(The necessity of identity is also derivable from the full substitutivity of identicals, and from there, using the S5 or B propositional base, so is the necessity of non-identicals (see [25], pp. 312 - 314).) As we can see, Gödel’s positivity axioms are sufficient to transform free second-order modal logic with non-rigid terms to a classical variant of second-order logic.

Gödel gave two interpretations to his system – one is moral-aesthetic (from the standpoint of moral-aesthetic ontology) and the other is attributive (from the standpoint of ontology proper). From both standpoints,
modal collapse seems to deny freedom (moral, aesthetic, ontic), to imply determinism, and as such seems to be hardly acceptable. Therefore, several ways have been proposed to emend the system in order to exclude modal collapse. Probably the best known is Anderson's given in [2] and modified in [3]. Andersonian systems were further explored and critically discussed and modified, for instance by Hájek [21] - [23] and Szatkowski (e.g., [38] and [39]). Sobel's proposal is to exclude modal collapse by deleting Axiom 5 ([34] and [35], preventing the provability of Theorem 1.4, too). Another approach was proposed by Hájek [21], consisting in the weakening of the ontological system to a belief system with the KD45 propositional base, where Theorem 1.4 is not provable. Fitting proposed a change from intensional to extensional types of variables, preserving the validity of Theorem 1.4. There are still other proposals, such as to modify Axiom 5 (Koons [28]), or simply to restrict the comprehension/\lambda-conversion schema (Koons [28], Kovač [29]; see Sobel in [36] and [37]). Good bibliographies on Gödel's ontological proof can be found in [10] and [12].

In the next part of the paper, we show that modal collapse is what Gödel, most probably, intended to have as a result, and we put modal collapse in the broader context of Gödel's philosophy. Thereafter, we propose a redefinition of Gödel's system from the standpoint of reinterpreted justification logic in a way that does not exclude modal collapse, but can give it an explicit, causal sense.

2 Is modal collapse in Gödel's ontology incidental?

2.1 Confirmations of modal collapse in Gödel's text

We aim to show that Gödel's texts and reflections confirm that modal collapse was intended and is part of Gödel's general philosophical view. As a confirmation of this intention of Gödel, we have referred in [29], p. 582, and in [30] to page 435 of [19], where Gödel says that

\[ \phi(x) \rightarrow \Box \phi(x) \]  \hspace{1cm} (2.1)

\(^6\)In [29] we proposed a restriction on \lambda-abstraction to block the provability of modal collapse, but, at the same time, stated that modal collapse was Gödel's intention (and put it in analogy with Gödel's cosmological collapse of time).

\(^7\)On Adams' discussion on this point see [1] (and below).
should only be derived from the existence of God, and not vice versa, the existence of a thing “for every compatible system of properties” (including God) from (2.1). According to Gödel, the proof from assumption (2.1) is “the bad way” (“der schlechte Weg”, translated in [19] somewhat misleadingly as “the inferior way”). Gödel’s approach is obviously, first, to prove the existence of God from ontological axioms, and only thereafter to prove modal collapse.

In the cited place (p. 435, “Ontological proof”, nr. 4), Gödel compares two assumption candidates from which the existence of God is derivable. The first was at that time adopted by him as a crucial axiom:

$$\mathcal{P}(X) \rightarrow \mathcal{P}(\lambda x. \Box X(x))$$

(2.2)

that is, “the necessity of a positive property is positive”. Gödel mentions (in a footnote to a text several lines above) the dual form, too, for this assumption

if $M\phi$ is a perfective, then $\phi$ is too

i.e., $\mathcal{P}(\lambda x. \Diamond X(x)) \rightarrow \mathcal{P}(X)$. Here, “perfective” is only a special way (corresponding to the later “moral aesthetic” interpretation) of how to interpret the “positive” (another acceptable interpretation of “positive” being “assertion”).

An alternative, unsatisfactory way is to assume (2.1), as provable from the essence of $x$. In fact, the provability of (2.1) from the essence of $x$ would not be welcomed by Gödel, since this would have as a result the “bad way” of proving the existence of God simply from modal collapse. The concept of essence is not mentioned in the axiomatic outline on p. 435, but has been defined in other places: in an earlier outline of the ontological proof (p. 431) as well as in the last sketch (from 1970, with a slight difference). It is this definition of essence (p. 431, Scott) with respect to which Sobel’s refutation of the provability of (2.1) from the essence of $x$ holds (Sobel [36] and the reply in the correspondence with me on 7 February 2004). We may note that the inconclusiveness proved by Sobel need not be an error on the side of Gödel, but something that blocks the “bad” ontological proof. Why does Gödel name such an ontological proof

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8We have referred to this place also in the correspondence with Sobel on 6 February 2004. In the reply (7 February), Sobel allowed that Gödel, at least in 1956, “was easy” with the idea of modal collapse. In 2006 [36] and referring to [1], Sobel, too, expressed the view that there is a strong evidence that, for Gödel, modal collapse was a “welcomed feature” of his metaphysics.
a “bad way”? Gödel obviously wanted necessitarianism (and individual essences) eventually to be founded on God and on the “whole”. Hence, he had to exclude the provability of modal collapse directly from separate individual essences, and to exclude the provability of the existence of God from such (naturalistic or fatalistic) modal collapse.

It appears R. M. Adams [1], p. 400, was the first to quote (in 1995) from most probably the same place in Gödel’s notes, and this directly from the manuscript (see [36], p. 402). He quotes what he calls Gödel’s thesis that “for every compatible system of properties there is a thing”. However, taking into account a suggestion of Ch. Parsons, he thinks that the interpretation of the thesis is “not obvious”, although it “looks strongly necessitarian” (Gödel’s necessitarianism as a “somewhat speculative” suggestion, p. 401).

There is still another place confirming Gödel’s adoption of modal collapse. Gödel says:

“The positive and the true sentences are the same, for different reasons …” ([16], p. 433)

Closed sentences are zero-place properties. If God is defined as a being having all positive properties, and God necessarily exists, then whatever is true is necessarily true. It is hard to imagine that Gödel would not be aware of this consequence. The above quotation expresses, in fact, our Theorem 1.6, with ‘truth’ instead of ‘being’.

2.2 Modal collapse and modal rise in Gödel’s philosophy

Gödel’s adoption of modal collapse suits perfectly well his general philosophical views as documented in his manuscripts, in reports by Hao Wang [41] and [42], and in a particular way corresponds to Gödel’s philosophy.
Modal Collapse in Gödel’s Ontological Proof

of time (see interpretations of his philosophy of time by P. Yourgrau, e.g., [43] and [44]).

As Wang reports, according to Gödel, the separation of force (wish) and fact (wish is force “as applied to thinking beings, to realize something”), and the overcoming of this separation (in the “union of fact and wish”), are the “meaning of the world” [42], pp. 311 and 309. This separation and overcoming stand under the “maximum principle for the fulfilling of wishes”, in the direction of building up the (Leibnizian) best possible world [42], p. 312. Hence, the overcoming of the separation of force (wish) and fact should be perfect, in the sense of all possibilities being realized: “... as many beings as possible will be produced - and this is the ultimate ground of diversity ...” [19], p. 433. In these views it is not hard to recognize modal collapse in terms of the possibility: $\diamond \phi \rightarrow \phi$.

Far from being disappointed with the perspective of modal collapse, since it means the realization of all possibilities in the perfect world, Gödel states: “our total reality and total existence are beautiful and meaningful” (where “the short period of misery may even be necessary for the whole”) [42], p. 317. The collapse of modalities is for Gödel, in fact, the rise of modalities to the perfect being.

Besides, despite the (final) modal collapse/rise, there is in Gödel another, clearly distinguishable, although inferior, view on modalities: strong “separation” of force and fact as characteristic of some fragments of the whole (“this world”), along with the “superior” view from the standpoint of “the whole” (where modalities unite).

Such a reading of Gödel’s philosophical remarks is strengthened by Gödel’s much more precisely elaborated cosmology and philosophy of time. Gödel contributed to cosmology through his discovery of the possibility (in the sense of natural laws) of rotating universes without definable “absolute time”. Further, in non-expanding rotating universes “time travel” would be physically possible. On the ground of the mere physical possibility

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11 “... [since there are so many unrealized possibilities in this world, it must be a] preparation for another world” [42], p. 312. Also, according to Gödel, our “very imperfect life ... may be necessary for and adequately compensated for by[, ] the perfect life afterwards” [42], p. 317.

12 On the correspondence between modal collapse in Gödel’s ontological proof and the temporal collapse in Gödel’s cosmology, see briefly in Kovač [29], p. 582, and in [30], pp. 160 - 161. This is strongly connected with some of the Yourgrau’s reflections (see footnote 14).

13 For a recent physicist’s account of Gödel’s cosmology, see W. Rindler [33].
of rotating universes, and on the ground of the reflection that it is highly philosophically unsatisfactory to think of time as dependent on the (accidental) arrangement of matter and motion in the universe, Gödel endorsed the view that there is no objective lapse of time (or change), and that time is only subjective (“an illusion or appearance due to our special mode of perception”), without any objective lapse. Hence, there is objectively no future realization of presently non realized possibilities, but each possibility is “already” realized in cosmological spacetime. Yourgrau especially elaborated this side of Gödel’s philosophy, but stressing the resulting dual perspective of the world: “from within and also sub specie aeternitatis”.

Let us add two remarks. In Gödel’s non-expanding rotating universes, time behaves, from the same basic logical viewpoint, like modalities in his ontological proof – it satisfies the conditions of the S5 propositional base (reflexive euclidean models). This can be clearly seen from Gödel’s following statement on his cosmological results about rotating universes:

“. . . it is possible in these worlds to travel into any regions of the past, present, and future, and back again, exactly as it is possible in other worlds to travel to distant parts of space.” ([17], p. 205)

Second, there is another interesting analogy between the ontological and the cosmological proof of modal collapse which we can reconstruct. The “bad way” in cosmology would be first to assume that there is no lapse of time, and then from there to prove the existence of relativistic spacetime. Gödel proceeded the other way around, first presupposing natural laws, and coming from there to relativistic spacetime, he constructs his rotated universes, by means of which he concluded that time collapses.

14Yourgrau points out one characteristic general feature of Gödel’s reasoning, which consists in the transition from the possible to the actual. It is manifest in mathematics (possible mathematical objects are as such mathematically actual), in cosmology (from the possible to the actual non-existence of time), as well as in the ontological argument (from the possible to the actual existence of God) [43], p. 44, and [44], p. 130.

15“Yet, somehow, we, the individual selves, must be able to support both perspectives. Then what are we? ‘We do no know what we are (namely, in essence and seen eternally)’ (Gödel, in Wang 1987, p. 215)” [43], p. 191. A. Ule has emphasized that, beside our experience of the lapsing time, we also have temporary non-lapsing time consciousness [40].
3 Modal collapse and causality

Since time and modalities collapse, the question about what we are then left with in ontology remains. Gödel’s answer is: causality. In his timeless ontology (“time is no specific character of being” [42], p. 320), time structure is replaced by causal structure:

“The real idea behind time is causation; the time structure of the world is just its causal structure.” ([42], p. 320)

Of course, the causal structure is itself unchanging:

“Causation is unchanging in time and does not imply change. It is an empirical – but not a priori – fact that causation is always accompanied by change.” ([42], p. 320)

What is the ontological status of collapsing modalities in general? According to Gödel, they should be (“perhaps”) derived from causality, too:

“The fundamental philosophical concept is cause. It involves: will, force, enjoyment, God, time, space. . .

. . . Perhaps the other Kantian categories (that is, the logical [categories], including necessity) can be derived in terms of causality, and the logical (set-theoretical) axioms can be derived from the axioms for causality. [Property = cause of the difference of the things]” ([19], pp. 433 - 434)

Force (“connected to objects”), as well as concepts (“being general”), should be explained by means of causation (see [42], p. 312, nr. 9.4.5).16 Let us add that, while (the lapse of) time disappears in the objective causal structure of the world and remains in ontology only subjectively, as the “frame of reference” for the mind [42], p. 319, space is for Gödel the “possibility of influence” ([19], pp. 434 - 435) – where possibility is conceived as “a weakened form of being”, “synthesis of being and nonbeing” ([42], p. 313).

Gödel’s reflections give some hints as to how to understand ontological concepts in the causal, fundamental perspective. As we see from the quotation above, properties cause the difference of things. Further, according to Gödel, a mathematical theorem causes its consequences [42], p. 320. It can be understood that positive properties, in “attributive” interpretation,

16 In [42], p. 297, Gödel says: “Force should be a primitive term in philosophy”.

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are the properties that affirm being,\(^{17}\) and that the “affirmation of being” is the \textit{causal} meaning of positiveness:

“The affirmation of being is the cause of the world.” ([42], p. 433)

Finally, God should be the last cause of the world, a “necessity in itself”, which does not require any further cause.\(^{18}\)

This much could be sufficient as a corroboration of the thesis that modal collapse is a constituent part of Gödel’s ontology and philosophy in general, and, secondly, that causation is for Gödel the real and fundamental concept behind modalities.

4 Causality and justification logic

Let us now see in more detail how modality in Gödel’s ontological proof could be replaced by causality.

The formulation “if \(\phi\), then necessarily \(\phi\)” is very general. It is not precise about the \textit{kind} of necessity, except that it is understood that this is an \textit{ontological} necessity, defined by the propositional S5 base and by Gödel’s specific axioms of positivity. As remarked in [7] (though from the epistemic perspective), there are two ways of reading \(\Box\): as a universal quantifier over possible worlds (truth in all possible worlds), and as an existential quantifier over reasons (truth for a reason). The existential reading of \(\Box\) in modal collapse theorem of GO formally expresses Leibniz’s principle of sufficient reason: each truth has its sufficient reason. But which reason in each particular case is it? Justification logic defines “realization” algorithms for the replacement of each occurrence of \(\Box\) by a particular reason (“proof term”, evidence) in such a way that each theorem of, e.g., propositional S4 or S5, and of first-order S4 (FOS4), after realization, remains (in a modified form) a theorem of the corresponding justification logic LP, LPS5 and FOLP, respectively (see Artemov [5], Fitting [14], Artemov and Yavorskaya [8]).

Actually, “reason” indicates a special modality beside possibility and necessity. From Leibniz on, and especially from Kant, we can trace a distinct-

\(^{17}\)See Gödel’s attributive interpretation of positiveness, “as opposed to ‘privation’” – the disjunctive normal form of a positive property contains a member without negation [19], p. 404.

\(^{18}\)... according to the Principle of Sufficient Reason the world must have a cause. This must be “necessity in itself” (otherwise it would require a further cause).” ([19], p. 431)
tion between the principles of non-contradiction (possibility, “problematic judgments”), of sufficient reason (existence, reality, truth, “assertions”), and of excluded middle (necessity, “apodeictic judgments”) (see [27], B 99 - 101 and 106\textsuperscript{19}). The existential reading of $\Box$ is in fact “assertoric” (“existential”) modality, which can be obtained through a sort of reduction of necessity each time to some existing reason. “It is necessary that $\phi$” is reduced (in a propositional setting) to “$t$ is a reason for $\phi$, $t : \phi$, for some existing reason $t$. Since the principle of excluded middle is intuitionistically not valid, it is quite natural that Gödel came to the idea to replace S4 modal propositional logic, as a modal formalization of intuitionism, with a corresponding justification logic.\textsuperscript{20} Gödel saw that in S4 with $\Box$ conceived simply as “provability”, the contradiction with his second incompleteness theorem follows.\textsuperscript{21}

“Reason” is a term general enough to cover the epistemological as well as ontological sense of assertoric modality. It is conceived in these two senses, for example, by Leibniz as well as by Kant (who makes a sharper distinction between them). An appropriate and established ontological sense of “reason” is causality.\textsuperscript{22} And, as already mentioned, causality is Gödel’s explicitly intended primary ontological concept, from which all other “categories” should be derived. Therefore, causality is the best candidate to be conceived as reason in the “justificationally” transformed Gödel’s ontological system. Accordingly, each occurrence of $\Box$ in Gödel’s ontological proof should be read as an (ontological) cause, and $\Box\exists x G x$ should mean not merely that God necessarily exists, but should explicitly name the cause for which God exists. Similarly, modal collapse in Gödel’s ontology should not simply mean that every fact is necessarily true, but should explicitly name the cause (“sufficient reason”) for which the truth obtains. Let us add that the modal collapse theorem discriminates ontological justification logic from the epistemological one in that in the epistemological case not

\textsuperscript{19}Kant gives a nice explanation in his letter to Reinhold from 12. 5. 1789 [26], vol. 9. Also, see [31].

\textsuperscript{20}This proposal of Gödel was published posthumously in his “Zilsel lecture” in 1995 [20]. At that time, Artemov had already independently worked out the same idea, and published it the same year in [4].


\textsuperscript{22}For example, for Kant, to the three logical principles of non-contradiction, sufficient reason and excluded middle, there correspond three ontological categories (“relations”): substance – accident, causality, and community (mutual influence), respectively [27], B 106.
every truth should be evident (and therefore epistemological justification logic can be intuitionistically appropriate), whereas in the ontological case, at least from some traditional viewpoint, we naturally expect each truth to have its cause.

Besides, taking into account Gödel’s distinction between the moral-aesthetic and the attributive (assertive) interpretations of positivity, we can in a natural way distinguish between moral-aesthetic causes, related to the affirmation or negation of what is “purely good” ([19], p. 433), of an objective moral or aesthetic value ([18], p. 375) and, on the other side, attributive causes, related simply and generally to the affirmation or negation of being.

To illustrate how a causal ontological proof might look, we describe the system CGO, a “justificationally” transformed GO in the causal sense. To that end, we extend the first-order justification logic FOLP by Artemov and Yavorskaya, and give it (informally) a causal interpretation. CGO is an extension to a second-order logic with first-order self-identity, with the justificational S5 propositional base (see Fitting [14]) and with modified Gödel’s ontological axioms. In vocabulary, there are cause variables and cause constants, and otherwise no constant terms except first-order = and \( \mathcal{P} \). Connectives other than \( \neg \) and \( \rightarrow \), as well as existential quantifiers, are defined in the classical way. Justification terms receive causal meaning in the following way: \( t + s \) means cause \( t \) or \( s \), \( t \cdot s \) means the affirmation (activation) of the cause \( t \) by means of cause \( s \), \( !t \) is the affirmation of a cause \( t \) having some specific effect, \(?t \) is the affirmation of a cause \( t \) not having some specific effect, and \( \text{gen}(t) \) is the general application of a cause. We add \( \text{lam}(t) \) as a property maker, and introduce two further ontological cause constants:

\[
\begin{align*}
g & \quad \text{cause of (“moral-aesthetic”) positivity,} \\
e \text{exs}(t) & \quad \text{cause of existence, “affirmation of being”}.
\end{align*}
\]

Justification formulas are built as in FOLP, with the addition of second-order variables in \( \mathcal{X} \) of \( t: \mathcal{X} \phi \).

The abbreviations of GO, except for the predicate \( G \), are slightly transformed, so that essence and necessary existence are defined in the following way:

\[
\begin{align*}
\mathcal{E}_t(X, x) &= \text{def} \ X(x) \land \forall Y(Y(x) \to t: \{X,Y\} \forall y(X(y) \to Y(y))) \quad (\text{essence})
\end{align*}
\]

\(^{23}\)The possibility that \( t \) in \( t: \phi \) could mean “something like set of causes or counterfactuals” is mentioned by Artemov in [6], p. 478
Ntx = def ∀Y(ℰt(Y, x) → exs(t):{Y} ∃xY(x)) (necessary existence)

Axioms are the axioms of classical first-order logic, self-identity x = x, and the following ones (axiom schemes (CVCons) – (C4) and (C∀) are second-order generalizations of the corresponding schemes of FOLP [8]):

(∀2a) ∀X(φ) → φ(T/X) (T is substitutable for X in φ)
(∀2b) ∀X(φ → ψ) → (φ → ∀Xψ), X does not occur free in φ
(λConv) (λx.φ)(y) ↔ φ(y/x), y is substitutable for x in φ
(CVCons) t:χ,φ → t:χ,φ, y does not occur free in φ
 t:χ,Y φ → t:χ,φ, Y does not occur free in φ
(CVMon) t:χ φ → t:χ,Y φ t:χ φ → t:χ,Y φ
(CMon) s:χ φ → (s + t):χ φ t:χ φ → (s + t):χ φ
(CK) s:χ (φ → ψ) → (t:χ φ → (s · t):χ ψ)
(CT) t:χ φ → φ
(C4) t:χ φ → !t:χ t:χ φ
(C5) ¬t:χ φ → ?t:χ ¬t:χ φ
(C∀) t:χ φ → genx(t):χ ∀xφ, X ≠ X t:χ φ → genx(t):χ ∀Xφ,
(Cλ) t:χ φ(x/y) → lamx(t):χ (λy.φ)(x)

Gödel’s ontological axioms (CGA1) - (CGA5) are the same as (GA1) - (GA5), respectively, with the exception of the second (which is, in fact, an axiom scheme):

(CG2) ∀X∀Y(P(X) ∧ t:⟨X,Y⟩ ∀x(X(x) → Y(x)) → P(Y)),

and the fourth:

(CG4) ∀X(P(X) → g:⟨X⟩ P(X)).

Rules are modus ponens (MP), first- and second-order generalization (gen1) and (gen2), and axiom necessitation (ANec): if ⊢ φ, then ⊢ c: φ, where φ is an axiom, and c a cause constant (“axiom causation”).

Technical metatheoretical results about CGO will be presented in a separate paper. Here, we prove in CGO the theorems of Gödel’s ontological argument. Corresponding to Gödel’s argument, we start from proving the positivity of self-identity. In the annotations and indices, PL is propositional logic, and FOL first-order logic with self-identity and λ-abstraction.

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Proposition 4.1 \( \mathcal{P}(\lambda x. x = x) \) (the same as 1.1).

Proof: Start from the axiom \( x = x \), apply axiom necessitation, for example, with the cause constant \( c \), to get \( c: x = x \); by \((C\lambda)\) and \((MP)\), we obtain \( \text{lam}_x(c): (\lambda x. x = x)(x) \); enter the axiom \( (\lambda x. x = x)(x) \rightarrow ((\lambda x. \neg x = x)(x) \rightarrow (\lambda x. x = x)(x)) \), and derive, by axiom necessitation, \( d: ((\lambda x. x = x)(x) \rightarrow ((\lambda x. \neg x = x)(x) \rightarrow (\lambda x. x = x)(x))) \); therefore, by \((CK)\),

\[
d: ((\lambda x. x = x)(x) \rightarrow ((\lambda x. \neg x = x)(x) \rightarrow (\lambda x. x = x)(x))) \rightarrow (\text{lam}_x(c): (\lambda x. x = x)(x) \rightarrow (d\cdot \text{lam}_x(c)): ((\lambda x. \neg x = x)(x) \rightarrow (\lambda x. x = x)(x)))
\]

By \((MP)\), \((d\cdot \text{lam}_x(c)): ((\lambda x. \neg x = x)(x) \rightarrow (\lambda x. x = x)(x)) \) and, by \((\forall)\) and \((MP)\), \( \text{gen}_x(d\cdot \text{lam}_x(c)): \forall x ((\lambda x. \neg x = x)(x) \rightarrow (\lambda x. x = x)(x)) \). Finally, by \((CGA2)\), \((CGA1)\), we derive \( \mathcal{P}(\lambda x. x = x) \).

Theorem 4.1 \( \forall X (\mathcal{P}(X) \rightarrow \neg u : \{X\} \forall x \neg X(x)) \)

Proof:

1. \( c_{\text{FOL}} : \{X\} (\forall x \neg X(x) \rightarrow \forall x (X(x) \rightarrow (\lambda x. \neg x = x)(x))) \)
   Cause term \( c_{\text{FOL}} \) is the result of the successive application of \((ANec)\) to first-order logic axioms (with axioms for self-identity and \(\lambda\)-abstraction), and of the successive combination of the obtained cause terms into complex causal terms ("polynomials").

2. \( u : \{X\} \forall x \neg X(x) \rightarrow (c_{\text{FOL}} \cdot u : \{X\} \forall x (X(x) \rightarrow (\lambda x. \neg x = x)(x))) \)
   \( (CK), (MP) \)

3. \( (\mathcal{P}(X) \land (c_{\text{FOL}} \cdot u : \{X\} \forall x (X(x) \rightarrow (\lambda x. \neg x = x)(x))) \rightarrow \mathcal{P}(\lambda x. \neg x = x) \)
   \( (CGA2), (JVCons) \)

4. \( \mathcal{P}(X) \land u : \{X\} \forall x \neg X(x) \rightarrow \mathcal{P}(\lambda x. \neg x = x) \)
   \( (2), (3), (PL) \)

5. \( \mathcal{P}(X) \rightarrow (u : \{X\} \forall x \neg X(x) \rightarrow \mathcal{P}(\lambda x. \neg x = x)) \)
   \( (4), (PL) \)

6. \( \neg \mathcal{P}(\lambda x. \neg x = x) \)
   \( (CGA1), \text{Prop. 4.1} \)

7. \( \mathcal{P}(X) \rightarrow \neg u : \{X\} \forall x \neg X(x) \)
   \( (5), (6), (PL) \)

8. \( \forall X (\mathcal{P}(X) \rightarrow \neg u : \{X\} \forall x \neg X(x)) \)
   \( (7), (\text{gen2}) \)

\( u \) is a cause variable.

Corollary 4.1 \( \neg u : \forall x \neg G(x) \)
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**Proof:** From (CGA3) \((\mathcal{P}(G))\) and from Theorem 4.1, \(\neg u:_{\{x\}} \forall x \neg G(x)\) and hence (by \((JVC\text{Cons})\)) \(\neg u: \forall x \neg G(x)\) follow.

**Corollary 4.2** \(\neg ((c_{PL} \cdot a) \cdot ?\text{exs}(c_{SOL} \cdot g)) : \forall x \neg G(x)\)

**Proof:** In Theorem 4.1, we replace \(u\) by \(((c_{PL} \cdot a) \cdot ?\text{exs}(c_{SOL} \cdot g))\) (this causal justification will be used in Theorem 4.4). Hence, \(\neg ((c_{PL} \cdot a) \cdot ?\text{exs}(c_{SOL} \cdot g)) : \forall x \neg G(x)\) follows.

**Proposition 4.2** \(\forall x (G(x) \rightarrow \forall X(X(x) \rightarrow \mathcal{P}(X)))\) (the same as Proposition 1.2).

**Proof:** We leave it as an exercise.

**Theorem 4.2** \(\forall x (G(x) \rightarrow \varepsilon_{c_{SOL} \cdot g}(G, x))\)

**Proof:** Left as an exercise. Let us note that in the first part of the proof, we use second-order logic axioms and their necessitation, to come to \(c_{SOL} :_{\{x\}} (\mathcal{P}(X) \rightarrow \forall y (G(y) \rightarrow X(y)))\), where \(c_{SOL}\) is a combination of necessitations of second-order logic axioms (logical causality, preserving consistency of properties). We then derive \(g :_{\{x\}} \mathcal{P}(X) \rightarrow (c_{SOL} \cdot g) :_{\{x\}} \forall y (G(y) \rightarrow X(y))\), and, by Proposition 4.2, deduce the theorem.

**Theorem 4.3** \(\exists x G(x) \rightarrow \text{exs}(c_{SOL} \cdot g) : \exists y G(y)\)

**Proof:**

1. \(G(x) \rightarrow \forall X(\mathcal{P}(X) \rightarrow X(x))\) \(\text{Def. } G, \text{ (Theorem: } \phi \rightarrow \phi\)
2. \(G(x) \rightarrow (\mathcal{P}(N) \rightarrow N(x))\)
3. \(\mathcal{P}(N)\) \(\text{(CGA5)}\)
4. \(G(x) \rightarrow N(x)\) \(\text{(3), (4), (PL), (MP)}\)
5. \(G(x) \rightarrow \forall Y(\varepsilon_{c_{SOL} \cdot g}(Y, x) \rightarrow \text{exs}(c_{SOL} \cdot g) :_{\{Y\}} \exists y Y(y))\) \(\text{Def. } N\)
6. \(G(x) \rightarrow \varepsilon_{c_{SOL} \cdot g}(G, x)\) \(\text{Th. 4.2}\)
7. \(G(x) \rightarrow \text{exs}(c_{SOL} \cdot g) :_{\{Y\}} \exists y G(y)\) \(\text{(5), (6), SOL}\)
8. \(G(x) \rightarrow \text{exs}(c_{SOL} \cdot g) : \exists y G(y)\) \(\text{(7), (CVCons)}\)
9. \(\forall x (G(x) \rightarrow \text{exs}(c_{SOL} \cdot g) : \exists y G(y))\) \(\text{(8), (gen1)}\)
10. \(\exists x G(x) \rightarrow \text{exs}(c_{SOL} \cdot g) : \exists y G(y)\) \(\text{(9), FOL}\)

**Theorem 4.4** \(\text{exs}(c_{SOL} \cdot g) : \exists y G(y)\)
Proof:

(1) \( a: (\exists x G(x) \rightarrow \text{exs}(c_{\text{SOL}} \cdot g) : \exists y G(y)) \)

Abbreviation \( a \) stands for the causal term that results from the transformation of the proof of Theorem 4.3 by the cumulative necessitation (based on \((ANec)\)) of the axioms used.

(2) \( c_{PL}: ((\exists x G(x) \rightarrow \text{exs}(c_{\text{SOL}} \cdot g) : \exists y G(y)) \rightarrow (\neg \text{exs}(c_{\text{SOL}} \cdot g) : \exists y G(y) \rightarrow \neg \exists x G(x))) \)

\( c_{PL} \) is the abbreviation of a causal term obtained by the cumulative necessitation of propositional logic axioms used.

(3) \( (c_{PL} \cdot a): (\neg \text{exs}(c_{\text{SOL}} \cdot g) : \exists y G(y) \rightarrow (\neg \exists x G(x)) \quad (1), (2), (CK), (MP) \)

Informally, Theorem 4.4 says that the affirmation of logic and positivity is the cause of the existence of a God-like being. In some way, this should be understood as the explication of the “necessity in itself”, without “further cause”, and as somehow (in analogy with Th. 1.6, Def. of \( G \)) contained in the God-like being itself (see footnote 18).

**Theorem 4.5 (Modal collapse)** \( \forall z \forall X (X(z) \rightarrow ((c_{\text{SOL}} \cdot g) \cdot \text{exs}(c_{\text{SOL}} \cdot g)) : \{x, z\} X(z)) \)

Proof: Similar to axiomatic non-justificational proof, see [13], pp. 163 - 164.

(1) \( G(x) \rightarrow (G(x) \land \forall Y (Y(x) \rightarrow (c_{\text{SOL}} \cdot g) : \{Y\} \forall y (G(y) \rightarrow Y(y)))) \quad \text{Th. 4.2} \)

(2) \( G(x) \rightarrow \forall Y (Y(x) \rightarrow (c_{\text{SOL}} \cdot g) : \{Y\} \forall y (G(y) \rightarrow Y(y))) \quad (PL) \)

(3) \( G(x) \rightarrow \forall Y (Y(x) \rightarrow (c_{\text{SOL}} \cdot g) : \{x, y\} \forall y (G(y) \rightarrow Y(y))) \quad (CVMon) \)
Modal Collapse in Gödel’s Ontological Proof

(4) $G(x) \rightarrow ((\lambda x.Xz)(x) \rightarrow (c_{SOL} \cdot g)_{\{X,Y,z\}} \forall y(G(y) \rightarrow (\lambda x.Xz)(y)))$

(5) $G(x) \rightarrow ((\lambda x.Xz)(x) \rightarrow (c_{SOL} \cdot g)_{\{X,z\}} \forall y(G(y) \rightarrow (\lambda x.Xz)(y)))$

(6) $G(x) \rightarrow (X(z) \rightarrow (c_{SOL} \cdot g)_{\{X,z\}} \forall y(G(y) \rightarrow X(z)))$

(7) $\forall x G(x) \rightarrow (X(z) \rightarrow (c_{SOL} \cdot g)_{\{X,z\}} \forall y(G(y) \rightarrow X(z)))$

(8) $\exists x G(x) \rightarrow (X(z) \rightarrow (c_{SOL} \cdot g)_{\{X,z\}} (\exists y G(y) \rightarrow X(z)))$

(9) $\text{exs}(c_{SOL} \cdot g) : \exists x G(x)$

(10) $\exists x G(x)$

(11) $X(z) \rightarrow (c_{SOL} \cdot g)_{\{X,z\}} (\exists y G(y) \rightarrow X(z))$

(12) $X(z) \rightarrow (\text{exs}(c_{SOL} \cdot g)_{\{X,z\}} \exists y G(y) \rightarrow ((c_{SOL} \cdot g) \cdot \text{exs}(c_{SOL} \cdot g))_{\{X,z\}} X(z))$

(13) $\text{exs}(c_{SOL} \cdot g)_{\{X,z\}} \exists x G(x)$

(14) $X(z) \rightarrow ((c_{SOL} \cdot g) \cdot \text{exs}(c_{SOL} \cdot g))_{\{X,z\}} X(z)$

(15) $\forall z \forall X(X(z) \rightarrow ((c_{SOL} \cdot g) \cdot \text{exs}(c_{SOL} \cdot g))_{\{X,z\}} X(z))$

The above-given axiomatic outline gives an explicit causal, although rather metaphysical (theological), answer to the meaning of modal collapse in Gödel’s ontological proof.

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What kind of necessary being could God be?

Richard Swinburne

I shall understand by ‘metaphysical necessity’ the strongest kind of necessity there is, and by ‘metaphysical impossibility’ the strongest kind of impossibility there is, and so by ‘metaphysical possibility’ the weakest kind of possibility there is. My concern in this paper is with whether it is metaphysically possible that God be a metaphysically necessary being in these senses.

A substance or event is metaphysically necessary (or whatever) iff it is metaphysically necessary (or whatever) that it exists; and since we can have no discussable knowledge of whether it is metaphysically necessary (or whatever) that the substance or event exists, except (at least in part) by reflecting on features of the sentence which asserts this, it will be more convenient to speak of necessity etc as belonging to the sentence. I shall come back later to the issue of whether these modal properties belong primarily to entities of some other kind, such as propositions, and consider the consequences which would follow if they did.

1

I begin with general considerations about what determines the meanings, in the sense of the truth conditions, of the sentences of a human language, that is the conditions in which they are true and the conditions in which they are false and so which other sentences they entail and are entailed by. Sentences of a language mean what its speakers (or – in the case of technical terms – some group of experts, e.g. physicists) mean by them. Each of us learns the meanings of certain sentences by being shown many
observable conditions under which those sentences are regarded as true or as false, and by being told of other sentences to which a speaker is regarded as committed by uttering those sentences, and other sentences which are such that someone who utters them is regarded as committed to the former sentences. We learn the meaning of a word by being taught the difference to the meaning of a sentence made by that word playing a certain role in the sentence. By being taught the meanings of individual words and of sentences of various forms, we may then come to an understanding of the meaning of a sentence in which those words are arranged in a certain way, even if we have not been shown observable conditions under which that sentence is regarded as true or as false. Showing ‘observable conditions’ may involve pointing to them or describing them by terms already introduced. For example, we learn the meaning of ‘there is a cat over there’ by being shown observable circumstances under which this sentence is regarded as true, and observable circumstances under which it is regarded as false; and by being told that someone who utters this sentence is regarded as committed to ‘there is an animal over there’, and someone who utters ‘there are two cats over there’ is regarded as committed to the original sentence. We learn the meaning of ‘there is a dog over there’ in a similar way. Thereby we come to know the meanings of ‘cat’ and ‘dog’, and so the kind of meaning possessed by sentences of the form ‘there is a F over there’. We need to observe many different examples of observable conditions under which a sentence containing a certain word in various roles is regarded as true or false, and of the commitments speakers who use sentences containing that word in various roles are regarded as having; and this allows us to acquire an understanding of the conditions under which some new sentence containing that word would be regarded as true or false. Examples of different observable conditions under which some sentence is true or false, and of sentences to which we are not committed by a given sentence also illustrate which conditions do not rule out the sentence being true. We extrapolate, that is, from a stock of supposedly paradigm examples (of observable conditions and relations of commitment) to an understanding that the sentence would be regarded as true (or false, as the case may be) under conditions sufficiently similar in certain respects to most of the paradigm examples.\(^1\)

\(^1\)Note however that the sense of a word which we get from this process may be such as to rule out a few of the supposedly paradigm examples as examples of things to which that word applies. Thus we may derive from many supposedly paradigm examples by which we are taught the meaning of the word ‘cat’ a sense of ‘cat’ which
Because humans have very similar psychologies determining how they learn meanings, and because members of a language group are exposed to very similar paradigm examples (of observable conditions and rules of commitment), members of the same language group normally acquire a very similar understanding of the meanings of words and sentences. This common understanding may be reinforced by dictionary compilers and philosophers who ‘tidy up’ language by laying down rules for correct usage, usually by codifying most people’s actual usage. The rules give general descriptions of the observable conditions under which various sentences of the language are true and of the observable conditions under which various sentences of the language are false, and of the kinds of other sentences to which a sentence of a given kind commits the speaker and by which sentences of other kinds a speaker is committed to a sentence of a given kind. The rules of the syllogism for example are rules of this latter kind; ‘all A’s are B’ and ‘All B’s are C’ commits one to ‘All A’s are C’. But such rules can in the end only be understood by examples of observable ‘conditions’ and ‘kinds’ of sentences. One couldn’t understand the stated rule of the syllogism without being shown some things which have some property, and some things which have another property, and examples of things which constitute ‘all’ members of a class. This programme of ‘tidying up’ language aims to secure uniformity of use. To the extent to which it is successful in a language group, there is a correct use of language, and it is an objective matter to what one is and to what one is not committed by some sentence.

Words and sentence forms may be ambiguous, and new words and sentence forms enter language; but I shall count the language as having a correct use, so long as speakers can be got to recognize the ambiguity or novel meaning. This can often be achieved by philosophical discussion forcing a speaker to admit that in one sense of a word ‘W’, ‘S is W’ is true, whereas in another sense ‘S is W’ is false. Because of a lack of sensory or cognitive apparatus, some speakers do not have the capacity to extrapolate from any paradigm examples or inferential rules to the applicability of sentences in new situations. Some people are colour-blind, and so unable to understand the sense of ‘green’ to which they have been introduced by examples of green objects and so apply it to new instances. Other people do not have the cognitive apparatus to recognize some philosophical or rules out one of these examples as being a cat at all; it might turn out to have been a baby tiger instead. I ask the reader to understand future uses of ‘paradigm’ as short for ‘supposedly paradigm’.

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mathematical concept such as ‘tensor product’ or ‘internal negation’, to which they have been introduced, and so apply it to new instances. But so long as those who purport to be able to extrapolate from paradigm examples can be got to agree how to do, I shall count the expressions as having an objective meaning in the language. I shall call the assumption that all sentences of the language would have, in consequence of these procedures an objective meaning, the ‘common language assumption’. I shall call a rule for what one is objectively committed to by a sentence, a rule of mini-entailment: \( s_1 \) mini-entails \( s_2 \) if and only if anyone who asserts \( s_1 \) is thereby (in virtue of the rules for the correct use of language) committed to \( s_2 \); and \( s_1 \) entails \( s_n \) iff they can be joined by a chain of mini-entailments, such that \( s_1 \) mini-entails some \( s_2 \), \( s_2 \) mini-entails some \( s_3 \) and so on until we reach a sentence which mini-entails \( s_n \). I shall call a rule for what one is objectively not-committed to by a sentence or its negation a ‘compatibility rule’: \( s_1 \) is compatible with \( s_2 \) iff \( s_1 \) does not entail not-\( s_2 \). If a sentence \( s_1 \) is compatible with \( s_2 \), it is of course compatible with all the entailments of \( s_2 \).

2

Among metaphysical necessities etc are ones discoverable a priori, that is discoverable by mere reflection on what is involved in the claim made by the sentence. I’ll call these logical necessities etc. (They include both ‘logical’ necessities etc in a narrow sense, and ‘conceptual’ necessities etc.). The obvious examples by which we learn the meaning of ‘logically impossible sentence’ are self-contradictory sentences and ones which entail self-contradictions. A self-contradictory sentence claims both that something is so and also that it is not so, for example, ‘he is taller than 6ft and it is not the case that he is taller than 6 ft’. For such a sentence could only be true if that something was so, and the sentence asserts that it is not so. No sentence could be more obviously or more strongly impossible than such a sentence; and any sentence which entails a self-contradiction is as strongly impossible as a self-contradiction. And the natural understanding which most of us get from these examples is that a logically impossible sentence just is one which entails a self-contradiction; and so any logically necessary sentence is one whose negation entails a self-contradiction, and any logically possible sentence is one which does not entail a self-contradiction.

Purported examples of logically necessary sentences whose negations do not entail a self-contradiction, turn out, I suggest, on examination,
either to be such that their negations do entail a self-contradiction or not to be nearly as strongly necessary as ones whose negations do entail a self-contradiction. And there is a general reason for denying that there are any logically impossible sentences other than ones which entail a self-contradiction (that is, any sentences which are as strongly impossible as those which entail a self-contradiction, and whose impossibility is detectable a priori, but which do not themselves entail a self-contradiction). The reason is that any such sentence must have the form of a declarative sentence, in which the component words already have a sense in the language. It will be a subject-predicate sentence, an existential generalization, or some other one of many recognized forms of declarative sentence. It will – to put the point loosely – assert something about some substance

\[\text{Robert Adams has one example of what, he writes, 'seems to be a necessary truth': 'Everything green has some spatial property' (see, [1], pp. 213 - 214). He claims that this sentence cannot be shown to be 'analytic'. 'Analytic' may be understood in different ways, but one way which Adams mentions is being true 'solely in virtue of the meanings of its terms'; and he claims that this account is 'so vague as to be useless'. But if 'analytic' is spelled out in terms of the negation of the sentence entailing a self-contradiction, the notion is clear. I suggest that being 'green' can be understood in two possible ways, and that the cited sentence with 'green' understood in either of these ways can be shown to be such that its negation entails a self-contradiction. Being green is a property of a thing. One can understand the word 'green' in such a way that a thing being 'green' entails that thing being a publicly visible thing. A publicly visible thing must have a spatial extension – for what one sees one sees as occupying a region of space. In that case the negation of the cited sentence clearly entails a self-contradiction. But one can understand 'green' in a sense in which (not merely a public visible thing, but also) a private thing experienced by only one person, the content of a mental event such as a sense-datum (or, less controversially, an after-image), could be 'green'. Clearly what it would be for that private thing to be green is to have the same visual appearance in respect of colour as a green public object. It must look like a surface or a volume which is green; and so must have the visual appearance of a spatial thing. For a private object to have a visual appearance of a spatial thing entails it looking as if it occupies a region of public space, and it can only do that if it occupies a spatial region of one’s visual field. So again, even if one allows the existence of private objects which are green, the negation of 'everything green has some spatial property' entails a self-contradiction. Adams’s example does not disconfirm my claim that the logically necessary is simply that the negation of which entails a self-contradiction, and that similar equivalences hold for logical possibility and impossibility.\]
or property or event or whatever that it has or does not have some property or relation to some other substance, property etc; or that there are or are not certain substances, properties or whatever. Words have a sense in so far as it is clear what are the criteria for an object, property or whatever to be that object, property or whatever – they therefore delimit a boundary to the sort of object or property it can be or the sort of properties it can have. Hence it will be inconsistent to affirm that an object picked out by some expression is of a kind ruled out by the very criteria for being that object. And the form of a sentence $s_1$ will exclude some alternative $s_2$; and so it will be inconsistent to affirm ($s_1$ and $s_2$). It follows therefore that sentences exemplifying what used to be called ‘category mistakes’, e.g. ‘Caesar is a prime number’ or ‘this memory is violet’ are – in my sense – logically impossible sentences. If a sentence is not impossible for these reasons, then it will be making a claim about the world which does not entail a self-contradiction, a coherent claim. And plausibly no coherent claim can be as strongly impossible as a self-contradictory claim.

Given the common language assumption, we should all be able to agree – within a finite time – about many sentences that they entail self-contradictions, and so are logically impossible; and about many sentences that they are such that their negations entail self-contradictions, and so are logically necessary. Compatibility rules also allow us to recognize many logically possible sentences; and so, since any sentence entailed by a logically possible sentence is itself logically possible, to recognize many more logically possible sentences. Of course a philosophical discussion often begins with disagreement about what entails what or what is compatible with what. The way to resolve a disagreement about whether $p$ entails $q$ is to find a route of mini-entailments from $p$ to $q$, or – alternatively – a route of mini-entailments from ($p$ and not-$q$) to a self-contradiction. The way to resolve disagreements about whether $p$ is logically possible is to find some sentence (normally some long conjunction describing a circumstance) which disputants agree to be logically possible, which entails $p$), or to find some self-contradiction entailed by $p$. But prolonged failure to resolve disagreements in these ways is evidence of a failure in the common language assumption. That failure would mean that the examples and rules by which a word or sentence form has been given a sense has led to different unshareable concepts, different incommunicable understandings of that word or sentence form by different disputants. This may happen

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3I take these examples from the article on ‘Category mistake’ by Jack Meiland in [6].
either because the two groups differ in their sensory life or in their cognitive abilities.

Here is one example. Suppose that the only noises humans could hear were noises produced by strings vibrating with different frequencies; and they then describe the noises produced by the more frequent vibrations as having a higher ‘pitch’. The two groups might have two very different concepts of higher ‘pitch’. One group’s concept of a higher pitch might be simply the concept of being caused by a string vibrating more frequently; ‘higher pitch’ means more vibrations. The other group’s concept might be that of a quality of a noise contingently caused by string vibration. Both groups would allow that everyone normally judges correctly which strings are vibrating more frequently, but that sometimes members of both groups make mistakes. Yet the two groups would describe the ‘mistakes’ differently. For the first group the mistakes are simply (bare) mistakes about how frequently the strings are vibrating, while for the second group the mistakes are mistakes caused by vibrations occasionally causing notes of different pitch from the ones they normally cause. For the first group the ‘deaf’ lack an ability to discriminate vibrations by means of their ears; for the second group the ‘deaf’ lack an ability to have auditory sensations. The different concepts of ‘pitch’ have different entailments. For the first group ‘the string is now vibrating more frequently’ entails ‘it is producing a higher note’: for the second group, it does not. And so on. The first group reports that it ‘cannot make sense of’ much of what the second group is claiming.

The difference in the concepts inculcated by the teaching process may have arisen because the second group has sensations which the first group doesn’t have. (The first group suffer from the auditory equivalent of blind sight). But it may have arisen from a cognitive failure on the part of one or other group. It may be that both groups have sensations, but the first group doesn’t have the ability to distinguish its sensations from its beliefs. Or it may be that neither group has sensations, but the second group suppose that they must be having sensations because they convince themselves from the example of vision that all perceptual beliefs must be mediated by sensations.

A different kind of example shows how lack of cognitive abilities alone may lead to different concepts. We are all taught by the same kinds of example what is a ‘straight line’. Some people come to understand thereby simply a line (which can be extended indefinitely) such that the shortest distance between any two points on the line lies on the line, however far
the line is extended. But others, while allowing that as a possible meaning for ‘straight line’ may acquire a more sophisticated understanding, that a ‘straight line’ is a line (which can be extended indefinitely) such that for any point \( P \) on the line there is some point \( Q \) on the line such that the shortest distance between \( P \) and any point on the line closer to \( P \) than \( Q \) lies on the line. This second understanding allows for the possibility of all straight lines (in this second sense) eventually returning to their starting point, and so there being no straight lines in the first sense, and so of space being unbounded but closed. Some people simply ‘cannot make sense’ of this possibility; for them being a ‘straight line’ in the second sense entails being a ‘straight line’ in the first sense. Other people can make sense of this possibility, and so deny the entailment. And, although I myself can make sense of it, it is always (epistemically) possible that I am deceived.

The only way to attempt to overcome such conflicts is to continue to pursue the methods described earlier – try harder to agree on logically possible sentences which entail a disputed sentence, or to find a route by which it leads to a self-contradiction. But it may be that some of us simply lack the ability to recognize certain modal truths, or – alternatively – deceive ourselves into supposing that certain sentences are logically possible, when they are not. But where this doesn’t happen, there will be agreement about what is logically impossible etc, so long as we have the same understanding of the ‘logically impossible’ as that which entails a self-contradiction.

3

But not all metaphysical impossibilities or necessities are logical impossibilities or necessities. S. Kripke in [4] and H. Putnam in [7] drew our attention to the fact that there are many sentences which are such that neither they nor their negations seem to entail any self-contradiction, but which seem to be necessarily true or necessarily false with a necessity as strong as that of logical necessity, but whose truth or falsity are discoverable only a posteriori. These sentences were said to be metaphysically but not logically, necessary or impossible. Thus, to modify an example used by Kripke to illustrate this class of sentences, suppose that in days

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4I interpret the claims of Kripke and Putnam about necessity etc. as claims about the necessity of sentences. Kripke makes it clear that his concern is with sentences, and writes that he has no ‘official doctrine’ of how his account applies to ‘propositions’ ([4], pp. 20 - 21).
long before people knew the geography of the Himalayas, explorers named a mountain of a certain visual appearance seen from Tibet ‘Everest’, and a mountain of a certain different shape seen from Nepal ‘Gaurisanker’, and used these names as rigid designators of the mountains. (A ‘rigid designator’ is a word which picks out the same object, however the object may change in respect of its non-essential properties.) These mountains are the same mountain; and being the mountains they are, they are – by the necessity of identity – necessarily the same mountain; and so – it seems – ‘Everest is Gaurisanker’ is necessarily true, with as hard as necessity as any logically necessary sentence. However – we may suppose – the explorers did not know this, and clearly would not have been able to discover its truth by mere a priori means. Hence it is not a logically necessary truth. Or consider Putnam’s example of ‘water is $H_2O$’, ‘water’ being understood – as Putnam supposes that it was in the early nineteenth century – as a rigid designator of the transparent drinkable liquid in our rivers and seas. What makes the stuff that stuff is its chemical essence – being $H_2O$. Having that essence, it could not not have that essence. So ‘water is $H_2O$’ is metaphysically necessary, but again not so discoverable a priori. Hence is must be an a posteriori metaphysical necessity.

What has made these necessary sentences a posteriori is that the sentence contains at least one rigid designator of which we learn the meaning by being told that it applies to certain particular things (especially substances and kinds of substances) having certain superficial properties, but where – we are told – what makes a thing that thing (that substance or a substance of that kind) is the essence (of which we may be ignorant) underlying those properties. In ignorance of the latter, we do not fully understand what we are saying about a substance when we say that it is that substance or a substance of that kind. Hence I shall call such designators ‘uninformative designators’.

I define a rigid designator of a thing as an ‘informative designator’ if and only if someone who knows what the designator means (that is, has the linguistic knowledge of how to use it) knows a certain set of conditions necessary and sufficient (in any logically possible world) for a thing to be that thing (whether or not he can state those conditions in words.) I define a rigid designator as an ‘uninformative designator’ if and only if these conditions are not satisfied. To ‘know’ these conditions for the application of a designator – as I shall understand this expression – just is to be able (when favourably positioned, with faculties in working order, and not subject to illusion) to recognize where the informative designator (or, if it is defined
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in words, the words by which it is defined) applies and where it does not and to know the mini-entailments of sentences in which it occurs. Having the ability to recognize something when favourably positioned with faculties in working order and not subject to illusion, involves knowing what that thing is. In the case of technical terms, it is experts in the relevant field whose knowledge of the relevant necessary and sufficient conditions determines the meaning of a term. Thus it is physical scientists, whose knowledge determines the meanings of ‘quark’ or ‘electron’.

Many of the words – for example ‘red’, ‘square’, ‘has a length of 1 metre’ – by which we pick out properties are informative designators; they are such that if we know what the words mean we can recognize (subject to the stated restriction) where they do or do not apply, and can make the requisite inferences. Other words by which we pick out properties can be defined by words for which those conditions hold. For example ‘has a length of $10^{-15}$ metres’ can be defined in terms of the informatively designated property ‘has a length of 10 metres’ and the informatively designated relation of ‘being shorter by $1/10^{th}$ than’ (used 15 times).

So the reason why the claims originally made by the sentences ‘Everest is Gaurisanker’ and ‘water is $H_2O$’ are necessary with as hard as necessity as ‘logical necessity’ is that they are logically necessary, but the use of uninformative designators has the consequences that speakers did not know fully what these claims were until they had done some a posteriori investigation. When we know fully what we are talking about, mere a priori considerations can show whether some sentence is metaphysically necessary or impossible. Hence there is available a definition of a sentence as metaphysically necessary (impossible or possible) iff it is logically necessary (impossible or possible) when we substitute co-referring informative designators for uninformative designators. This definition will capture as metaphysically necessary (impossible or possible) almost all the examples of the ‘metaphysically necessary’ (‘impossible’ or ‘possible’) offered by Kripke, Putnam, and others. And so from these examples we derive a sense of metaphysically necessary in which a sentence is metaphysically necessary (impossible or possible) iff it is logically necessary (impossible or possible) when informative designators are substituted for uninformative designators. And, given the earlier understanding of ‘logical impossibility’, and so the understanding of metaphysical impossibility as in reality logical impossibility, it would seem that no sentence could be as strongly impossible metaphysically as one which is in reality logically impossible; and so there can be no metaphysically impossible (necessary, or possible)
sentences apart from ones of the kind analysed in this section.

4

So, given some a posteriori logically contingent information (e.g. about which are the molecules of which whatever is the transparent stuff in our rivers and seas are made) which determine which sentences are metaphysically necessary, there should be no scope for disagreement about modal metaphysical status of any sentence – given that the common language assumption applies to the words and sentence forms of the language – and that we have the same understandings of ‘logical’ and ‘metaphysical’ impossibility.

Now I have introduced the term ‘metaphysically’ impossible as the strongest kind of impossibility which a sentence can have; and defined the ‘logically’ impossible in terms of the metaphysically impossible. I have filled out what it is for a sentence to be impossible in this ‘strongest’ sense by examples of self-contradictions, entailments, and compatibilities; and by means of Kripke-Putnam type examples, which I have described in my own way by means of the concept of an ‘informative designator’. The particular examples could form the basis of any philosophy student’s introduction to the concepts of metaphysical and logical impossibility. From these examples I have derived sharp usable senses of ‘logically impossible’ (‘entailing a contradiction’) and ‘metaphysically impossible’ (reducible in the stated way to ‘logically impossible’), and thereby of the other modal concepts. I call these senses of the terms ‘logically’ and ‘metaphysically’ the narrow senses, and I will assume them for the rest of this section.

Others may purport to derive from the paradigm examples wider understandings of these terms. They may suppose (for example) that there are impossibilities in the strongest sense detectable a priori which do not entail contradictions; or necessities in the strongest sense which do not reduce to sentences whose negations entail contradictions when we substitute informative for uninformative designators. The issue then arises whether it

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5 Gendler and Hawthorne write, that ‘the notion of metaphysical possibility ... is standardly taken to be primitive’, adding in a footnote ‘in contemporary discussions at any rate’ ([2], p. 4). It doesn’t help me to understand this notion for them to say that it is the most basic conception of ‘how things might have been’ (ibid pp. 4 - 5). For since this ‘most basic conception’ is supposed to be wider than logical possibility (as defined by me), it is unclear how the latter is to be widened unless in the way I have analysed. ( In one book I myself unhelpfully used ‘metaphysically necessary’ to
is logically possible (on my understanding of this concept) for there to be such impossibilities or necessities. And the only way to resolve any disagreement about this is by the methods described earlier: putting forward examples which we can both recognize do not entail self-contradictions, or showing by a route that we can both recognize that purported examples do entail self-contradictions. But I am pessimistic about the chances of my reaching agreement with many of my opponents on this matter within any finite time. That will show that one or other of us suffers from some (sensory, or much more likely) cognitive deficiency.

5

However, given my understanding of these concepts together with my assumption that the modal properties are properties of sentences, it seems fairly implausible to suppose that a (positive) existential sentence (a sentence claiming that there exists some thing or things of a certain kind) can be logically necessary – for to be so it would need to be such that its negation entails a self-contradiction. The negation of an existential sentence has the form ‘¬∃xφ(x)’; it claims that a certain property (or conjunction of properties) is not instantiated. A self-contradictory sentence, of a kind not containing modal operators, claims that the actual world has a contradictory quality, and that will be so either because some object within it has such a quality or because it both does and does not contain an object of a certain kind. So it will have the form or entail a sentence of the form ‘∃xφ(x) ∧ ¬∃xφ(x)’ or the form ‘∃x(φ(x) ∧ ¬φ(x))’. Either way it will include or entail a positive existential sentence. But plausibly the mere non-existence of anything of some kind cannot entail the existence of anything. It may be suggested that the contradiction has the form of sentence in which there are modal operators, where the contradiction arises from its modal features; for example a sentence of the form ‘it is possible that something is φ, and it is not possible that something is φ’, ‘◊∃xφ(x) ∧ ¬◊∃xφ(x)’. But it is not easy to see how the mere non-existence of anything of some kind could entail a modal sentence which would not be entailed by the existence of a thing of that kind.

mean (roughly) whatever is the ultimate cause of things or is entailed by the existence of that ultimate cause; and so the ‘metaphysically possible’ is whatever is compatible with the existence of the actual ultimate cause (see, [10], pp. 118 - 119). But this is certainly not the sense which most writers who use the term have in mind, and not the sense in which I am using in this paper.)
The plausible suggestion that the mere non-existence of anything of some kind cannot entail a contradiction, and so no positive existential sentence can be a necessary truth is of course due to Hume.\(^6\) It will hold whether the thing is of a concrete or an abstract kind. So the supposed necessary existential truths of arithmetic do not constitute an exception. The negation of, for example, such a supposed necessary truth as ‘There are prime numbers greater than 3’, ‘There are no prime numbers greater than 3’, does not by itself entail a contradiction; it does so only when conjoined with some existential axiom of arithmetic (e.g ‘There is a number 1’ and ‘Every number has a successor’). So my suggestion must hold also for the special case where ‘Θ’ designates any conjunction of properties of a kind supposed to constitute a definite description of God – e.g. ‘omnipotent’, ‘omniscient’, ‘perfectly good’, and ‘eternal’ – and so God cannot be a logically necessary being.

However some people claim (in effect) that a particular negative existential sentence of this kind does entail a contradiction, and some of them claim to have demonstrated this. If this disagreement persists after serious attempts to clarify the issues, this indicates another case where my ‘common language assumption’ is mistaken. The entailment must depend on understanding sentences in different senses from the way the rest of us understand them, senses which are not equivalent to any which we can grasp. Yet if someone claims that ‘there is no x such that Θ(x)’ (where ‘Θ’ designates a definite description of God of any traditional kind) entails a contradiction, they will need to hold that innumerable other sentences of very different kinds to each other entail contradictions, when the rest of us hold that they are obviously logically possible. For example they will need to hold that ‘Once upon a time there were no rational beings’, and ‘No one knows everything’, and ‘No one is perfectly good’, and – among sentences evidently describing non-actual worlds – ‘The only substances are four mutually repelling steel balls’, and ‘No one knows what is happening outside a sphere of 1 mile diameter surrounding their body’ all entail contradictions, because – if any one of them does not entail a contradiction – there would not be a God in anything like a traditional sense. Someone could only derive a contradiction from all such sentences if they understood an enormous number of predicates – not just one or two technical philosophical terms – in different senses from the rest of us, or if they understood one or more formal terms such as ‘exists’, ‘not’, or ‘all’ differently from the rest of us. In view of the similarity in respect of psychological make-up and the

\(^6\)There is no being ... whose non-existence implies a contradiction’ ([3], Part 9).
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process of language acquisition between humans who believe in ontological arguments and those who don’t, I do not find it very plausible to suppose that the former understand all these predicates in different senses from the rest of us. But if they do understand the predicates in the same senses as the rest of us, they will have to admit that ‘there is no God’ does not entail a self-contradiction, and so ‘there is a God’ cannot be logically necessary in the sense in which I have spelled it out.

But if there is no logically necessary sentence of the form ‘there is an $x$ such that $\Theta(x)$’ where ‘$\Theta$’ designates a definite description of God, no substitution for ‘God’ in ‘God exists’ of a co-referring designator will yield a logically necessary sentence, and so ‘God exists’ cannot be metaphysically necessary in the sense in which I have spelled it out.

6

My arguments so far assume that the primary bearers of modal properties are sentences of human languages. Because many human languages have very similar structures to each other – words introduced by the same observable circumstances, the same types of sentences (subject-predicate sentences, existential sentences, and so on) with parallel inference patterns between them, some sentence in one language often means the same as some sentence in another language; they are inter-translatable. And so in order to talk about the claim which would be made by any such sentence, or any other sentence meaning the same which might be uttered in a language not yet invented, it is useful to suppose that – even before such a claim is made – there is a common thing which they all express, a proposition, the content and logical consequences of which can be discussed independently of the particular language in which it is expressed. This however is merely a useful fiction. There is no reason to suppose that there really are such things as propositions, existing independently of the sentences which express them. We cannot interact with propositions, nor do we need to postulate them in order to explain what we observe – the behaviour of humans who utter sentences. And if the necessary truths were truths about eternal propositions, there would be no easy explanation of how we are in a position to know which such propositions are necessarily true. Why should we trust our intuitions about this Platonic realm? Whereas if necessary truths are truths about human language, there is a ready explanation of how we are in a position to know about them: we learn them in learning language. So there is no reason to deny that ordi-
nary talk about ‘propositions’ (of a kind that does not imply their eternal existence) can be analysed as talk about human sentences.

If however we suppose that propositions are real timeless entities which have a modal status independently of any human sentences which might express them, then there is some plausibility in the claim that the proposition expressed (imperfectly) by the sentence ‘there is a God’ might be a metaphysically necessary truth – even if we assume that logically impossible propositions are ones which entail a self-contradiction, and metaphysically impossible propositions are ones from which we can derive from logically impossible propositions by substituting co-referring informative for uninformative designators. For there will not be the slightest reason to suppose that there are only as many propositions as will eventually be expressed or even (in some sense) could be expressed. In that case there would not be any reason to suppose that all necessary propositions which can be expressed can be shown by us to be necessary, because the demonstrations thereof may depend on a deduction which proceeds by means of propositions of kinds which cannot be expressed and whose mini-entailments may be known only to superior beings. So maybe ‘God exists’ is necessarily true – even though we humans are totally unable to show that. That of course would not provide us with a sound ontological argument, but it does allow the possibility of there being one, unknowable by humans.

This way of thinking does however carry certain unwelcome consequences for theism. It looks as if all these other necessary propositions coexisting eternally with God, constrain how God can act. If for example it is a necessarily true proposition that God cannot make me exist or not exist at the same time, then this constrains what God can do – not merely what we can do with the English language without uttering a necessarily false sentence. That necessarily true proposition would limit God. One way of attempting to avoid this is to claim that necessarily these propositions are ideas in the mind of God, and so part of his nature. But that raises the question why these propositions rather than any other ones are part of God’s nature, and if the answer is that that it is just a brute fact that they are, that would make God essentially a very un-simple being. Alternatively one could suppose that God has a nature such that necessarily he creates just these necessary truths.⁷ On a normal theistic view God is necessarily good, and so the only necessary truths which it follows from his essence

⁷See, for example, Thomas Morris and Christopher Menzel [8]. The later pages of this paper (pp. 172 - 178) go on to defend a (to my mind) even more implausible correlate of this view – that God creates his own nature.
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That he will create will be good ones. But there are well-known difficulties in supposing that any agent (including God) can make actions (morally) good (except in virtue of some fundamental moral principle which lays down which actions an agent can make good under which circumstances – e.g. that it is good to use force to uphold a just law if commanded to do so by a just sovereign).\(^8\) Given that the fundamental principles about which actions are good (that is, would be good if they were performed) are independent of the actions of agents, the fundamental principles to which propositions it is good to award the status of necessary truth will be independent of the actions of God. On the propositional view these will be eternally necessary propositions (stating which propositions it would be a good action to make necessary) existing independently of the actions of God, and determining which actions he can do. But then there will be necessarily true eternal propositions independent of the will of God stating which actions (of creating necessary truths) these are. These propositions determine how God can act; they cannot be consequences of God’s actions. But if necessary truths (including the fundamental moral truths) are just truths of human language about human language (including the truth about which property ‘morally good’ designates, and so which properties, such as the property of feeding the starving, are entailed by it), there are no pre-existing things apart from God – although of course the states of affairs which human language is used to describe – e.g. that God is good – may exist before they are described by humans.

So for all these reasons we should regard logical necessity as belonging primarily to human sentences, and only to any other entities as a convenient fiction; and then, I suggest, it follows that God is not a metaphysically necessary being (in the sense analysed in this paper), because it is not logically possible (in my sense) that there be any metaphysically necessary being.\(^9\) But this fact has no relevance to the logical possibility of there

\(^8\)For example, it does rather look as if some of the same actions would be good or bad if there was no God, as are good and bad if there is a God. But if no action would be good or bad unless God had so willed it, that must be because God has some property which other persons lack (e.g. being our creator, or being omnipotent) so that God’s willing some action would make this difference whereas other persons willing it would not. But then there must be a fundamental principle independent of God, that it is good to do any action willed by someone having that property; and that principle couldn’t be true in virtue of being willed by God. For my account of the relation of God to morality see my [12] and [9], Chapters 1 and 8.

\(^9\)I very much doubt whether anyone earlier than Anselm thought that God is a
existing a being necessary in some other sense, e.g. a being essentially everlasting and essentially not causally contingent on the existence of any other being for its own existence, which is a property which all traditional theists have believed God to have. And this fact allows the possibility of there being a cogent inductive argument to the existence of such a being.\textsuperscript{10}

metaphysically necessary being in the sense being discussed. Aquinas did not use ‘necessary being’ in this sense. For in his *Summa theologiae* Ia.50.5ad3 ([13]) (as elsewhere) he clearly implies that, as well as God, angels (which are beings created by the voluntary act of God) are also necessary beings. He seems to think of a necessary being as one not subject to corruption, that is one which will go on existing forever unless caused not to exist by something else. He distinguished God from other necessary beings as a ‘being necessary through its own nature (per se) and not caused to be necessary by something else’ ([13], Ia.2.3) and so ‘unconditionally necessary’. Angels depend for their non-corruptibility on God and so are only ‘conditionally necessary’. However Aquinas also seems to claim (in effect) that only self-contradictions are absolutely impossible. (‘The impossible is that in which the predicate is incompatible with the subject’ – [13], Ia.25.3.) That might seem to suggest that he thought that the negation of ‘there is a God’ entailed a contradiction, and so he did – but that was because he thought that anything incompatible with what was already fixed entailed a contradiction; on his view what is absolutely possible changes with time. But God, as the eternal source of everything, is always fixed, and so – by Aquinas’s criteria – his non-existence is always impossible, and that is why he is absolutely necessary. (I am indebted for this analysis of Aquinas’s understanding of modal concept to Brian Leftow. See his paper “Aquinas, Divine Simplicity, and Divine Freedom” [5]) But this is not metaphysical necessity in the sense in which I have been discussing it, which derives from a sense of logical necessity in which ‘entailing a contradiction’ is something intrinsic to a sentence (with its meaning), and independent of what is or is not already fixed outside the sentence. Admittedly, Aquinas also thought that ‘God is the same as his own nature or essence’ ([13], Ia.3.3); but he goes on to claim that anything immaterial, not just God, is the same as its own nature. His point is simply that material things are individuated by the matter of which they are made, whereas immaterial things are individuated by their forms, that is natures. I know of nothing in Aquinas which should lead us to suppose that he thought that God’s existence is a metaphysically necessary truth (in the sense used in this paper). He certainly thought that on Anselm’s ‘definition’ of God the negation of ‘There is a God’ did not entail a self-contradiction ([13], Ia.2.1.ad.2), and I know of no reason to suppose that he thought that this would hold on any other ‘definition’ (in our sense) of ‘God’.

\textsuperscript{10} For my own inductive arguments to the existence of God of this kind, see my [11]
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Bibliography


Part VI

Ontological Proofs and Formal Ontology
On Grim’s Cantorian Anti-Ontological Argument

ROBERT E. MAYDOLE

1 The Cantorian Argument

Anti-ontological arguments are deductive arguments for the nonexistence of God from general a priori principles of metaphysics, logic, mathematics, and other assumptions about the nature or essence of God. They should be distinguished from arguments that purport to show that ontological arguments are flawed. The so-called paradox of the stone and Hume’s argument against the existence of necessary entities are anti-ontological. A very formidable recent example is Grim’s Cantorian argument against the existence of an omniscient being.

Grim sketches his Cantorian argument as follows:

“Were there an omniscient being, what that being would know would constitute a set of all truths. But there can be no set of all truths, and so can be no omniscient being ... [For] suppose there were a set $T$ of all truths, and consider all subsets [of] $T$ – all members of the power set [of] $T$. To each element of this power set will correspond a truth. To each set of the power set, for example, a particular truth $T_1$ either will or will not belong as a member. In either case we will have a truth: that $T_1$ is a member of that set, or that it is not. There will then be at least as many truths as there are elements of the power set of $T$. But by Cantor’s power set theorem we know that the power set of any set will be larger than the original ...” ([2], p. 267)
ON GRIM’S CANTORIAN ANTI-ONTOLOGICAL ARGUMENT

There are many possible ways to object to Grim’s Cantorian argument, the most notable being:

(1) Reject Cantor’s theorem outright by rejecting the very idea of different levels of infinity.

(2) Deny the existence of the power set of each set.

(3) Show it as a disguised Liar Paradox.

(4) Link omniscience to some vaguely defined multiplicity or totality of truths other than the set all truths.

(5) Redefine omniscience to avoid quantification over any kind of multiplicity, totality or collection.

(6) Adopt a non-classical logic without the principle of excluded middle.

(7) Use a set theory other than the set theory that undergirds Grim’s reasoning.

Even though Grim has responded forcefully to most of these objections, I believe that there is more to the story than he tells about whether his Cantorian argument is sound and stands up in the face of certain alternative set theories.

It will be useful to logically reconstruct Grim’s Cantorian argument by filling in the missing premises and putting it into standard logical form thus:

(G1) If a God of Theism exists, then something is omniscient.

(G2) If something is omniscient, then the class of all truths \( \exists \) exists.

(G3) If \( \exists \) exists, then the class of all subclasses \( \mathcal{P}(\exists) \) (the power class \( \mathcal{P}(\exists) \)) of \( \exists \) exists.

(G4) For every member \( x \) of \( \mathcal{P}(\exists) \), there is a truth \( y \) such that \( y \) is about \( x \) and not about any other member of \( \mathcal{P}(\exists) \).

(G5) If there is a truth \( y \) that is about every member of a class \( x \) and not about any other member of \( x \), then \( \exists \) has at least as many members as \( x \).

(G6) \( \exists \) has fewer members than \( \mathcal{P}(\exists) \).

(G7) For any classes \( x, y \) and \( z \), if \( x \) has fewer members than \( y \), and \( z \) has at least as many members as \( y \), then \( x \) has fewer members than \( z \).
(G8)  It is not the case that $\mathcal{S}$ has fewer members than $\mathcal{S}$.

Therefore,

A God of Theism does not exist.

This logical reconstruction of Grim’s Cantorian argument is clearly valid, and I have no reason to challenge premises (G1), (G2), (G5) and (G7). What I shall show first is that Grim fails to support (G4) and that (G4) is arguably false. Secondly, I shall argue contra Grim that (G3), (G6) and (G8) do not fare well in Quine’s axiomatic set theory ML (where ‘ML’ is short for ‘Mathematical Logic’) when it is slightly modified and/or extended to accommodate truths and classes of truths. Thirdly, I shall contend that because it has never been shown that the axiomatic set theory of Zermelo-Fraenkel (ZF) that undergirds Grim’s reasoning is more plausible than both ML and every other set theory, Grim’s Cantorian argument is not justifiably sound, and it fails to falsify the existence of the God of Theism.\(^1\)

2  The Basics of ZF and ML

ZF has the following constructive axioms: \(^2\)

Extensionality: Classes with the same members are identical.

Pairs: Every two classes are members of some class.

Union: For every class there is a class of the members of its members.

Power class: For any class there is a class of its subclasses.

Separation: For any class there is a class of its members that satisfy any condition $\Phi$.

\(^1\)If we were to construe Grim’s sentence “But there can be no set of all truths, and so can be no omniscient being” as a conjunction of modal absolutes, then a modal Cantorian argument could be either ‘$\Box((G1) \land (G2) \land ... \land (G8)) \therefore \Box(a \text{ God of Theism does not exist})$’, or ‘$\Box^{\text{ZF}}(G1), \Box(G2), \Box(G4), \Box^{(\text{ZF})}((G3) \land (G5) \land (G6) \land (G7) \land (G8))) \therefore \Box(a \text{ God of Theism does not exist})$’, where ‘$\Box^{\text{ZF}}$’ is ‘the conjunction of the axioms of ZF’.

\(^2\)A non-constructive axiom of choice can be consistently added as needed to ZF: For every class of exclusive non-empty classes, there is a class of with exactly one member from each of these non-empty classes.
On Grim’s Cantorian Anti-Ontological Argument

The following lexicon will be useful in discussing \text{MIL}:

\begin{itemize}
  \item \text{\textquotesingle} \alpha \in \beta \text{\textquotesingle} \ for \ \text{\textquotesingle} \alpha \ is \ a \ member \ of \ \beta \text{\textquotesingle},
  \item \text{\textquotesingle} \alpha \prec \beta \text{\textquotesingle} \ for \ \text{\textquotesingle} \alpha \ has \ fewer \ members \ than \ \beta \text{\textquotesingle},
  \item \text{\textquotesingle} \alpha \preceq \beta \text{\textquotesingle} \ for \ \text{\textquotesingle} \beta \ has \ at \ least \ as \ many \ members \ as \ \alpha \text{\textquotesingle},
  \item \text{\textquotesingle} \alpha \subseteq \beta \text{\textquotesingle} \ for \ \text{\textquotesingle} \alpha \ is \ a \ subset \ of \ \beta \text{\textquotesingle},
  \item \text{\textquotesingle} \mathbf{x''}\beta \text{\textquotesingle} \ for \ \text{\textquotesingle} the \ range \ of \ relation/function \ \mathbf{x} \ with \ domain \ \beta \text{\textquotesingle},
  \item \text{\textquotesingle} \{u : \Phi\} \text{\textquotesingle} \ for \ \text{\textquotesingle} the \ class \ of \ all \ \mathbf{u}'s \ such \ that \ \Phi \text{\textquotesingle},
  \item \text{\textquotesingle} \hat{u}\Phi \text{\textquotesingle} \ for \ \text{\textquotesingle} the \ set \ of \ all \ \mathbf{u}'s \ such \ that \ \Phi \text{\textquotesingle}.
\end{itemize}

\text{MIL} \ has \ three \ special \ notions:

\textbf{Stratification:} \ A \ formula \ is \ stratified \ if \ and \ only \ if \ it \ is \ possible \ to \ put \ numerals \ for \ its \ variables \ (the \ same \ numeral \ for \ all \ occurrences \ of \ the \ same \ variable) \ in \ such \ a \ way \ that \ \text{\textquotesingle} \in \text{\textquotesingle} \ comes \ to \ be \ flanked \ always \ by \ consecutive \ ascending \ numerals \ (\text{\textquotesingle} n \in (n + 1) \text{\textquotesingle}). \ ([3], \ p. \ 157)

\textbf{Sets:} \ A \ set \ is \ a \ class \ that \ is \ a \ member \ of \ anything.

\textbf{Ultimate Classes:} \ An \ ultimate \ class \ is \ class \ that \ is \ not \ a \ set.

Finally, \text{MIL} \ has \ three \ constructive \ axioms: \[\text{Extensionality:} \ Classes \ with \ the \ same \ members \ are \ identical.

\text{Sethood:} \ If \ \Phi \ has \ no \ free \ variables \ beyond \ u, \ \beta_1, \ \beta_2, ..., \ \beta_n, \ and \ is \ formed \ from \ a \ stratified \ formula \ by \ restricting \ all \ bound \ variables \ to \ sets, \ then \ \{u : \Phi\} \ is \ a \ set.

\text{Comprehension:} \ There \ is \ a \ class \ \beta \ such \ that, \ for \ every \ \alpha, \ \alpha \ is \ a \ member \ of \ \beta \ if \ and \ only \ if \ \alpha \ is \ a \ set \ that \ satisfies \ \Phi: \ \exists \beta \forall \alpha(\alpha \in \beta \leftrightarrow S(\alpha)&\Phi), \ where \ \text{\textquotesingle}S(\alpha)\text{\textquotesingle} \ is \ short \ for \ \text{\textquotesingle} \alpha \ is \ a \ set\text{\textquotesingle}.

\text{MIL} \ is \ a \ pure \ set \ theory, \ which \ as \ such \ admits \ only \ classes \ and \ classes \ of \ classes, \ etc. \ to \ its \ domains \ of \ discourse. \ Yet \ Grim’s \ Cantorian \ argument \ requires \ an \ ontology \ that \ includes \ truths \ and \ classes \ of \ truths, \ which \ are \ prima \ facie \ different \ kinds \ of \ things \ than \ classes. \ Truths \ by \ most \ accounts \ are \ individuals \ or \ atomistic \ ur-elements \ that \ can \ be \ elements \ but \ not \ have

\[A \ non-constructive \ axiom \ of \ choice \ for \ sets \ can \ be \ consistently \ added \ to \ \text{MIL}: \ For \ every \ class \ of \ mutually \ exclusive \ non-empty \ sets, \ there \ is \ a \ class \ of \ with \ exactly \ one \ member \ from \ each \ of \ these \ non-empty \ sets.\]

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them. But individuals might also be understood as unit sets of themselves ([3], p. 122). So Grim’s Cantorian argument requires us either to interpret truths as self-membered unit sets, or to extend $\mathbb{M}$L to accommodate classes of truths qua ur-elements that are based upon conditions that those atomistic truths satisfy.

The reason one might want to construe an individual $y$ as the self-membered unit class \{y\} would be to make sense of ‘$x \in y$’. This does not mean, however, that we should really believe that individuals are in fact self-membered unit classes. Rather, if our set theory countenances individuals, then, as Quine has said, “‘$x \in y$’ ... is to have as its full translation the following: ‘$x$ is a member of $y$ or is the same as $y$ according as $y$ is or is not a class’” ([3], p. 122). But $y = \{y\}$ if and only $\forall x (x \in y \leftrightarrow x \in \{y\})$, and $x \in \{y\}$ if and only if $x = y$. So $y = \{y\}$ if and only if $\forall x (x \in y \leftrightarrow x = y)$. So, if individuals are construed as self-membered unit classes, and $y$ is an individual, then ‘$x \in y$’ is equivalent to ‘$x$ is the same as $y$’. And, if individuals are construed as self-membered unit classes, and $y$ is not an individual, then ‘$x \in y$’ is equivalent to ‘$x$ is a member of $y$’. So individuals, and truths qua individuals, should not be thought of as really being self-membered unit classes. They only parade as such for the sake of simplicity, but at the risk of ambiguity.

Grim makes two suggestions about how to extend $\mathbb{M}$L in his attempt to show that $\mathfrak{S}$ is inconsistent with a $\mathbb{M}$L framework. He says that we could extend $\mathbb{M}$L by adding a truth predicate and either of the following comprehension principles as axioms:

**Sets of Truths:** There is a set $\beta$ such that, for every $\alpha$, $\alpha$ is a member of $\beta$ if and only if $\alpha$ is a set that satisfies $\Phi$: $\exists \beta \forall \alpha (S(\beta) \land (\alpha \in \beta \leftrightarrow T(\alpha) \& \Phi))$, where ‘$T(\alpha)$’ is short for ‘$\alpha$ is true’.

**Classes of Truths:** There is a class $\beta$ such that, for every $\alpha$, $\alpha$ is a member of $\beta$ if and only if $\alpha$ is a truth that satisfies $\Phi$: $\exists \beta \forall \alpha (\alpha \in \beta \leftrightarrow T(\alpha) \& \Phi)$.

What I now want to briefly show is that the **Sets of Truths** principle is problematic for three reasons. First, if in pure $\mathbb{M}$L only stratified conditions determine sets, it would seem that only stratified conditions on truths should determine sets in an extended $\mathbb{M}$L. Moreover, if **Comprehension** of $\mathbb{M}$L only guarantees the existence of classes of sets, it would seem that conditions on truths should only guarantee the existence of classes of truths.

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Second, it might be thought that omniscience only requires the existence of the set $\mathcal{S}^*$ of all truths. Grim contends, however, that $\mathcal{S}^*$ cannot exist anymore than $\mathcal{S}$ can. But this requires that it is provable in $\mathsf{ML}$ that $\mathcal{S}^*$ has fewer members than the class of all its subsets. In order to prove this variant of Cantor’s Theorem, one would have to prove that there exists a diagonal subset of $\mathcal{S}^*$ that is based upon a diagonalization condition that excludes the members of that diagonal subset from the subsets of $\mathcal{S}^*$ with which they are assumed paired for the purpose of a reductio ad absurdum. Since diagonalization conditions are not stratified, Grim therefore absolutely needs the stronger $\mathit{Sets}$ of Truths rather than $\mathit{Classes}$ of Truths in order to prove that there exists a diagonal subset of $\mathcal{S}^*$.

$\mathit{Sets}$ of Truths also makes it possible to prove that both (G3) and $\mathcal{S} < \hat{u}(u \subseteq \mathcal{S})'$ are true in $\mathsf{ML}$. $\mathit{Classes}$ of Truths does not. $\mathit{Sets}$ of Truths, but not $\mathit{Classes}$ of Truths, generates the subsets of $\mathcal{S}$, including a diagonal subset of $\mathcal{S}$ for any presumed pairing $w$ of $\mathcal{S}$ with $\hat{u}(u \subseteq \mathcal{S})$: $\hat{u}(u \subseteq \mathcal{S} \& u \notin w')$; and this plus Comprehension insures that all such subsets are members of $\hat{u}(u \subseteq \mathcal{S})$. Thus, Grim’s rationale for $\mathit{Sets}$ of Truths looks ad hoc. He needs it to ground his anti-ontological reasoning, and apparently for nothing else.

Third, a condition $\Phi$ determines a class $\beta$ if and only $\forall \alpha (\alpha \in \beta \leftrightarrow \Phi)$. Then the following valid argument shows that $\mathit{Sets}$ of Truths is false in $\mathsf{ML}$ if at least one class of truths is ultimate:

1. There is a set of precisely those truths satisfying any condition.
2. No set is an ultimate class.
3. No more than one class satisfies a condition.

Therefore,

No ultimate class of truths satisfies a condition.

The first premise of this argument is a restatement of $\mathit{Sets}$ of Truths. The second is a logical truth. And the third follows from Extensionality. But the conclusion of this argument is false, as I shall show a few pages hence.

3 On whether the Premises of the Cantorian Argument are True

We can use Grim’s $\mathit{Classes}$ of Truths principle to generate specific subclasses of $\mathcal{S}$. But we cannot couple $\mathit{Classes}$ of Truths with Comprehension...
of $\mathbb{M}\mathbb{L}$ to prove the existence of $\mathcal{P}(\exists)$, for Comprehension only can generate a class of sets, and Classes of Truths only generates classes of truths. Nor can Classes of Truths itself generate $\mathcal{P}(\exists)$. Classes of Truths cannot generate a class of classes of truths, only classes of truths. So (G3) does not seem to be provably true in $\mathbb{M}\mathbb{L}$ extended by Classes of Truths.

There are two reasons for questioning (G4). First, to the best of my knowledge the only justification for (G4) that has ever been given for (G4) by Grim or anyone else is to argue for it by begging the question and giving examples. Here is Grim’s typical argument for (G4), where his ‘$T$’ is our ‘$\exists$’ and his word ‘set’ names either a set or a class of $\mathbb{M}\mathbb{L}$:

“Suppose there is a set of all truths $T$: $T = \{t_1, t_2, t_3, \ldots\}$. And consider further all subsets of $T$, the elements of the power set $\mathcal{P}(T)$: $\emptyset, t_1, t_2, t_3, \ldots, \{t_1, t_2\}, \{t_1, t_3\}, \ldots, \{t_1, t_2, t_3\}, \ldots$. To each element of this power set there will correspond a truth. To each set of the power set, for example, $t_1$ either will or will not belong as a member. In either case we will have a truth: $t_1 \notin \emptyset, t_1 \in \{t_1\}, t_1 \notin \{t_2\}, t_1 \notin \{t_3\}, \ldots, t_1 \in \{t_1, t_2\}, t_1 \in \{t_1, t_2\}, \ldots, t_1 \in \{t_1, t_2, t_3\} \ldots$. There will then be as many truths as there are members of the power set $\mathcal{P}(T)$.” ([1], pp. 91 - 93)

This argument boils down to the following:

(1) To each member of $\mathcal{P}(\exists)$ there corresponds a distinct truth.

(2) For example, it is distinctly true that $t_1 \notin \emptyset$, that $t_1 \in \{t_1\}$, that $t_1 \notin \{t_2\}$, that $t_1 \notin \{t_3\}$, ..., that $t_1 \in \{t_1, t_2\}$, that $t_1 \in \{t_1, t_2\}$, ..., that $t_1 \in \{t_1, t_2, t_3\}$ ... .

Therefore,

There are as many distinct truths as there are members of the power set $\mathcal{P}(\exists)$.

Now this is clearly a fallacious argument. The first premise merely rephrases the conclusion; and the second premises merely lists a finite number of members of $\mathcal{P}(\exists)$, which is surely infinite, with members that are impossible to even list or enumerate.

Proving (G4) would require that ones set theory guarantee the existence of a correspondence function that maps $\mathcal{P}(\exists)$ onto some subclass of $\exists$. But $\mathbb{M}\mathbb{L}$ does not prove the existence of any such function. Indeed, since both $\mathbb{M}\mathbb{L}$ (and $\mathbb{Z}\mathbb{F}$) proves Cantor’s Theorem, $\mathbb{M}\mathbb{L}$ (and $\mathbb{Z}\mathbb{F}$) proves that there is no correspondence function between $\mathcal{P}(\exists)$ and some subclass of
On Grim’s Cantorian Anti-Ontological Argument

ℑ, howsoever many lists of subclasses of ℑ someone might produce to persuade us that there is.

This brings us to our second reason for challenging (G4): (G4) contradicts Cantor’s Theorem in ZF. That alone shows that to Grim’s Cantorian argument is not sound.

Since ℙ(ℑ) is not guaranteed to exist in MLL with Classes of Truths, (G6) cannot be inferred from Cantor’s Theorem. Thus, (G6) does not seem to be provable in MLL with Classes of Truths.

One might think that (G8) is the most intuitively obvious of the premises of the Cantorian argument. Neither Grim nor anyone else ever argues for it, to the best of my knowledge. Yet intuitions can sometimes be unreliable. Consider the argument below for why ‘ℑ ≺ ℑ’ might actually be true in MLL.

In set theory ‘α ≤ β’ usually means that there is a function that maps α onto a subclass of β. Since the range (image) of a function cannot be bigger than its domain (argument), equivalently, one can take ‘α ≤ β’ to mean that α is a subclass of the range of a function with domain β. Let ‘Funk(x)’ be short for ‘x is a function’. Then ‘α ≤ β’ is just ‘∃x(Funk(x)&α ⊆ x'β)’ ([4], p. 78).

Argument for ℑ ≺ ℑ:

(1) ¬∃x(Funk(x)&ℑ ⊆ x''ℑ) → ¬(ℑ ≤ ℑ).
(2) ¬(ℑ ≤ ℑ) → (ℑ ≺ ℑ).
(3) If ℑ is a class of individuals that are not classes then ¬∃x(Funk(x)&ℑ ⊆ x''ℑ) holds.
(4) ℑ is a class of individuals that are not classes.
(5) (Therefore) ℑ ≺ ℑ.

This argument is clearly valid. The first two premises are logical truths, and we showed earlier that the fourth premise is true. So let us now take up the third premise.

The truth of ‘∃x(Funk(x)&α ⊆ x''β)’ is equivalent in MLL to there being a function that maps α onto a subclass β. So ¬∃x(Funk(x)&ℑ ⊆ x''ℑ) equivalent in in MLL to there being a function that maps ℑ onto in ℑ. So, does MLL insure that there is such a function?

The answer is no. Not even the identity function can map ℑ onto ℑ in MLL. Functions in MLL only apply to sets lest paradox ensue. But no member of ℑ is a set, because truths are ur-elements, not classes, and
hence not sets. Therefore, the third premise of the argument for $\emptyset \prec \emptyset$ is true, and the argument is sound. Moreover, neither Comprehension, nor Sets of Truths, nor Classes of Truths are powerful enough to augur otherwise.

(G3), (G4), (G6), and (G8) all fail in our extended $\mathbb{ML}$ because of a failure of being able to prove the existence of certain classes. Quine makes a related observation with respect to Cantor’s Theorem (law) and the existence of the classes of all classes, $\mathbb{V}$:

“We must beware ... of dissociating Cantor’s law from its comprehension premise. Some theorems that carry comprehension premises can reasonably be contemplated in abstraction from those premises, because those premises are ones that would eventually be sustained, in general or in typical cases, by any body of comprehension axioms that we are likely to settle for. But Cantor’s law falls immediately into paradox except as held in check by comprehension failures. Unhedged, ‘$\alpha \prec \{x : x \subseteq \alpha\}$’ gives ‘$\mathbb{V} \prec \{x : x \subseteq \mathbb{V}\}$’, briefly ‘$\mathbb{V} \prec \mathbb{V}$’, which is known as Cantor’s paradox.” ([4], pp. 91 - 93)

Grim’s Cantorian argument is likewise held in check by the fact that $\mathbb{ML}$ fails to have comprehension axioms strong enough to support the existence requirements of its premises. And, as we have seen, Grim’s suggested comprehension remedies also fail: Sets of Truths is ad hoc and too strong, while Classes of Truths is too weak.⁴

4 On whether $\mathbb{S}$ an Ultimate Class

The distinction between sets and ultimate classes serves the purpose in $\mathbb{ML}$ of dodging paradoxes. Take, for example, Russell’s paradox of the class of all classes that are not members of themselves, $\{u : u \notin u\}$. Clearly, $\{u : u \notin u\}$ cannot exist; otherwise, ‘$(\{u : u \notin u\} \in \{u : u \notin u\}) \iff (\{u : u \notin u\} \notin \{u : u \notin u\})’ would be true. But Comprehension does insure the existence of the class of all sets that are not members of themselves, $\check{u}(u : u \notin u)$; and we are free to count ‘$(\check{u}(u : u \notin u) \in$

⁴We need not worry whether the comprehension axioms of an extended $\mathbb{ML}$ are strong enough to generate $\mathbb{S}$. Our purpose here has not been to prove that $\mathbb{S}$ exists, but only that its nonexistence is not necessarily proven by Grim’s Cantorian argument. And if we were interested in proving that $\mathbb{S}$ exists, perhaps a more fruitful path to follows would be one that started with omniscience instead of set theory per se.
\( \hat{u} (u : \ u \not\in u) \leftrightarrow (S(\hat{u}(u : \ u \not\in u)) \& (\hat{u}(u : \ u \not\in u) \in \hat{u}(u : \ u \not\in u))) \)' as true by forcing \( \hat{u}(u : \ u \not\in u) \) to be an ultimate class.

It is also paradoxical that \( \emptyset \not\in \emptyset \). Unlike Russell’s paradox, however, this one is only veridical: it leads to a very surprising truth by sound reasoning. Still, we must wonder whether \( \emptyset \) is an ultimate class. Stratified conditions are what determine sethood in \( \mathbb{M} \). One would therefore expect that if \( \emptyset \) is set then its conditions for membership should be stratified. In other words, it would have to be the case that, for every proposition \( t \), \( t \) is true if and only if the truth conditions for \( t \) are stratified. Moreover, if \( \emptyset \) were finite we could express its condition for membership disjunctively. But surely the truth conditions for many truths are not stratified. For example, the truth condition \( '(x \in x) \leftrightarrow (S(x) \& x \not\in x)' \) for the truth \( '(\hat{u}(u : \ u \not\in u) \in \hat{u}(u : \ u \not\in u)) \leftrightarrow (S(\hat{u}(u : \ u \not\in u)) \& (\hat{u}(u : \ u \not\in u) \in \hat{u}(u : \ u \not\in u)))' \) is clearly not stratified. It is not even possible to specify necessary and sufficient conditions for many truths. Gödel’s First Incompleteness tells as much for the truths of arithmetic. And, if a truth condition cannot be specified, it can hardly be stratified. Therefore, \( \emptyset \) does not look much like a set, but rather like an ultimate class.

Grim claims, however, that \( \emptyset \) does not appear to qualify as ultimate. “Wouldn’t such a class be a member of the class of classes of propositions? Of the class of classes containing one or more truths” ([1], p. 110). \( \mathbb{M} \)'s answer to these rhetorical questions is no. \( \mathbb{M} \)'s Comprehension does not guarantee the existence of the class of all classes of propositions, which is just the class of all subclasses of the class of all propositions; and it does not give the class of all classes containing one or more truths, which is just the class of all subclasses of the class of classes containing one or more truths. Assuming that the notion of a proposition is expressible in an extension of \( \mathbb{M} \), Comprehension gives the class of all sets of propositions, which is just the class of all subsets of the class of all propositions; and it gives the class of all sets containing one or more truths. Let \( \pi \) be the class of propositions. For example, Comprehension only gives this: \( \exists \beta \forall \alpha ((\alpha \in \beta) \leftrightarrow (S(\alpha) \& \alpha \subseteq \pi)) \). So, if \( \emptyset \) were not a set, but an ultimate class, it should not be provable in \( \mathbb{M} \) that \( \emptyset \) is a member of the class of sets of propositions, and it is not.

Grim also says, “With an eye to the issue of omniscience, consider also the class of things known by existent beings. Wouldn’t what God knows be a member of that class?” ([1], p. 110). Again, \( \mathbb{M} \) answers no. Let \( K \) be the class of classes of things known by God. Then Comprehension only gives this: \( \exists \beta \forall \alpha ((\alpha \in \beta) \leftrightarrow (S(\alpha) \& \alpha \subseteq K)) \). So again, if \( \emptyset \) were not
a set, but an ultimate class, it should not be provable in $\mathbf{ML}$ that $\mathfrak{S}$ is a member of the class of sets of things known by existent beings, and it is not.\footnote{It would be interesting to tell a deeper story about truth, ultimate classes and omniscience. This story might say that ultimate classes are exceptionally large infinite classes, and that $\mathfrak{S}$ is equinumerous with the biggest class there is. It might say that only an infinite mind could know the full content of $\mathfrak{S}$ and the truth value of propositions with non-stratified truth conditions. But such a story remains to be told.}

5 Conclusion

Given that $\mathbf{ML}$ and $\mathbf{ZF}$ are incompatible, if we could show that $\mathbf{ML}$ is equally as plausible as $\mathbf{ZF}$, we would then be able to infer that Grim’s Cantorian argument is not justifiably sound, and that it fails to justifiably falsify the existence of the God of Theism. Conversely, if Grim could show that $\mathbf{ZF}$ is more plausible than every other set theory, then he would be able to infer that his Cantorian argument is justifiably sound, and that justifiably the God of Theism does not exist. But it has never been shown that $\mathbf{ZF}$ is more plausible than both $\mathbf{ML}$ and every other set theory.\footnote{The plausibility of a set theory might be judged by the following criteria among others:}

(1) Its consistency.

(2) Its capacity to ground mathematics.

(3) How useful it might be outside of mathematics.

(4) Its degree of constructiveness.

(5) The degree to which it generates ‘large’ classes.

(6) The ease with which it dodges paradoxes.

(7) Its degree of elegance and simplicity.

(8) The economy of its axioms and proofs.

(9) Its degree of entrenchment.

(10) Its popularity among users.

$\mathbf{ZF}$ fares better than $\mathbf{ML}$ with respect to some of these criteria, and $\mathbf{ML}$ better than $\mathbf{ZF}$ with respect to others. Both ground mathematics equally well, but $\mathbf{ZF}$ is appears more popular among mathematicians than $\mathbf{ML}$, and $\mathbf{ML}$ among philosophers.
Moreover, in the absence of any compelling argument for ZF being more plausible than every other set theory, we cannot infer that ZF is necessarily true. If ZF is not necessarily true, and not justifiably more plausible than ML, then we have no justifiable reason I can think of for believing that all the premises of Grim’s Cantorian argument are necessarily true, and, therefore, no reason to think that the logic of Grim’s analysis could possibly show that the premises and the conclusion of Grim’s Cantorian argument are all necessarily true. So the best that we can conclude is that the status quo holds sway: God’s existence and omniscience remain possible. And if you couple that conclusion with the logic of a sound modal ontological argument then God exists and He is omniscient.

**Bibliography**


However, a complete adjudication of ZF and ML with respect to even these criteria is well beyond the place and scope of this paper.
The concepts of proof and the formalized arguments *ex gradibus perfectionis*

**Edward Nieznański**

Surely, the claim of Heinrich Scholz that "either a proof is valid for everybody or it is not a proof at all"\(^1\) is exaggerated. Some people regard a proof as formally valid when they are sure that all its premises are true, while others when they are not sure if some of the premises are false. The aim of the present paper is to settle, by means of the method of formalization, the matter of sequitur in so-called "ways" *ex gradibus perfectionis* by Saint Anselm and Saint Thomas Aquinas and to examine its logical values.

1 Introductory remarks

The medieval philosophy adopted Aristotle’s tradition of logic from the (Prior and Posterior) Analytics and the ontological knowledge from Metaphysics. Hence any ”something” or ”everything” was understood as ”whatever specific” within perception or imagination. Not all essences understood in such a way are objects. In Physics Aristotle writes: ”by ‘subject’ I mean what is affirmatively expressed” ([2], 225 a, p. 305) and Jan Łukasiewicz rephrases the Aristotelian conception as follows: ”by ‘object’ only something that can not have and at the same time not have the same characteristic should be understood” ([5], pp. 110). According to the other

\(^1\)the original reads: ”Ein Beweis ist verbindlich für jedermann, oder es ist überhaupt kein Beweis” ([7], p. 64).
definition, ”an ‘object’ is everything free from contradiction” ([5], pp. 111).²

A medieval philosopher knew that objects are non-contradictory essences, both existent and non-existent, that they are either particulars or universals and ”all beings worthy of the name are individuals by their definition”.³ Every being is an individual object, but not all individual objects are beings (the ones which do not actually exist are not beings).

Let ”a” be an individual constant and ”E” the predicate ”really exists”. Then, the real existence of the essence a can be proved in two ways:

(1) Assuming that individual variables represent individual objects, the theorem $E(a)$ or $\exists x (x = a \land E(x))$ can be proved, i.e., that a certain individual object identical to the essence a really exists.

(2) Assuming that individual variables represent individual objects, the theorem $E(a)$ or $\exists x (x = a)$ can be proved, i.e., that a certain individual object identical to the essence a really exists.

The first way is characteristic of the ancient and medieval tradition, while the second of the modern tradition. The first approach shall be used in the formalization of Anselm’s argument. Henceforth the term ”object” shall be used to refer only to individual objects.

Also, it shall be noted that the medieval texts will provide only inspirations for the proof and we shall guess the intentions of the authors without attempting to translate their ambiguous statements into the symbolic language faithfully. The formalizations shall be based on the classical first-order predicate calculus with identity. We take this calculus characterized in style of natural deduction.

2 The formalism inspired by Anselm

Let the individual variables: $x$, $y$, $z$,... represent objects. Apart from the logical connectives: $\land$, $\rightarrow$, $\leftrightarrow$, $\neg$, and the standard quantifiers: $\forall$ and $\exists$, we shall use the individual quantifier: $\exists_1$ with the meaning: $\exists_1 x \Phi(x) \leftrightarrow \exists x \Phi(x) \land \forall x \forall y (\Phi(x) \land \Phi(y) \rightarrow x = y)$, the predicates: $E$, $=$ and $<$ with the meaning: ”$E(x)$” is read as ”$x$ exists”, ”$x = y$” is read as ”$x$ is identical

² the translation has been based on Polish: ”przez ’przedmiot’ należy rozumieć tylko coś takiego, co nie może zarazem mieć i nie mieć tej samej cechy”. ”’Przedmiotem’ według drugiej definicji nazywamy wszystko, co nie zawiera sprzeczności”.

³ the translation has been based on Polish: ”wszelki byt godny tej nazwy jest z definicji swej jednostkowy” (see, [4], p. 43.)
to $y$, and “$y < x$” is read as “$x$ may be thought as greater (more excellent) than $y$”.

The basis for constructing the formalized proof of the existence of God shall be Anselm’s well-known text:

”Ergo Domine, qui das fidei intellectum, da mihi, ut, quantum scis expedire, intelligam, quia es sicut credimus, et hoc es quod credimus. Et quidem credimus te esse aliquid quo nihil maius cogitari possit. An ergo non est aliqua talis natura, quia ”dixit insipiens in corde suo: non est Deus” (Ps 13,1; 52,1)? Sed certe ipse idem insipiens, cum audit hoc ipsum quod dico: ’aliquid quo maius nihil cogitari potest’, intelligit quod audit; et quod intelligit, in intellectu eius est, etiam si non intelligat illud esse. Aliud enim est rem esse in intellectu, alium intelligere rem esse. Nam cum pictor praecogitat quae facturus est, habet quidem in intellectu, sed nondum intelligit esse quod nondum fecit. Cum vero iam pinxit, et habet in intellectu et intelligit esse quodiam fecit. Convincitur ergo etiam insipiens esse vel in intellectu aliquid quo nihil maius cogitari potest, quia hoc, cum audit, intelligit, et quidquid intelligitur, in intellectu est. Et certe id quo maius cogitari nequit, non potest esse in solo intellectu. Si enim vel in solo intellectu est, potest cogitari esse et in re; quod maius est. Si ergo id quo maius cogitari non potest, est in solo intellectu: id ipsum quo maius cogitari non potest, est quo maius cogitari potest. Sed certe hoc esse non potest. Existit ergo procul dubio aliquid quo maius cogitari non valet, et in intellectu et in re.”

In the translation:

”And so, Lord, do you, who do give understanding to faith, give me, so far as you know est it to be profitable, to understand that you are as we believe; and that you are that which we believe. And indeed, we believe that you are a being than which nothing greater can be conceived. Or is there no such nature, since the fool has said in his heart, there is no God? (Ps 13,1; 52,1). But, at any rate, this very fool, when he hears of this being of which I speak - a being than which nothing greater can be conceived - understands what be hears, and what he understands is in his understanding; although he does not understand it to exist. For, it is one thing for an object to be in the understanding, and another to understand that the object exists. When a painter first conceives of what he will afterwards perform,

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he has it in his understanding, but he does not yet understand it to be, because he has not yet performed it. But after he has made the painting, he both has it in his understanding, and he understands that it exists, because he has made it. Hence, even the fool is convinced that something exists in the understanding, at least, than which nothing greater can be conceived. For, when he hears of this, he understands it. And whatever is understood, exists in the understanding. And assuredly that, than which nothing greater can be conceived, cannot exist in the understanding alone. For, suppose it exists in the understanding alone: then it can be conceived to exist in reality; which is greater. Therefore, if that, than which nothing greater can be conceived, exists in the understanding alone, the very being, than which nothing greater can be conceived, is one, than which a greater can be conceived. But obviously this is impossible. Hence, there is doubt that there exists a being, than which nothing greater can be conceived, and it exists both in the understanding and in reality."

In Anselm’s argument there are two axioms:

\( (A1) \quad \exists x \neg \exists y \ x < y \)

Informally: There is exactly one object such that nothing greater - more excellent - is thinkable (Aliquid quo nihil maius cogitari possit).

\( (A2) \quad \forall x (\neg E(x) \rightarrow \exists y (x < y \land E(y))) \)

Informally: For each non-existent object \( x \) there is a certain existent object - being - \( y \) such that \( y \) is thinkable as a greater - more excellent - than \( x \) (Enim vel in solo intellectu est, potest cogitari esse et in re).

These axioms imply the following theorems:

\( (T1) \quad \neg \exists y \ b < y \), where \( b \) is an individual constant - read: God, introduced on the basis of the axiom (A1).

\( (T2) \quad \neg E(b) \rightarrow \exists y \ b < y \), because of (A2)

Informally: If the one God does not exist, is not a being, then there is at least one object that is greater than he.

\( (T3) \quad E(b) \), because of (T1) and (T2).

\[ ^5 \text{see, in [1]} \]

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The sequence of (A1), (A2), (T1), (T2) and (T3) may be considered as a proof of the existence of the one God.

The premises (A1) and (A2) imply the conclusion (T3) which in turn is in accordance with the previously adopted aim. Thus, the deduction is free from the non sequitur and fallatio elenchi.\textsuperscript{6} The further assessment of the proof depends on the qualification of the axioms. By common consent, a proof should preclude any material fallacy, thus no adopted axiom should be false, or rather the axiom should not be accepted if it is known to be false. What follows is that first it shall be checked if the axiomatics is consistent. And they are so when there is a semantic model in which every axiom is true.

Let $P = \{a, b, c, d\}$, $E = \{b, d\}$, $E^- = P - E = \{a, c\}$, $R = \{< a, b >, < a, d >, < c, b >, < c, d >, < c, b >, < d, b >\}$. It is easily observable that both axioms (A1) and (A2) are true in the model $M = \langle P, E, E^-, R, b \rangle$, which means that the axiomatics is consistent.

Let the intended model be such a relational system whose all components are designated in accordance with the meanings present in Anselm’s text, i.e., the universe $P$ is the range of variability of individual variables, thus the set of all objects. $E$ is the set of all existent objects, thus all beings. $E^-$ is the set of all non-existent objects. $R$ is the relation between pairs of objects $< x, y >$ such that $x$ is thought as greater (more excellent) than $y$. Finally, $b$ is the only object such that it is not an element of the left domain of the relation $R$. Thus, the intended model takes the form: $M = \langle P, E, E^-, R, b \rangle$. Are the axioms true in the model $M$? Indeed, it is well founded to assert that in relation to every non-existence it is possible to designate a being such that it is thinkable as better than the non-existence. The problem lies in the logical value of the axiom (A1). To solve it the following is needed:

1\textsuperscript{0}. $\exists x (x \in P \land \neg \exists y (y \in P \land xRy))$, thus the relation $R$ has maximal elements,

and

\textsuperscript{6}In the case of attempting to prove $\exists (B(x) \land E(x))$ - where 'B(x)' is read 'x is God' by analogy to $E(b)$, the deduction would indeed gain a weakened axiom (A1) (no assumption that there is only one God), however it would suffer from fallatio elenchi, allowing unintended polytheism. This is a sufficient reason for rejecting this way of formalization.
The concepts of proof and the formalized arguments ...

\[ \forall x \forall z (x \in P \land z \in P \land \neg \exists y (y \in P \land x R y) \land \neg \exists y (y \in P \land z R y) \rightarrow x = z), \]

thus the relation \( R \) has at most one maximal element.

However, such knowledge is beyond us. It is neither known if such one maximal element exists nor that it does not exist. But since the adopted axioms are true in the possible model \( M \), the truth of the axiom (A1) is not precluded, which means that the axiom may be accepted suppositionally, yet without assertion. How does it affect the assessment of the value of the proof? The solution depends on the strength of beliefs demanded to accept the axioms. It seems that the past demands concerning this matter were stronger than the present ones. Aristotle writes in Posterior Analytics:

"If a man sets out to acquire the scientific knowledge that comes through demonstration, he must not only have a better knowledge of the basic truths and a firm conviction of them than of the connection which is being demonstrated: more than this, nothing must be more certain or better known to him." ([3], 72a, p. 99)

Nowadays an axiom is almost universally defined as a statement in a deductive system that was accepted without proof. What is of importance, however, is that the author of the proof does not believe that the conclusions are derived from false premises (material fallacy) and does not imply stronger positive beliefs concerning the conclusion than those they have themselves as regards the premises (petitio principii). Anselm’s proof is carried out according to a posse ad esse, it is free from any formal or (obvious) material fallacy, thus does not fall into fallatium elenchi. The only possible objection could be the petitionis principii fallacy, which would require a sufficient justification of the axiom (A1) if such a justification were possible at all.

If the high standards relating to the validity of a proof set by Aristotle were to be met, then Leibniz’s advice would be of use:

"I should add that even principles that aren’t completely certain can have their uses, if we build on them purely demonstratively. Although all our conclusions from them would then be merely conditional, and would be worth having only if the principle in question were true, nevertheless the very fact that this connection holds would have been demonstrated, as would those conditional assertions. That is, even if \( P \) is false, deductively deriving \( Q \) from it shows that \( P \) and \( Q \) are connected in that way, and shows that ‘If \( P \) then \( Q \) is true’. So it would be a fine thing if many books were written in this way." ([8], Book IV, Chapter XII.)

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Since only (A2) can be asserted, the proper implication, also asserted and in accordance with Leibniz’s postulate, is the implication: (A1) ⇒ (T3), i.e., ∃x¬∃y x < y → E(b)? As a result Aristotelian process of acquiring scientific knowledge through demonstration is obtained, however, the proof is burdened with fallatium elenchi since it deviates from the line of thought in order to prove the thesis E(b).

3 The formalism inspired by Thomas Aquinas

Summa Theologiae reads:

"Quarta via ex gradibus qui in rebus inveniuntur. Inveniuntur enim in rebus aliquid magis et minus bonum, et verum, et nobile; et sic de alis hujusmodi quod appropinquant diversimode ad aliquid quod maxime est: sicut magis calidum est, quod magis appropinquat maxime calido. Est igitur aliquid quod est verissimum, et optimum, et nobilissimum, et per consequens maxime ens." ([9], p. 56.)

In the translation:

"The fourth way is taken from the gradation to be found in things. Among beings there are some more and some less good, true, noble and the like. But ’more’ and ’less’ are predicated of different things, according as they resemble in their different ways something which is the maximum, as a thing is said to be hotter according as it more nearly resembles that which is hottest; so that there is something which is truest, something best, something noblest and, consequently, something which is uttermost being” ([10], Question 2, Article 3)

Since Thomas Aquinas speaks not of objects but of ”the world of things”, contemporary reading of the text may be translated into the symbolic language, thus individual variables: x, y, z, ... can be treated as representing only beings.

In the formalism based on the classical first-order predicate calculus with identity that shall encompass the suggestions of ”the fourth way”, the following logical connectives shall be used: ∧, ∨, →, ¬. Also, the quantifiers: ∀ and ∃; the identity predicate: =; together with its negation: ≠; and the predicate: > where x > y is read: x is more excellent than y.

In Thomas’s text there are two axioms:

(Ax1) ∀x∀y(x > y → ¬(y > x))
The concepts of proof and the formalized arguments ...

Informally: The relation of a higher excellence is asymmetrical,

and

(Ax2) \( \exists y \forall x (x \neq y \rightarrow y > x) \)

Informally: There is a being more excellent than all others.

The axiom (Ax1) implies that there is at most one such being:

(Th1) \( \forall y \forall z (\forall x (y \neq x \rightarrow y > x) \land \forall x (z \neq x \rightarrow z > x) \rightarrow z = y) \), because

(1) \( \forall x (y \neq x \rightarrow y > x) \) assumption
(2) \( \forall x (z \neq x \rightarrow z > x) \) assumption
(3) \( z \neq y \) assumption of indirect proof
(4) \( y > z \) from (1) and (3)
(5) \( z > y \) from (2) and (3)
(6) \( \neg(y > z) \) from (Ax1) and (5)
(7) contradiction (4) and (6)

Since the axiom (Ax2) ascertains the existence of the highest being and the theorem (Th1) claims that there is at most one such being, thus there is one and only one such being, then the being can be a named by constant "b" (the one God) and the theorem (Th2) can be proved:

(Th2) \( \forall x (b \neq x \rightarrow b > x) \)

Informally: God is a being more excellent than all others (the most excellent).

When the variables in a language represent only what exists, the existence of God is translated into: \( \exists y \ y = b \), i.e., that in the universe of beings there is an element identical to the one God.\(^7\) Also, what is of importance is the theorem:

(Th3) \( \forall x (b \neq x \rightarrow b > x) \rightarrow \exists y \ y = b \), because:

Informally: God defined as the only most excellent being actually exists

\(^7\)W. V. O. Quine claims in [6] that the thesis "\( \forall x \ x \ exists \)" is "trivially true", for "to be is to be the value of a variable". Hence the problem lies not in the existence of God but in God’s nature.
(1) $\forall x (b \neq x \rightarrow b > x)$ assumption

(2) $\neg \exists y y = b$ assumption of indirect proof

(3) $\forall y y \neq b$ from (2)

(4) $b \neq b$ from (3)

(5) $b = b$ from the principle $\forall x x = x$

(6) contradiction (4) and (5)

The whole procedure ends with the theorem:

(Th4) $\exists y y = b$, because of (Th2) and (Th3)

Informally: A certain being is identical to God, thus God exists.\(^8\)

The sequence of (Ax1), (Ax2), (Th1), (Th2), (Th3) and (Th4) is the "proof" of the conclusion (Th4), on the basis of the obvious axiom (Ax1) and the permissible axiom (Ax2). It is free from any formal or material fallacy. Also, it is not burdened with fallatium elenchi.

In order to avoid petitionis principii, it shall be assumed that the conclusion (Th4) is permissible to the same degree as the axiom (Ax2). Consequently, the proof shall be devoid of its assertive power, which, on the other hand, can be restored by following Leibniz’s proposal - at the cost of fallatium elenchi it is accepted that the conclusion of the deduction is only the implication $(Ax2) \Rightarrow (Th4)$, i.e., $\exists y \forall x (y \neq x \rightarrow y > x) \rightarrow \exists y y = b$.

The language of Thomas’s proof has some advantages over the discourse of Anselm. It is less ambiguous. It makes use of a more precise universe of speech. Also, it assumes the existence of the most excellent being (and allows to prove the uniqueness), while Anselm’s premise is twofold: it assumes the existence and the uniqueness of the maximal element in the relation of understanding what object is greater. Finally, it seems that Anselm’s and Thomas’s deductions speak in favour of two propositions. The former strengthens the existential proposition, while the latter the proposition of the uniqueness of the most excellent being.

Whereas, if Thomas’s discourse were not limited only to beings, and Anselm’s universe of all objects were adopted, then apart from Thomas’s axioms (Ax1) and (Ax2) (applying now to all objects) additional two axioms would be needed:

\(^8\)(Th4) can be obtained directly from the fact that $b = b$, but we wanted to keep the way of Thomas’s argumentation.
The concepts of proof and the formalized arguments ... 

(Ax3) \( \exists x \ E(x) \)

Informally: There are also objects that are beings.

and

(Ax4) \( \forall x \forall y(y > x \land E(x) \rightarrow E(y)) \)

Informally: That what is more excellent than the existing one also exists.

Then the proofs for (Th1) and (Th2) would be the same as before, however the theses (Th3) and (Th4) and their proofs would change the form:

(Th3') \( \forall x (b \neq x \rightarrow b > x) \rightarrow E(b) \), because:

(1) \( \forall x (b \neq x \rightarrow b > x) \) assumption

(2) \( \neg E(b) \) assumption of indirect proof

(3) \( b \neq x \rightarrow b > x \) from (1)

(4) \( b = x \lor b > x \) from (3)

(5) \( b = x \rightarrow \neg E(x) \) from (2)

(6) \( b > x \rightarrow \neg E(x) \) from (Ax4) and (2)

(7) \( \neg E(x) \) from (4), (5) and (6)

(8) \( \forall x \neg E(x) \) from (7)

(9) \( \neg \exists x E(x) \) from (8)

(10) contradiction (Ax3) and (9)

and

(Th4') \( E(b) \), because (Th2) and (Th3).

Then a deduction a posse (Ax2) ad esse (Th4), similar to Anselm’s, is obtained.

Translated by Magdalena Tomaszewska

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Ontological Logical Melioration

Jerzy Perzanowski†1

1 Introduction

0. Critical, epoch-making, ways of reasoning, like the real diamonds, are rather rare. Which are their sources, their origin, if any? Quite often they came from philosophy, sometimes from theologic, in a few cases from mathematical practice. In many cases they emerged from paradoxes, being theirs realized opportunity.2 In particular:

- Zeno’ paradoxes of change, motion, space and time led through Bolzano’s paradoxes of infinity to methods of infinite division, approaching limits, characterization of infinity, etc.
- The Liar Paradox of Eubulides bore the diagonal method of Cantor.
- Last but not leads, the melioration principles of Augustine and Anselm led immediately to maximalization principles of contemporary mathematics.

†1When the late Professor Jerzy Perzanowski was writing the present text, he was battling an incurable illness. He circulated the text among his friends and colleagues, asking them for suggestions and critical remarks. As a result, there can be found texts entitled “Ontolo\Logical Melioration” that differ from each other in some small and unimportant details. The text published in the present volume is the paper which I received from Prof. Jerzy Perzanowski and thoroughly discussed with him. – Mirosław Szatkowski.

2Recall Whitehead’s dictum “Inconsistency is not a failure, but opportunity”.

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1.1 Anselmian source

1. The melioration theory which follows is drawn from an elementary, first-order, formalization of the memorable argument put forward Anselm’s “Proslogion 2”. My considerations belong therefore to theo\logic.\footnote{According to Aristotle (Metaphysics, \textit{E}, 1026a), theo\logic is the core of metaphysics, simply the true first philosophy.} They belong also to the general theory of relations, which itself is the central part of the borderland between algebra, set theory, logic and ontology. As will be seen, they are done for any given class and a relation on it, both chosen quite arbitrary.

Theological intuitions are indeed necessary to build a theory of God’s essence based on a purely relational theory of God’s existence, where we must select proper orders, and hence proper meliorated conditions as well, to describe beings (or in some cases a being) which are the most melior. Certainly, this selection, like any, needs intuition. From the outset, I will therefore read and interpret my formulas in theo\logical way. The reader, however, can use a purely mathematical reading, if he or she prefers.

2. Anselm’s argument in “Proslogion 2” looks like, and indeed is, a paradox.

- The argument is based on a premise transcending the realm of our everyday experience, the realm of concreteness. It is therefore, to some extend, surprising. It is short but fruitful, and rich in consequences.

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- On the other hand, despite its apparent brevity, it is surprisingly complex.

- As a matter of fact, it is based on a quite unusual and very powerful Principle of Onto\logical Melioration. In what follows, I shall set out the Principle and discuss its meaning, power and beauty.

Let me add that from the history of the Anselm’s Argument, especially from the history of its reception, it is also easy to see that the Argument is indeed a paradox, at least to a common-sensical, narrow-minded, standpoint.
3. The argument is short, powerful, difficult to grasp, and surprising. It produces several important observations and ideas, but also quite a large amount of rubbish.

We shall see that, in a certain sense, it resembles the Liar Paradox.

1.2 Maximals or God-like beings

Let an arbitrary but fixed logical frame $\mathcal{U} = \langle U, M \rangle$, where $U$ is the universe of all objects (beings) under consideration, whereas $M$ is a binary relation on $U$ (to be better, melior or greater), be given.

God

Deus, or the most perfect being, therefore a maximal element (in the space of all beings)

4. A god-like being is any object which is $M$-maximal in $\mathcal{U}$, in frames without supremum - coatom:

$$(G) \quad G(x) \overset{\text{df}}{=} \neg \exists y (y \neq x \land xM y).$$

Immediately we have:

$$G(x) \leftrightarrow \forall y (xMy \rightarrow x = y), \text{ i.e.}, [x] \subseteq \{x\}.$$

Hence, $x$ is an $M$-maximal object.

5. It is sometimes convenient to consider cosimples as god-like beings:

$$(G^*) \quad G^*(x) \leftrightarrow \neg \exists y xMy.$$

A cosimple, or god*-like, being is an object without objects superior to/predominant over it. Clearly,

$$G^*(x) \rightarrow G(x), \text{ but not vice versa.}$$

6. The above logical understanding of $M$-coatoms, $G(x)$, or $M$-cosimples, $G^*(x)$, obviously depends on the reading of $M$ as “to be melior”. They are therefore the most melior beings, respectively improper or proper.

If, however, we consider $M$ to be a relation being subsitution of a situation, then $M$-coatoms are possible worlds in an appropriate universe of

\footnote{God of philosophers: Plato, Aristotle, Seneca, Augustine, Anselm, Thomas, Duns Scotus, Descartes, Spinoza, Leibniz, Cantor, Gödel and others. Its definition was given by Seneca.}

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situations. The important lesson which follows from this is that consequen-
tial claims preserve their validity in any relational frame; their meaning
can, however, change from one class of frames to another.

7. By the way, notice that $G$, considered as a functor, is implicitly a modal
operator. To see this, take irreflexive frame $\langle U, M \rangle$ and put $M_y(x) \overset{\text{df}}{=} xMy$. Now,

\[(3) \quad G(x) \leftrightarrow \neg \diamond M_y(x) \quad \text{or} \quad G(x) \leftrightarrow \Box \neg M_y(x).\]

2 Meliorations’ Sextet

2.1 Anselm’s Melioration

1. Consider the following definition of the basic ontological modality of
melioration,

\[(M) \quad MA \overset{\text{df}}{=} \forall x(\neg A(x) \rightarrow \exists y(A(y) \land xMy)).\]

The formula (condition) $A(x)$ is *meliorated* iff to be *not* $A$ for $x$ is
meliorated by being $A$ for some $y$, which is greater than $x$.

Hereafter $A(x)$ means, as is common among scholars, that $x$ is the only
variable with free occurrence in $A$. Similarly for $A(y)$, etc.. Sometimes we
will consider the parameterized version of $(M)$:

\[(MA) \quad MA(x) \leftrightarrow (\neg A(x) \rightarrow \exists y(A(y) \land xMy)).\]

which should be carefully distinguished from $MA$.

Formulas’ melioration clearly depends on the underlying frame-relation
$M$, because the condition $(M)$ is stated for a given frame $U = \langle U, M \rangle$.
The meaning and power of $(M)$ therefore change from frame to frame!

A few preliminary examples

2. To gain an idea of melioration, consider the following four paradigmatic
examples (the fifth, Anselmian, I am going to discuss later).

Example I
Tautologous conditions, formulas (sentences), are undoubtedly meliorated,
but contradictions are not.\(^5\)

Example II (Metalogical)
Let $\mathcal{L} = \langle TH, \leq \rangle$ be a Lindenbaum bundle, i.e., $TH$ is the family of all

\(^5\)For any frame, simply by logic!
consistent overtheories of a given theory, with its usual order of inclusion. Take $A(x) \overset{\text{df}}{=} "a \text{ consistent theory } x \text{ is maximally consistent}"$. Now $MA$ is simply Lindenbaum’s Lemma: If $x$ is not maximally consistent (but consistent), then there exists its maximally consistent overtheory.

**Remark**
From the case of Lindenbaum’s Lemma an important lesson follows: Usually, for a given condition we need to prove that it is meliorated in a frame under consideration, which in many cases is non trivial.

**Example III**
Now take the property $G(x)$ - “being a maximal of $U$”, or “being god-like”. Clearly,

1. $MG$ holds in $U$ iff $U$ is coatomic, i.e., such that each element of $U$ is covered by a coatom (i.e., a maximal object in $U$).

As a matter of fact $MG \leftrightarrow \forall x(\neg G(x) \rightarrow \exists y(G(y) \land xM y))$, hence melioration of $G$ means that each element which isn’t coatom is below a coatom, which means that $U$ is coatomic, as required.

**Example IV**
The ladder hierarchy is the family of all natural numbers with its standard ordering: $\omega = \langle \{0, 1, 2, \ldots\}, \leq \rangle$. In this case,

2. $MA$ holds in $\omega$ iff $\{ x : A(x) \}$ is infinite.

Put $\{ A \} \overset{\text{df}}{=} \{ x : A(x) \}$. If $MA$, then $\forall x(\neg A(x) \rightarrow \exists y(A(y) \land x \leq y))$. Clearly, if $\{ A \}$ is finite, then it has the largest element $n$. Thus $A(n)$, but (i) $\forall y > n \neg A(y)$. In particular, $\neg A(n + 1)$. Therefore, by $MA$, there is $y > n$, such that $A(y)$, which is in contradiction with (i). For the reverse implication, suppose that $\{ A \}$ is infinite. Without lose of generality, we can assume that $\{ A \}$ is not full. Now take $n$ such that $\neg A(n)$. For contradiction, suppose $\neg \exists y(A(y) \land n \leq y)$. Hence, $\forall y(A(y) \rightarrow y < n)$. $\{ A \}$ thereby is finite, which again is in contradiction with our assumption. Therefore $MA$, as required. This observation has quite clear theo\logical meaning, which I shall try to explain later.

**GM Lemma**
3. Meliorated formulas are truly Anselmian, since we can prove the following fundamental Lemma, which is Anselmian in spirit:
(GM) \( MA \land G(x) \rightarrow A(x). \)

Proof:

(i) Assume \( MA: \forall x(\neg A(x) \rightarrow \exists y(A(y) \land xMy)). \)

Hence,

(ii) \( \neg A(x) \rightarrow \exists y(A(y) \land xMy). \)

Multiplying it with \( \neg A(x) \rightarrow \neg A(x) \), we obtain

(iii) \( \neg A(x) \rightarrow \neg A(x) \land \exists y(A(y) \land xMy). \)

By transfer laws for quantifiers, we have

(iv) \( \neg A(x) \rightarrow \exists y(\neg A(x) \land A(y) \land xMy). \)

But, by Leibniz's Axiom of Extensionality

(v) \( \neg A(x) \land A(y) \rightarrow x \neq y. \)

And, by standard laws of Classical Logic, we accept

(vi) \( \neg A(x) \land \exists y(x \neq y \land xMy). \)

Hence,

(vii) \( \neg A(x) \land \neg G(x), \) i.e.,

(viii) \( G(x) \rightarrow A(x), \) as required.

Thus, by \((GM)\), for any frame \( \mathcal{U} \) meliorated conditions hold for maximals (God-like-beings): \( MA \rightarrow (G(x) \rightarrow A(x)). \) They are therefore good candidates for being (semi)characteristic formulas of maximals, that is, for being their (semi)perfections.

4. Clearly, by (2) of Introduction, we have

(G^\ast M) \( MA \land G^\ast(x) \rightarrow A(x). \)

\((GM)\) is indeed the basic lemma for Anselm-type reasonings. With its help, we immediately obtain the following generalizations of Anselm’s claims including his theorem (3):

(3) Assume \((G)\) and \((MA)\). Then \( G(x) \rightarrow A(x) \) and \( \exists xG(x) \rightarrow \exists xA(x). \)

If, in addition, we assume (or better prove) \( \exists xG(x) \), then under the same assumptions we obtain

(4) \( \exists xA(x). \)
In other words,

(5) If \((G)\) and \((MA)\), then \(\exists xG(x) \rightarrow \exists x(G(x) \land A(x))\), hence

If \((G)\), \((MA)\) and \(\exists xG(x)\), then \(\exists x(G(x) \land A(x))\).

**Anselm’s Argument Revisited**

5. Recall, first, **Seneca’s and Anselm’s Definition of God**: God is a being such that nothing greater (more meliorated) than it can be imagined. In short: \(QNMCP\) (quo nihil maius cogitari posit). In Anselm’s words: *Et quidem credimus te esse aliquid quo nihil maius cogitari posit*. By definition, a God-like-being is \(QNMCP\):

\[
(G.CP) \quad G(x) \overset{df}{=} \neg CP(\exists y(y \neq x \land xMy)).
\]

Our condition \((G)\) is an extensional version of the above modal concept \((G.CP)\). An important modality can be imagined is omitted in it.

6. Recall also **Anselm’s Insight**: To be real is greater than to be not real. *Si enim vel in solo intellectu est, potest cogitari esse et in re, quod maius est. (For if it exists only in the mind, it can be thought to exist in reality as well, which is greater.)* (tr. E. Anscombe [1])

\[
(G.ESI) \quad G(x) \land ESI(x) \rightarrow CP(\exists y(G(y) \land ER(y) \land xMy)), \text{ or }
G(x) \land ESI(x) \land \neg ER(x) \rightarrow CP(\exists y(G(y) \land ER(y) \land xMy)).
\]

7. In the elementary, first-order version of Anselm’s reasoning, after canceling \(CP\), we immediately obtain our definition of God-like beings as maximals of frames under consideration (including the real world)

\[
(G) \quad G(x) \overset{df}{=} \neg \exists y(y \neq x \land xMy).
\]

and, as extensional version of (6), the claim

\[
(G.ESI) \quad G(x) \land ESI(x) \rightarrow \exists y(G(y) \land ER(y) \land xMy), \text{ or }
G(x) \land ESI(x) \land \neg ER(x) \rightarrow \exists y(G(y) \land ER(y) \land xMy).
\]

The above formula suggests the following **Anselmian Principle of Melioration**:

\[
(MAn) \quad \forall x(\neg ER(x) \rightarrow \exists y(ER(y) \land xMy)).
\]
Everything which is not real is meliorated by something real: Better to be real than to be not real! Better be than not be. It is quite common-sense principle. Its meaning depends however on its context.\textsuperscript{6}

8. Anselmian Melioration Principle (MA\textit{n}) is a particular case of the general principle (M). Indeed

\begin{equation}
(\text{MA}n) \leftrightarrow M(ER), \quad \text{for}

M(ER) \leftrightarrow (\neg ER(x) \rightarrow \exists y(ER(y) \land xMy)).
\end{equation}

The principle (MA\textit{n}) thus states that the condition to be real is meliorated. In this way, Anselm’s Principle shows its deeply modal character.\textsuperscript{7}

9. Assuming, after Anselm, the principle (MA\textit{n}), i.e., M(ER), by (GM), we obtain

\begin{equation}
G(x) \rightarrow ER(x), \quad \text{hence} \quad \exists xG(x) \rightarrow \exists xER(x).
\end{equation}

Multiplying it, in the first step, by $EI(x) \rightarrow EI(x)$ and, in the second one, by $G(x) \rightarrow G(x)$, we get

\begin{equation}
EI(G(x)) \rightarrow EX(x), \quad \text{hence} \quad \exists xEI(G(x)) \rightarrow \exists xEX(G(x)).
\end{equation}

Now, by empirical assumption of the Argument (concerning the Fool, i.e., a typical unbeliever) we obtain that

\begin{equation}
\exists xEI(G(x)).
\end{equation}

Finally, we have Anselm’s conclusion

\begin{equation}
\exists xEX(G(x)), \quad \text{hence} \quad \exists x(EI(x) \land G(x) \land ER(x)).
\end{equation}

10. Meaning of Anselm’s Argument. The above modern version of Anselm’s Argument shows clearly that:

1. It is a case of Cosmological Argument. It is based on the underlying, hidden, order $M$; on investigation of their meliorations.

2. One of them, existence, is encapsulated by the principle (MA\textit{n}) (or in a more general way by (M)) and is used next to prove (14), (15) and finally (17).

\textsuperscript{6}Cf., B. Russell’s notorious dictum: Better red than dead! Or much more reasonable Polish proverb: Lepszy wróbel w garści niż orzeł na dachu! (A bird in the hand is worth two in the bush!)

\textsuperscript{7}Cf., Subsection 3.1
3. It is also based on the empirical assumption about the mind (or rather heart) of the Fool to prove consistency of the cosmological concept of Deity. In this way, it meets requirements of Leibniz in his well-known critique of Cartesian version of the argument.

4. Last but not least, it introduces, and is based on, a very powerful Principle of Melioration which is certainly the greatest achievement of Western metaphysics.

5. Finally, it would be better to prove the Anselmian Principle (MAn) than to assume it as an axiom. As, for sure, to prove is better than to choose and accept.\footnote{Is provability a meliorated condition?}

Equivalents

11. Let me list two noticeable equivalents of (MA). First, its existential form:

\[(13) \ \forall x \exists y (\neg A(x) \rightarrow (A(y) \land xMy)).\]

Thus (MA) itself implicitly and the above equivalent explicitly are $\forall \exists$ formulas. The most important equivalent is, however:

\[(14) \ \forall x (\forall y (xMy \rightarrow \neg A(y)) \rightarrow A(x)).\]

MA says thereby that $A$ is strongly non-inductive, i.e., not hereditary for $M^{-1}$:

\[(15) \ \forall x (\forall y (yM^{-1}x \rightarrow \neg A(y)) \rightarrow A(x)).\]

This explains, to some extent, counterintitiveness of meliorated conditions. Our common-sense intuition is based on finite, uniform fields, which are inductive. We will see in §26 that melioration entails transition or transcendence, in particular Cantorian transmelioration.

2.2 Augustine’s Melioration

13. To show the power of the condition (MA), let me illustrate its logical and historical antecedent. In a quite natural and rather intuitive form, it was introduced in more general way by St. Augustine in his Confessiones: “... melius esse incorruptibile quam corruptibile ...”; “... uncorrupted items are better (greater or melior) than corrupted ones” (Augustine [3], 7.4):
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\( (MAu) \) \( Cor(x) \land Incor(y) \rightarrow xMy. \)

The following definition of Augustine’s melioration is an obvious generalization of the condition \( (MAu) \):

\( (M^*) \quad M^*A \overset{df}{=} \forall x\forall y(\neg A(x) \land A(y) \rightarrow xMy) \)

\( A \) is meliorated if it is better to be \( A \) than non-\( A \).

\( M^*A \) is therefore a general formula of the form “\( \forall \forall \)”.

Filtration

13. The general melioration of Augustine is closely connected with the usual filtration condition:

\( (F) \quad FA \overset{df}{=} \forall x\forall y(xMy \land A(x) \rightarrow A(y)) \)

which is also of the type “\( \forall \forall \)”.

14. Making the underlying connection explicit, observe that Augustine’s melioration is, in fact, a case of a \textit{dual} filtration. As a matter of fact, putting \( x - My \overset{df}{=} \neg(xMy) \) with reading: \( \neg M \) is the complement of \( M \), we have:

\( (16) \quad F_M A \leftrightarrow M^*_{-M}(\neg A) \) and \( M^*_MA \leftrightarrow F_{-M}(\neg A) \).

Indeed, to check the second equivalence observe:

\[
M^*_MA \leftrightarrow \forall x\forall y(\neg A(x) \land A(y) \rightarrow xMy)
\leftrightarrow \forall x\forall y(\neg xMy \rightarrow (A(y) \rightarrow A(x)))
\leftrightarrow \forall x\forall y(x - My \rightarrow (A(y) \rightarrow A(x)))
\leftrightarrow \forall x\forall y(x - My \rightarrow (\neg A(x) \rightarrow \neg A(y)))
\leftrightarrow \forall x\forall y(x - My \land \neg A(x) \rightarrow \neg A(y))
\leftrightarrow \quad F_{-M}(\neg A).
\]

The first equivalence can be calculated in a quite similar way.

15. Recall that filtration is the crux of two recent ontological proofs: Gödel’s argument ([4]) and Perzanowski’s one ([5]). Gödel had worked
with positive ultrafilters, whereas I was gambling with a restricted F-consistency, i.e., with the condition \((F)\ (x \text{ is consistent}): \text{consistent objects forms the filter.}\)

Melioration therefore unifies in a general way two main types of ontological arguments. General, purely mathematical, ontological proofs, which are based on the idea of an order filter determined by logic, are strictly connected, as we see, with classical proofs which follow the line of Augustine and Anselm.

**Comparison**

16. Generally, both meliorations considered thus far are independent.

(17) *Neither \(MA\) implies \(M^*A\) nor \(M^*A\) implies \(MA\).*

For the first claim, consider, for example, the ladder hierarchy from Example IV. It is clear, that the condition “\(x\) is odd” is meliorated in Anselm’s sense in this frame, but not in Augustine’s one, for 4, being even, is not odd, but some odd numbers are not greater than it. For the second claim take simply inconsistent: \(M^*A\), but \(\neg MA\).

17. Under quite natural consistency proviso, however, Augustine’s melioration \(M^*A\) is stronger than \(MA\). Indeed, we have

(18) \(M^*A \rightarrow MA, \text{ provided } A \text{ is consistent.}\)

**Proof:** Assume \(M^*A, \exists yA(y)\) and \(\neg A(x)\). By the second assumption, we have for some \(a\), \(A(a)\). By the first, we obtain \(\neg A(x) \land A(a) \rightarrow xMa\). But \(\neg A(x) \land A(a)\), hence \(xMa\). Therefore \(A(a) \land xMa\), thus \(\exists y(A(y) \land xMy)\), as required.

18. Immediately we obtain \(G\)-Lemma for Augustine’s melioration.\(^{10}\)

**G-Lemma:** \(G^*M, M^*A, \exists xA(x) \vdash G(x) \rightarrow A(x)\).

**Proof:** By (18) and GM.

Observe that

(19) \(GM \vdash GM^*\) for consistent \(A\).

\(^{10}\text{Convention: For all meliorations considered in the paper, denoted } \mu, G\text{-lemma for the melioration } \mu \text{ means the implication: } \mu A \land G(x) \rightarrow A(x). \text{ Similarly, } \mu, G^*\text{-Lemma means } \mu A \land G^*(x) \rightarrow A(x).\)
Existential weakening

19. Observe that the existential form of Augustine’s melioration is too weak. It indeed is much weaker than the condition of Anselmian melioration, because, in fact, it is a theorem of classical quantifier calculus.

\[ \forall x \exists y (\neg A(x) \rightarrow (A(y) \rightarrow xM y)). \]

**Proof:** By simple logical calculations we obtain that the formula under consideration is equivalent to \( \forall x (\neg A(x) \rightarrow \exists y (A(y) \rightarrow xM y)) \), which in turn is equivalent to \( \forall x (\neg A(x) \rightarrow \exists y (\neg A(y) \lor xM y)) \). This, however, is equivalent to \( \forall x (\neg A(x) \rightarrow \exists y (\neg A(y)) \lor \exists y (xM y)) \), which is a theorem of classical logic.

Special cases

20. We immediately obtain that both inconsistent and tautological formulas are \( M^* \)-meliorated. Also, for a given frame \( \mathcal{U} \), any formula valid in it is \( M^* \)-meliorated:

\[ \text{If } \mathcal{U} \models A, \text{ then } \mathcal{U} \models M^* A. \]

**Proof:** Indeed, if \( A \) is valid in \( \mathcal{U} \), then \( \{ \neg A \} \) is empty in it. Therefore any implication having as its antecedent \( \neg A(x) \) (alone or as one of its conjunct) must be also valid in \( \mathcal{U} \).

Observe that analogous implications generally hold, for each of six meliorations treated in the present paper.

21. We say that a frame \( \mathcal{U} = \langle U, M \rangle \) is nearly full if \( M \cup \Delta \) is full, i.e., all elements of \( U \) are related by \( M \cup \Delta \). Now, taking into account obvious connection of the antecedent of the implication in the condition \( M^* A \) with statement that \( x \neq y \), given by the Leibniz’s Principle of Identity

\[ \text{(LP) } x \neq y \leftrightarrow \exists A (\neg A(x) \land A(y)), \]

we obtain:

\[ \text{(22) For any } A, \text{ } M^* A \text{ iff } M \text{ is nearly full: } \forall x \forall y (x \neq y \rightarrow xM y). \]

**Proof:** Observe that for any \( A \), \( M^* A \leftrightarrow \forall A \forall x \forall y (\neg A(x) \land A(y) \rightarrow xM y) \leftrightarrow \forall x \forall y (\exists A (\neg A(x) \land A(y)) \rightarrow xM y) \), which, by Leibniz’s Principle LP is in turn equivalent to \( \forall x \forall y (x \neq y \rightarrow xM y) \).

22. On the other hand, it is easy to see that
(23) \( M^*(x = y) \iff \forall x \forall y (x \neq y \rightarrow xMy). \)

Therefore,

(24) \( \text{For any } A, \ M^*A \iff M^*(x = y). \)

To conclude, in nearly full frames, and only in them, Augustine’s melioration is trivial. It is nontrivial only if its underlying frame order properly elements of its universe: \( \exists x \exists y (x \neq y \land \neg(xMy \land yMx)). \)

2.3 Overmelioration

23. By the way, the general form of Anselmian melioration (which, as will be recalled, is an existential formula) analogous to Augustine’s melioration is quite strong. To see this define:

\[
(M') \quad M' A \overset{\text{df}}{=} \forall x \forall y (\neg A(x) \rightarrow A(y) \land xMy).
\]

It is easy to see that \( M' \) is overtrivial:\(^{11}\)

(25) \( M' A \iff A \text{ is full}: \forall x A(x). \)

**Proof:** Implication to the left is obvious. For the reverse direction, assume \( M' A \), hence by logic we have \( \neg A(x) \rightarrow A(x) \land xMy \). Therefore, \( \neg A(x) \rightarrow A(x) \), hence \( A(x) \), and \( \forall x A(x) \), as required.

24. \( M' A \) is common strengthening of both \( MA \) and \( M^*A \). Thus, it deserves to be called "overmelioration". To see this, notice first, that

(26) \( M' A \implies MA, \text{ but not reversely.} \)

**Proof:** The first claim, concerning implication, is obvious. To the second claim, consider again the ladder model of Example V and the condition “to be even”. \( M(x \text{ is even}) \) holds, but \( M'(x \text{ is even}) \) does not.

Next, consider \( M^*A \). Analogously,

(27) \( M' A \implies M^*A, \text{ but not reversely.} \)

The implication holds for conjunction in the antecedent implies implication. For the second claim, consider, for example, an inconsistent \( A \) or any non-full \( A \).

\(^{11}\)To be overtrivial means to be full or inconsistent, whereas to be trivial means, as usual, to hold for all formulas.
2.4 Cantor’s Transmelioration

25. Thus far, we have found two conditions which are stronger than $MA$, namely $M'A$ in general and, under proviso of consistency, $M^*A$. However, we still lack a natural, well-motivated, condition weaker than it. It would therefore be profitable to find an *interesting* melioration operator *weaker* than $M$.

26. The obvious candidate is the following Cantorian Principle of Transcendence:

$T_{A} \overset{df}{=} \forall x (\neg A(x) \rightarrow \exists y (A(y) \leftrightarrow xMy))$. 

Or, in an equivalent form:

$(28) \quad T_{A} \leftrightarrow \forall x (\forall y (xMy \leftrightarrow \neg A(y)) \rightarrow A(x))$. 

The meaning of $T_{A}$ is obvious, clear and transparent. Consider, for instance, the case of Cantorian transfinization. Let $A(x)$ be “$x$ is infinite”, whereas $xMy$ iff $y < x$. In this case $x$ such that $A(x)$ is greater than any finite set; therefore, it transcends the domain of finite sets. Its name therefore is “infinity”.

27. Observe that $T_{A}$ being by definition common weakening of $MA$ and $M^*A$ is indeed weaker:

$(29) \quad \text{Neither } T_{A} \rightarrow MA \text{ nor } T_{A} \rightarrow M^*A$. 

To see this, consider the Boolean frame 2 from Example VI of Section 3.

2.5 Diagonalization

28. On the other hand, $(CT)$ is powerful enough, simply by substitution and distribution of $\forall$ with respect to $\rightarrow$, to furnish an essential part of the diagonal law of the Classical Quantifier Calculus.

29. Recall first that Cantor’s diagonal method is an application of following Cantor’s law of the first-order logic:

$\vdash \neg \exists x \forall y (xMy \leftrightarrow \neg (yMy))$, 

---

$^{12}$Or Transition, or Transmelioration, or Transfinitization. The principle was in fact used by Georg Cantor several times.
which, in a sense, is a canonic case of the diagonal law of the first-order logic:
\[ \vdash \neg \exists x \forall y (A(x, y) \leftrightarrow \neg A(y, y)). \]

**30.** Now, transmelioration of the reflexivity condition gives as a case the irreflexive case of Cantor’s law:

\[(30) \ T(xMy) \rightarrow (\neg \exists x \ xMx \rightarrow \neg \exists x \forall y (xMy \leftrightarrow \neg (yMy))). \]

**Proof:** By definition, \( T(xMx) \overset{\text{df}}{=} \forall x(\neg(xMx) \rightarrow \exists y(xMy \leftrightarrow yMy)). \) Hence, \( \forall x(\neg(xMx) \rightarrow \forall x \exists y(xMy \leftrightarrow yMy) \), therefore \( \neg \exists x(xMx) \rightarrow \neg \exists x \forall y(xMy \leftrightarrow \neg (yMy)), \) as required.

Claim (30) offers, in fact, a reason for the diagonal argument in the case of irreflexive relations, which is their transcendence of reflexivity:
\[ T(xMx) \not\!
ot\!\not\not= \]

To be exact, notice that (30) is trivial, because its antecedent is a theorem of Classical Logic. Its proof, however, shows that the case of diagonalization under investigation is implicitly contained in the transcendence claim \( T(xMx). \)

**31.** Notice an immediate application of (30) in the realm of all founded sets \( V = \langle V, \in \rangle. \) Obvious transcendence of the condition “\( x \in y \)” is the condition “\( x \not\in y \).” Recall \( V \models \neg \exists x(x \not\in x). \) Hence, by (30) applied to \( A_x(y) \overset{\text{df}}{=} x \in y, \) we obtain

\[(31) \ V \models T(x \in x) \rightarrow \neg \exists x \forall y (y \in x \leftrightarrow \neg (y \not\in y)). \]

Therefore, in the presence of \( T(x \in x), V \) is not a set.

### 2.6 Nearly trivial and trivial meliorations

**32.** Finally, consider two natural weakenings of \( TA: \)

\[(T') \quad T'A \overset{\text{df}}{=} \forall x(\neg A(x) \rightarrow \exists y(A(y) \rightarrow xMy)), \quad \text{and} \]
\[(T^*) \quad T^*A \overset{\text{df}}{=} \forall x(\neg A(x) \rightarrow \exists y(xMy \rightarrow A(y))). \]

Notice first that \( T^*A \) is weak indeed, for it simply is trivial:

\[(32) \ T'A, \text{ for any } A \text{ and any frame } \mathcal{U}. \]

**Proof:** Observe that

\[ T'A \leftrightarrow \forall x(\neg A(x) \rightarrow \exists y(A(y) \rightarrow xMy)) \]
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\[ \forall x(\neg A(x) \rightarrow \exists y(\neg A(y) \lor xMy)) \]
\[ \forall x(\neg A(x) \rightarrow \exists y(\neg A(y)) \lor \exists y(xMy)), \]
which is a theorem of classical logic.

33. On the other hand, \( T^* \) is nearly trivial, for

\[ (33) \quad T^* \iff A \text{ is consistent or } \forall x\exists y \neg (xMy). \]

Proof: As a matter of fact

\[ T^* A \iff \forall x(\neg A(x) \rightarrow \exists y(xMy \rightarrow A(y))) \]
\[ \iff \forall x(\neg A(x) \rightarrow (\forall y(xMy) \rightarrow \exists yA(y))) \]
\[ \iff \forall x(A(x) \lor \exists y\neg(xMy) \lor \exists yA(y)) \]
\[ \iff \forall x(\exists y\neg(xMy) \lor \exists yA(y)) \]
\[ \iff \forall x\exists y\neg(xMy) \lor \exists yA(y). \]

34. Now it is easy to check that

\[ (34) \quad T' A \text{ implies } T^*, \text{ but not the reverse.} \]

Also

\[ (35) \quad \text{No implication from } T^* \text{ or } T'A \text{ to other melioration under consideration holds.} \]

This is so because of the triviality of both weakenings of Cantor’s melioration \( T \), which itself, together with two its strengthenings, is usually quite non-trivial.

2.7 Sextet of Meliorations

35. To resume, we find six meliorations which form strictly decreasing chain of conditions. Let \( \Rightarrow \) denote here not-reversible implication. In general, we have

\[ (36) \quad M' A \Rightarrow M^* A \Rightarrow MA \Rightarrow TA \Rightarrow T^* A \Rightarrow T'A. \]

Two of them are trivial, one overtrivial (banal). Three, however, are quite non-trivial. In hands of Augustine, Anselm, Cantor and others, they show their power and beauty, their glory as crowning achievements of Western civilization.
3 Melioration Cases

The Boolean 2

1. To learn more we will examine carefully the irreflexive Boolean frame 2.

Example VI
Let 2 \( \equiv \langle \{0, 1\}, \{< 0, 1>\} \rangle \). Here \( xMy \) iff \( x = 0 \) and \( y = 1 \). We will calculate in turn the three meliorations involved.

- **Augustine’s one.** Recall \( M^*A \equiv \forall x \forall y(\neg A(x) \land A(y) \rightarrow xMy) \). Put \( B(x, y) \equiv \neg A(x) \land A(y) \rightarrow xMy \). By usual elimination of quantifiers in finite domains we have: \( M^*A \leftrightarrow B(0, 0) \land B(0, 1) \land B(1, 0) \land B(1, 1) \). Now, by Duns Scotus’ law, \( \vdash B(0, 0) \land \vdash B(1, 1) \). Also, \( 2 \models B(0, 1) \) and \( 2 \models B(1, 1) \leftrightarrow (A(0) \rightarrow A(1)) \), for \( \neg(1M0) \). Therefore, \( M^*A \leftrightarrow (A(0) \rightarrow A(1)) \). The last implication in our model is equivalent to \( \neg A(0) \lor A(1) \). Hence, in set-theoretical terms, to be \( M^* \)-meliorated in frame 2 means to contain 1 or not to contain 0.

- **Anselm’s one.** Recall \( MA \equiv \forall x(\neg A(x) \rightarrow \exists y(A(y) \land xMy)) \). Using similar procedure to the one used above we obtain: \( MA \leftrightarrow B(0) \land B(1) \). Here \( B(0) \) is equivalent to \( \neg A(0) \rightarrow \exists y(A(y) \land 0M1) \), which in turn is semantically (in the model 2) equivalent to \( \neg A(0) \rightarrow A(1) \), i.e., to \( A(0) \lor A(1) \). On the other hand, \( B(1) \) is equivalent to \( \neg A(1) \rightarrow \exists y(A(y) \land 1My) \), which in our model (where \( \neg(1My) \) for each \( y \)) is equivalent to \( A(1) \). In sum, \( 2 \models A(1) \lor (A(0) \land A(1)) \), therefore \( 2 \models MA \leftrightarrow A(1) \). Hence, in set-theoretical terms, to be \( M^* \)-meliorated in frame 2 means to contain 1.

- **Finally, consider Cantor’s transcendence.** Now, \( TA \equiv \forall x(\neg A(x) \rightarrow \exists y(A(y) \leftrightarrow xMy)) \). Similarly, \( TA \leftrightarrow B(0) \land B(1) \), for appropriate formula \( B \). Here \( B(0) \) is equivalent to \( \neg A(0) \rightarrow \exists y(A(y) \leftrightarrow 0M1) \), which in turn is semantically (in the model 2) equivalent to \( \neg A(0) \rightarrow \neg A(0) \lor A(1) \), which is a tautology. On the other hand, \( B(1) \) is equivalent to \( \neg A(1) \rightarrow \exists y(A(y) \leftrightarrow 1My) \), which in our model (where \( \neg(1My) \) for each \( y \)) is equivalent to \( \neg A(1) \rightarrow \exists y\neg yA(y) \), which also is a theorem of logic. In sum, \( 2 \models TA \), for every \( A \).
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To conclude, in the model $2$ the only \textit{a priori possible} implication remaining valid is that of (II.$§$ 27) and of (II.$§$ 26) (under its usual consistency proviso): $M^*A \rightarrow MA$ and $MA \rightarrow TA$, but not reversely (for the first implication we need to assume that $A$ is consistent!).

2. It would be illuminating, for sure, to compare also behavior of the three melioration operators under investigation in chosen extreme, sometimes pathological, situations.

Tautologies

3. For tautologies we have that

$$(1) \quad \text{Tautologies are meliorated ever, for every formula and in each variation of the word.}$$

For, simply by logic, $M^*(A \lor \neg A)$. Hence, by (II.$§$ 34) both $M(A \lor \neg A)$ and $T(A \lor \neg A)$ hold as well.

Inconsistencies

4. Inconsistencies differentiate meliorations:

$$(2) \quad \begin{align*}
(i) & \quad M^*(A \land \neg A), \text{ but } \\
(ii) & \quad \neg M(A \land \neg A), \text{ whereas } \\
(iii) & \quad T(A \land \neg A) \leftrightarrow \neg \exists x \forall x(xMy).
\end{align*}$$

Claim (i) is by logic alone, (ii) was checked above. To check(iii) observe that $T(A \land \neg A) \leftrightarrow \forall x(\neg(A \land \neg A) \rightarrow \exists y(xMy \leftrightarrow A \land \neg A))$. The right side of the above equivalence is equivalent to $\forall x \exists y \neg(xMy)$, hence $\neg \exists x \forall y(xMy)$. Each step in the above reasoning can be reversed, which entails (iii).

Sentences

5. Proper meliorations in frames under consideration are formulas; with at least one free variable, they are not sentences. Meliorated sentences are, for sure, improper. Why? Observe first that for a given sentence we have semantically two situations only: either it is valid in a given frame or not, i.e., its negation is valid. Also both quantifiers involved are void. Therefore

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3. Let $A$ be a sentence. Then

(i) $M^* A$

(ii) $MA \iff A$, whereas

(iii) $TA \iff (\exists x \forall y (xMy) \rightarrow A)$

**Proof:** If $A$ is a sentence, then it doesn’t depend on variables and each its quantification is void. Therefore $M^* A \iff \forall x \forall y (A \wedge \neg A \rightarrow xMy)$, which is a tautology. Similarly $MA \iff \forall x (\neg A \rightarrow \exists y (A \wedge xMy))$, which in turn is equivalent to $A \vee (A \wedge \forall x \exists y (xMy))$, hence to $A$ as well. Finally, $TA \iff \forall x (\neg A \rightarrow \exists y (A \leftrightarrow xMy)$, which in turn is equivalent to $\forall x \exists y (\neg A \rightarrow (A \leftrightarrow xMy))$, next to $\forall x \exists y (xMy \rightarrow A)$, hence to $\forall x \exists y (xMy) \rightarrow A$.

**Emptiness**

6. Consider now the case $M = \emptyset$, i.e., any discrete frame $U = \langle U, \emptyset \rangle$. In this case we immediately obtain the following claim:

(4) (i) $M^* A$ iff $A$ is inconsistent or tautological, hence $\{A\}$ is empty or full.

(ii) For any $A$, $MA$ iff $\forall A(x)$ (or $\{A\}$ is full).

(iii) For all $A$, $TA$ does hold.

**Proof:** For (i) observe that if $M = \emptyset$, then $M^* A$ is equivalent to $\forall x \forall y (\neg A(x) \wedge A(y))$, hence to $\forall x \forall y (A(y) \rightarrow A(x))$, which in turn is equivalent to $\exists y A(y) \rightarrow \forall A(x)$. Hence $\{A\}$ is either empty of full, as required.

Claim (ii) is immediate, whereas in our case (iii) $TA \iff \forall x (\neg A(x) \rightarrow \exists y \neg A(y))$, which is a theorem of logic, hence $TA$.

**Set Theory**

6. Let $V$ be the family of all sets and $xMy \overset{df}{=} y \in x$. Take $\mathcal{V} = \langle V, M \rangle$. Now

(5) $\mathcal{V} \vDash T(x \text{ is infinite}) \ (T(x \geq \omega))$.

**Proof:** To see this observe, that in the frame $\mathcal{V}$, $T(x \text{ is infinite})$ means: $(\forall y (y \in x \iff y \text{ is finite}) \rightarrow x \text{ is infinite})$, hence $(x = \{y : y \text{ is finite}\} \rightarrow x \text{ is infinite})$, and in turn $(x = \{y : y < \omega\} \rightarrow x \geq \omega)$, which is a theorem of $\mathbb{Z F}$. In a similar way we can obtain quite a lot of powerful axioms of infinity.
4 Melioration Modal Logics

4.1 Modalities

**Hidden modalities**

1. Anselm’s Argument in its full, modal, version has at least four modalities. The first, epistemological modality of Anselm and Leibniz $CP$, is transparent in the original, Anselm’s formulation.

The other two are, however, hidden. The second one is our melioration modality $M$, whereas the third (denoted $M$) is the standard relational modality defined with respect to our basic ontological frame $\mathcal{U} = \langle U, M \rangle$ by means of the consequent of the implication in the definition of $MA$:

\[ (M) \quad MA(x) \overset{df}{=} \exists y(A(y) \land xMy). \]

Thus Anselm’s theory of God, even in the elementary version, is indeed deeply modal.

**Melioration and frame modalities**

2. Observe that both $M$-modalities are interconnected, $M$ is the onto-counterpart of the logical modality $M$, and reversely.

On the other hand, melioration $M$ is an algebraic majorization of the relational modality $M$. Indeed,

\[ (1) \quad MA \leftrightarrow A \lor MA. \]

3. Recall that the formula $A \lor MA$ is the counterpart of $MA$ in the positive decomposition of modalities in the sense of my topography of modal logics [5]. For a given modality $L$ and its dual $M$ we put there:

\[ \begin{align*}
L^+ A & \overset{df}{=} LA \land A \quad \text{and} \quad M^+ A \overset{df}{=} A \lor MA.
\end{align*} \]

New operators ($L^+, M^+$) are projections of the old ones ($L, M$) in the topography mentioned above. Also, we see that

\[ (2) \quad MA \leftrightarrow M^+ A \quad \text{and} \quad LA \leftrightarrow L^+ A. \]

Thus melioration $M$ taken as modality is simply the projection of $L$!

4. Observe also that $MA$ is the operator of Diodorean type: $LA \lor A \lor MA$. Thus Anselmian modality is Diodorean and reversly.
4.2 Logics

5. On the other hand the formula $M^+A$, hence $MA$, is the axiom of the smallest negative logic $L^-$ of the topography outlined in [5]. Hence it has quite negative character.

Notice also that accepting the axiom $M^+A$ we have that

(3) Any consistent formula is meliorated: $\forall A MA$.

Hence

(4) From a negative point view each consistent condition is meliorated.

6. **Warning:** We should be careful with reading the quantifier condition $MA$ and its reading as modality. We know that $M(A \land \neg A)$ is inconsistent according the first reading, whereas it is consistent according the second one.

7. Notice also close connections between usual $M$ - conditions and $M$ - conditions between their logics.

(5) The following conditions characterize modal melioration logics:

(i) $M(A \lor B) \leftrightarrow MA \lor MB$. $M$-logics (containing formulas of this type) are regular.

(ii) $A \rightarrow MA$. $M$-logics (containing formulas of this type) are $T$-logics (or positive).

(iii) $M$-logics are monotonic if $M$ - logics are monotonic.

(iv) $\neg M0$.

(v) $M$-logics are normal if their $M$ - counterparts are monotonic (a fortiori) extensional.

Therefore

(6) The modal logic of melioration $M$ is the logic of Gödel-Feys-von Wright $T$, provided that its $M$ - base is monotonic.

8. Let $MM$ denote the common logic of $M$ and $M$. By above observations we have

(7) $MM \vdash 0$, provided its conjugate $M$ - logic is monotonic.

\[13\]The phrase: “(containing formulas of this type)” in the conditions (i) and (ii) was introduced by M. Szatkowski.
Bibliography


Doomed to fail: The sad epistemological fate of ontological arguments

John Turri

For beings like us, no ontological argument can possibly succeed. They are doomed to fail. The point of an ontological argument is to enable nonempirical knowledge of its conclusion, namely, that God exists. But no ontological argument could possibly enable us to know its conclusion non-empirically, and so must fail in that sense. An ontological argument will fail even if it is perfectly sound and begs no questions.

1 Definition

I begin by defining key terminology.

An ontological argument’s nonempirical character distinguishes it from other theistic arguments. The principal goal of presenting or rehearsing an ontological argument is to promote the acquisition of inferential nonempirical knowledge that God exists, via acceptance of the conclusion based on competent deduction from the premises, which are themselves either non-empirically known or knowable. I will argue that, at least for beings like us, all ontological arguments must fail to achieve this goal.

By God I mean the god of classical western religious monotheism: the unique, eternal, omnipotent, omniscient, and omnibenevolent person who created and sustains the universe (Alston [2], ch 1; Tooley [11], section 1.1).

You inferentially know a proposition if and only if you know it based at least partly on inference from something else you accept. I use ‘accept’
broadly to include any kind of doxastic commitment, including belief, suspicion, faith, presupposition, etc. It is obvious that knowledge based on an argument is inferential.

You non-empirically know a proposition if and only if you know it in a way that is not even partly based on external perceptual experience. It is a harmless oversimplification to think of nonempirical knowledge in my sense as knowledge you could have, as they say, “from the armchair”. You empirically know a proposition if and only if you know it based at least partly on external perceptual experience. It is possible to simultaneously know a proposition empirically and non-empirically, so long as you have sufficient independent bases for it.

An external perceptual experience is an experience produced by a power of the mind to receive signals from things other than itself (an “external channel”). Ordinary sensory experience counts as external perceptual experience. An external perceptual experience needn’t have sensory content or any other sort of phenomenal or conscious character. Actual cases of blindsight involve external perceptual experience, as would a phenomenal zombie’s typical perceptual beliefs. Knowledge based on signals acquired through telepathy, clairvoyance and other forms of extra sensory perception, if such things exist, would count as empirical. By contrast, introspective awareness of one’s mental conditions and concepts counts as nonempirical on my definition. Introspection is not an external channel. This fits nicely with the history of ontological argumentation. Anselm appeals to what exists “in the understanding” of even a fool, and in his argument Descartes notes, “clearly the idea of God ... is one I discover to be ... within me.” We could try to reconstruct these arguments without reference to introspection, but my approach doesn’t force us to.

Innate knowledge counts as nonempirical on my definition. Perhaps it’s possible that some innate knowledge is based on reasons, depending on what counts as basing an attitude on a reason. But I doubt that innate inferential knowledge is possible, because inferential knowledge re-

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1 I don’t equate it with “exteroception” in the physiological sense, which pertains to detection of stimuli outside of the body. If the mind is identical to the body, then it would be equivalent to exteroception.

2 Block [4] dubs the processes “super-duper-blindperception”. But “blind” pertains specifically to vision, whereas zombies have no phenomenal experience, visual or otherwise. So a better general label would be “super-duper-blankperception”.

3 Meditation Five in [8].

4 Turri in [12] offers an account of basing.
quires completing an inference, and it’s doubtful that you could come into existence having completed an inference. In any case, the point of an ontological argument – indeed of any argument – is obviously not the innate acquisition of knowledge; it’s far too late for that by the time we’re in a position to consider the argument. For this reason, in what follows I will explicitly set aside innate knowledge.

Finally, it is perhaps worth noting that nonempirical knowledge in my sense is not the same thing as a priori knowledge in the standard sense. As it is often understood, a priori knowledge is knowledge based purely on rational intuition or understanding of concepts, perhaps along with inference from other things accepted similarly on the basis of rational intuition or understanding (Bonjour [5], Bealer [3]). Assuming a non-perceptual model of rational intuition, all a priori knowledge counts as nonempirical in my sense, but not all nonempirical knowledge in my sense counts as a priori. For example, introspective knowledge counts as nonempirical but not a priori.

2 Doom

Now I will present my main argument. The argument proceeds from three simple and intuitive premises. Remember that I am explicitly setting aside innate knowledge, so where I speak of knowing non-empirically, please understand me to mean knowing non-empirically and postnatally.

My first premise is that you cannot non-empirically know that another specific person exists now. You are the only specific person whom you can non-empirically know to exist now. You can know that other people exist now, of course, but you can know this only empirically. In order to know that others exist now, you must either (a) see them, hear them, touch them, smell them, or acquire some other sort of signal through an external channel, such as telepathy or clairvoyance, if such things are possible; or (b) remember things from previous external perceptual experiences, which somehow enables you to know that others presently exist. Such memories inherit their progenitors’ empirical character.

I expect that this first premise is apt to seem uncontroversial, verging on the obvious. Even so, a vivid thought experiment can help us appreciate the point. Imagine that at some point in the near future, humanity plunges into a nuclear holocaust. Jade is buffeted by the blast, knocked unconscious in the otherwise empty and nondescript bomb shelter in her backyard. She suffers complete loss of declarative memory. Jade wakes
up alone, numb and confused. It is completely dark, completely silent. She can see nothing, hear nothing, smell nothing, feel nothing, remember nothing. She wonders to herself, "Is there anyone else out there? Am I the only one?" It seems obvious that she can’t know whether anyone else exists until she acquires some empirical evidence. She cannot, just by reflecting on her concepts and current mental condition, learn that she isn’t alone, that other people currently exist.

Some philosophers have defended views about thought and language that might seem to threaten my first premise. Consider content externalism. Hilary Putnam asks, “Could we, if we were brains in a vat ... think that we were?” (Putnam [10], p. 31), and answers that we could not possibly do so. We couldn’t think of vats unless we had a proper “causal connection to real vats”, which no mere brain in a vat could have (Putnam [10], p. 37). And content externalism arguably generalizes to most or all of our concepts (Burge [6]). Moreover, content externalism is, like other philosophical theses, knowable non-empirically, if knowable at all. Grant that we do non-empirically know that content externalism is true. Thus you could non-empirically know that you have been causally connected to a community. You need only introspect that you have thoughts about a community, and reflect carefully enough to appreciate that this requires you to have had causal contact with a community. A community requires more than one person. So as long as you can non-empirically know that you are not more than one person, you can also non-empirically know that other people exist.

I’m willing to grant that you can know non-empirically that you are not more than one person, along with the other claims about what you can know non-empirically about your own thoughts and content externalism. But the objection under consideration still fails. Even supposing everything granted thus far, it doesn’t follow that you can non-empirically know that other people now exist. At most it follows that you can non-empirically know that other people have at some point in time existed. Compare Jade’s case. Let’s grant that she can non-empirically know that she has been in causal contact with others, based on the introspectible fact that she asked herself whether others exist. She still can’t knowledgeably infer that others exist now. And this has nothing special to do with the fact that her knowledge that others have existed is nonempirical. Adjust the case so that her amnesia isn’t complete: she remembers, and thereby knows, that other people have existed. Thus her knowledge that others have existed is empirical. Yet she still can’t knowledgeably infer
that anyone else exists now. That’s still an open question for her.

A related objection to my first premise might derive from Davidson’s triangulation theory of content. Davidson agrees with Putnam, Burge and others that content externalism is true. Moreover, Davidson agrees that content externalism has a dramatic anti-skeptical upshot: we can know, just by reflecting on the nature of content and language, that systematic perceptual error is ruled out. “Anyone who accepts [content] externalism knows that he cannot be systematically deceived about whether there are such things as cows, people, water, stars, and chewing gum” (Davidson [7], p. 201). Of course, this isn’t enough to help in Jade’s case because she doesn’t believe that others presently exist; she suspends judgment on that question. Moreover, due to her amnesia she lacks knowledge and beliefs about others. So she can’t infer that most of her beliefs about others are true.

But there is more to Davidson’s theory. Having granted that our beliefs are about what typically causes them, Davidson asks, “what has typically caused them?” Consider our beliefs about cows. Why think that cows cause these beliefs rather than, say, “events spatially closer to the thinker than any cow?” Davidson claims that there is a “social basis” for determining the answer. The relevant cause is picked out by how other people interpret the thinker. Thought requires an “essential triangle” of thinker, environment and interpreter: “The presence of two or more creatures interacting with each other and with a common environment” is required for thinking to occur (Davidson [7], pp. 202 - 203). Triangulation determines the relevant causal relations and, thus, the content of all thought.

Even if we grant everything Davidson says about triangulation, it doesn’t threaten my first premise. Suppose that you can non-empirically know that you’re having thoughts. And suppose that you can non-empirically know that your thoughts are typically about what causes them, and that this requires you to have interacted with another person in a common environment. You still can’t knowledgeably infer that another person exists now. Nothing in Davidson’s view requires that another person monitor you whenever you have a thought, or usually when you have a thought, or even recently when you’ve had a thought. (Not even Big Brother’s Thought Police could ensure that everyone was monitored whenever they had a thought.) Thus you can’t knowledgeably infer that others are now monitoring you, and so exist now. That’s still an open question for you.

My second premise is that if any ontological argument can succeed for you, then you can non-empirically know that God, a person, exists now.
The Sad Epistemological Fate of Ontological Arguments

For if it were to succeed, then you would non-empirically know that God exists. And it’s trivially obvious that if God exists, then God exists eternally, including now; this is a simple conceptual truth, which you can and do know non-empirically. And it’s trivially obvious that God is a person; this is a simple conceptual truth, which you also can and do know non-empirically.

My third premise is that you are not God. Disappointing as that may be, I trust that you will, at least upon cool reflection, agree that this premise needs no defense.

From those three premises it follows that no ontological argument can succeed for you. For if any ontological argument can succeed for you, then you can non-empirically know that God, a person, exists now. But you are the only person whom you can non-empirically know to exist now. So either you are God, or no ontological argument can succeed for you. But you are not God. So no ontological argument can succeed for you.

Moreover, there’s nothing special about you in this regard. The same is true for all humans. So no ontological argument can succeed for any of us. For us, ontological arguments are doomed to fail. In fact, if my argument is correct, then presumably God is the only person for whom an ontological argument could succeed. I accept this consequence of the argument.

If a more formal representation of the basic argument is desired, the following will suffice. (Note that the numbered premises in this formal rendition don’t correspond to the ordinally numbered premises of the informal presentation above.)

(Doomed)

(1) If you can nonempirically know that a certain person exists now, then you = that person. (Premise)

(2) If God exists, then God is a person. (Premise)

(3) So if you can nonempirically know that God exists now, then you = God. From (1) and (2)

(4) If any ontological argument can succeed for you, then you can non-empirically know that God exists now. (Premise)

(5) So if any ontological argument can succeed for you, then you = God. From (3) and (4)

(6) You ≠ God. (Premise)
(7) So no ontological argument can succeed for you. From (5) and (6)

(8) If no ontological argument can succeed for you, then no ontological argument can succeed for any of us.

(9) So no ontological argument can succeed for any of us. From (7) and (8)

3 Delimitation

This section emphasizes some limitations and unique features of my argument.

First, I have neither argued for nor committed myself to any of the following claims:

* God couldn’t possibly exist.
* God doesn’t exist.
* You can’t know that God exists.
* You can’t innately know that God exists.
* You can’t empirically know that God exists.
* You can’t know by testimony that God exists.
* You can’t know by revelation that God exists.
* You can’t nonempirically know that any god whatsoever exists.

Regarding the last item, nothing in my argument speaks against nonempirical knowledge of impersonal gods. For example, perhaps we can know nonempirically that an infinite impersonal substance exists, and perhaps that counts as divine. That is consistent with everything I say here.

Second, I have neither utilized nor committed myself to any of the following general grounds for rejecting ontological arguments:

* Ontological arguments are invalid.
* Ontological arguments have a false premise.
* Ontological arguments are unsound.
* Ontological arguments suffer refutation by parody.
* Ontological arguments beg the question.
Question-begging arguments must fail.

Persuasiveness to opponents is a criterion of a successful argument.

Persuasiveness to reasonable and neutral third parties is a criterion of a successful argument.

Nonempirical knowledge would have to be infallible, indefeasible, or certain.

Finally, I have neither assumed nor committed myself to any of the following specific claims which feature centrally in some traditionally influential responses to the ontological argument:

Existence is not a predicate.

Existence is not a first-order predicate.

Existence is not a perfection.

The concept of God is inconsistent.

4 Conclusion

In closing, I would like to consider one final objection. Suppose that a proponent of the ontological argument rejects my premise that you are the only person whom you can nonempirically know to exist now. And suppose that this proponent offers the ontological argument itself as a counterexample to my premise. In that case, I am content to let the entire matter rest on the comparative plausibility of the two competing claims: on the one hand, that knowledge of other people’s current existence requires empirical support, and on the other, that the ontological argument succeeds. I know which one seems more probable to me. Others will of course decide the matter for themselves.5

Bibliography


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The Premises of Anselm’s Argument

Paul Weingartner

1 Introduction

This paper considers two aspects of Anselm’s “Ontological Argument” in his Proslogion (Anselm [1]): (1) Does the argument contain empirical premises? and (2) Are the premises self-evident in the sense of Aquinas or analytic in the sense of Kant? The first question is answered with Yes, thus the widespread opinion that the Ontological Argument is an a priori argument is not correct. Moreover it will be shown that the suitable representation of the argument, when incorporating the empirical premises, is intuitionalistic logic. The second question can also be answered with Yes, therefore the objections of Thomas Aquinas and of Kant receive a more accurate explanation.

2 Does the Argument Contain Empirical Premises which have to be Used as Premises in the Argument?

The argument consists of two parts.\textsuperscript{1} The first part tries to show that the \textit{id quo maius cogitari non potest} (abbreviated as: \textit{QM})\textsuperscript{2}, the existence of which is to be proved, is understood and exists therefore in the mind. The second part consists of the indirect proof that the \textit{QM} also exists in reality.

\footnote{Anselm [1], ch. 2}

\footnote{We use this expression which is used in that part of the argument which is the indirect proof.}
2.1 The first part of Anselm’s Argument contains empirical statements

The first part starts with the statement of a belief and continues with a question and finally contains a partial answer to the question. The belief is expressed thus: “we believe that YOU are a QM, i.e. something than which no greater can be conceived”. The question is: Is it possible that such a thing does not exist? Because the fool says in his heart: there is no God. The partial answer to the question is: If the fool hears the words *id quo maius cogitari non potest*, then he understands what he is hearing; and what he understands is in his mind (even if he does not grasp that this thing exists).

Anselm then continues by underlining the distinction between existence in the mind and existence in reality. He says at the end of the first part that even the fool has to accept that there is – at least in the mind – something than which no greater can be conceived. After this statement the second part with the indirect proof begins.

We can summarise Anselm’s statements of the first part as follows:

1. Religious believers understand by God an ‘*id quo maius cogitari non potest*’, i.e. something than which no greater can be conceived.
2. Even the fool i.e. also the non-believer understands the words ‘*id quo maius cogitari non potest*’ (*QM*).
3. What the believers and the non-believers understand exists in their minds.

It is evident that these statements are empirical statements. This is also Thomas Aquinas’s understanding, since he gives a counterexample to the universalisation of statement (1) and (2): It also seems to be Anselm’s claim that by God all humans understand a *QM*, however against this claim Aquinas points out that some thought that God is a body.³ An example is Tertullian.

2.2 Is it necessary to use the empirical statements as premises in the argument?

It is disputable, whether these empirical statements are premises which have to be used in Anselm’s argument. This is a serious question, since

³Thomas Aquinas [2], I, qu.2, 1 ad 2.
there are interpretations of Anselm’s argument, which use only the second part, i.e. the indirect proof. Additionally, in the part of the indirect proof, there is no premise which assumes the existence of a $QM$ in someone’s mind. Thus it is understandable that if Anselm’s argument is merely understood as the indirect proof, it is called an a priori argument.

However, should we think that Anselm used the first part only as an introduction, which is superfluous for the proof which follows? Concerning an interpretation of Anselm’s argument with the help of modern logic, this question can also be put into the following form: Should the strength of the logical principles be such that – as a consequence – the statements of the first part need not be used as premises? Or more specifically: Is the indirect proof in the sense of an indirect existence proof a generally accepted method when interpreting philosophical or scientific texts? And as a consequence: Is there a way of interpreting Anselm’s argument in such a way that some or all empirical statements of the first part have to be used as premises? The answer to the last question is Yes. And this way of interpretation drops the indirect existence proofs. As is well-known, the logical systems which do not permit indirect existence proofs are intuitionistic logic and some intermediate logics. It will be shown subsequently that Anselm’s argument can be interpreted with the help of intuitionistic logic in such a way that one empirical premise is a necessary starting-premise of the argument.

The idea behind such an interpretation is this: Take the whole text seriously. Use as much as possible from the text, but keep the interpretational means at a minimum. In other words: Use the whole text, but use a logic as weak as possible. This methodological principle is the opposite of another principle, which is frequently applied: Use strong logical principles, for then you can drop a part of the text as redundant or superfluous. However the latter principle does not do justice to the text and its author.

3 Representation of Anselm’s Argument with Intuitionistic Logic

3.1 Indirect existence proofs and ‘tertium non datur’

In almost all versions of intuitionistic logic it holds: existence in mathematics and logic coincides with constructivity. According to this principle, a statement of the form $\exists x F(x)$ can only be proved by providing a construction of a number (concept) $k$ and a proof for $F(k)$. On the other
hand, in order to prove $\neg \exists x F(x)$, it is necessary to give a proof for $\neg F(n)$ for every particular $n$. If the domain of $x$ is finite, there is no problem with the validity of the tertium non datur:

$\exists x F(x) \lor \neg \exists x F(x)$, since the universal negation can be interpreted as a conjunction of a finite number of decidable statements. But if the domain is infinite, the tertium non datur is not generally provable, since neither of the two requirements above may be satisfied. Because of the universal validity of $\exists x F(x) \lor \neg \exists x F(x)$ in classical logics and mathematics, one gets an indirect existence proof whenever one succeeds to derive a contradiction from $\neg \exists x F(x)$ (viz. $\forall x \neg F(x)$ and consequently to prove $\neg (\neg \exists x F(x))$). Intuitionistically one cannot reduce the double negation, so that this only leads to the non-absurdity (or non-inconsistency) of $\exists x F(x)$, but not to $\exists x F(x)$.

Let us now apply these considerations to the concept id quo maius cogitari non potest ($QM$). That is we consider the validity of the following instance of the tertium non datur: $\exists x QM(x) \lor \neg \exists x QM(x)$.

According to intuitionistic means, this has to be interpreted thus: Either there is a conceptual construction of the object $k$ and a proof for $QM(k)$ or there is a uniform proof, which shows the falsity of $QM(n)$ for every $n$. It is rather plain that neither of the two is satisfied.

Concerning the first part of the disjunction a conceptual construction would mean to give a definition or at least necessary conditions for the definiens of a partial definition. But most of the great christian philosophers and theologians – beginning from Clemens of Alexandria through the Middle Ages to those of our time – hold that the essence of God cannot be known in this life and consequently an Aristotelian definition of the essence of God cannot be given. What is possible according to many of these thinkers is that the predicates or concepts we apply to God are taken from creation and creatures, i.e. from the world including man; and they have to be interpreted per analogiam. But the concept $QM$ is not one taken from creation or from the world, because it surpasses all the things of the world. On the other hand it would also be difficult to show the falsity of $QM(n)$ for each $n$. This would mean to show that every $n$ belongs to the complement class of $QM$ i.e. to the class of id quo maius cogitari potest. This class is rather indefinite concerning its extension. Thus the atheist is in a position at least as bad as the theist.

We might consider therefore the concept of something than which no greater can be conceived which exists in reality (abbreviated as: $QMR$). Here we have to distinguish two negations: (1) We drop the negation in
we accept things such that greater ones can be conceived and which exist in reality. This class of things can be finite and definite. But this is not the class Anselm has in mind. (2) We apply the negation to the second part i.e. we deny the existence in reality. This is Anselm’s negation, which he uses in the assumption of the indirect existence proof: ¬∃xQMR(x).

Thus we have to consider the validity of the following instance of the tertium non datur: ∃xQMR(x) ∨ ¬∃xQMR(x).

Here we are in a similar situation as concerning QM above: It will hardly be possible to provide a conceptual construction of an object k and a proof for QMR(k) or to give a uniform proof which shows the falsity of QMR(n) for every n.

It seems we are in a similar situation here as described by intuitionism concerning mathematics if it transcends the denumerable domain; since then the concepts are not always definite or precise and the range of variables running over transfinite numbers or over supernatural things is rather unclear.

3.2 Anselm’s Argument with Intuitionistic Means

As indicated by the above considerations, the validity of the tertium non datur applied to the concept id quo maius cogitari non potest is problematic. Consequently the application of the classical method of indirect existence proofs concerning such concepts is problematic. For these reasons it seems rewarding to formulate Anselm’s Argument with intuitionistic means. That is, exclusively intuitionistic principles are used for the proof. Also, the proof has the form of an indirect proof, deriving a contradiction from the premises plus the assumption which negates the real existence of a QM. It should be clear however, that the conclusion has to be weaker than the one derived with the help of classical logic. We can only prove the non-absurdity or the consistency of a QM, but not its real existence. However to prove the consistency or non-absurdity of the object of religion against claims to the contrary is still important. In fact, Thomas Aquinas holds this to be a very important task of theology: “The reasons employed by holy men to prove things that are of faith, are not demonstrations; they are persuasive arguments showing that what is proposed to our faith is not impossible”.4

QM(x) : x is that than which no greater can be conceived,

4Thomas Aquinas [2], II-II, 1, 5 ad 2
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\[ QMM(x) : \text{ } x \text{ is that than which no greater can be conceived which exists in the mind,} \]

\[ QMR(x) : \text{ } x \text{ is that than which no greater can be conceived which exists in reality,} \]

(1) \( QMM(a) \)  \hspace{1cm} Premise
(2) \( \forall x (QMM(x) \rightarrow QM(x)) \)  \hspace{1cm} Premise
(3) \( \forall x (QMM(x) \land \neg QMR(x) \rightarrow \neg QM(x)) \)  \hspace{1cm} Premise
(4) \( \neg \exists x QMR(x) \)  \hspace{1cm} Assumption of Indr. Proof
(5) \( QMM(a) \rightarrow QM(a) \)  \hspace{1cm} (2), Instantiation
(6) \( QM(a) \)  \hspace{1cm} (1), (5), Modus Ponens
(7) \( \forall x \neg QMR(x) \)  \hspace{1cm} From (4)
(8) \( \neg QMR(a) \)  \hspace{1cm} (7), Instantiation
(9) \( QMM(a) \land \neg QMR(a) \)  \hspace{1cm} (1), (8), Conjunction
(10) \( QMM(a) \land \neg QMR(a) \rightarrow \neg QM(a) \)  \hspace{1cm} (3), Instantiation
(11) \( \neg QM(a) \)  \hspace{1cm} (9), (10), Modus Ponens
(12) \( QM(a) \land \neg QM(a) \)  \hspace{1cm} (6), (11), Conjunction
(13) \( \neg \exists x QMR(x) \rightarrow QM(a) \land \neg QM(a) \)  \hspace{1cm} Conditional Proof
(14) \( \neg (QM(a) \land \neg QM(a)) \)  \hspace{1cm} Principle of Non-Contradiction
(15) \( \neg \neg \exists x QMR(x) \)  \hspace{1cm} (13), (14), Modus Tollens
(16) \( \exists x QMR(x) \)  \hspace{1cm} (15), Classical Logic

3.3 Commentary to the Premises

**Premise (1):** This is Anselm’s assumption that a certain a is a QMM, i.e. an *id quo maius cogitari non potest* which is in a concrete (individual) mind (say in the mind of a fool). Observe that starting with an existential statement of the form \( \exists x QMM(x) \) is intuitionistically not sufficient, since it could be proved only by a sentence of the form \( QMM(a) \).

**Premise (2):** This premise is a kind of enthymemic presupposition and leads via instantiation and modus ponens to \( QM(a) \) (step (6)). Another way of reading is a strengthening of premise (3) in such a way that premise (2) is no more needed. This is indicated by premise:  

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(2′) \( \forall x (QMM(x) \land \neg QMR(x) \rightarrow (QM(x) \leftrightarrow \neg QM(x))) \).

**Premise (3):** This is the essential premise of the argument, although premises (1) and (2) are also necessary if one takes the whole text into account. Premise (3) says that a \( QM \) which exists in the mind, but does not exist in reality is not a \( QM \); since then there would be a greater thing which exists both in mind and in reality. The decisive passage for premise (3) is the following:

\[
\text{“Si ergo id quo maius cogitari non potest, est in solo intellectu: id ipsum quo maius cogitari non potest, est quo maius cogitari potest.”}
\]

If we translate this passage word for word into the symbolic language, then we receive a somewhat longer premise (3∗) instead of premise (3):

(3∗) \( \forall x ((QM(x) \rightarrow (QMM(x) \land \neg QMR(x))) \rightarrow (QM(x) \rightarrow \neg QM(x))) \).

Premise (3∗) also leads to the same result as premise (3), namely to \( \neg QM(a) \) (step (11)). This is so, since also in this case the sentences \( QMM(a), \neg QMR(a) \) and \( QM(a) \) are sufficient intuitionistically; and the principle of addition and \( (\neg p \lor q) \rightarrow (p \rightarrow q) \) are intuitionistically valid. Although (3) seems to be the shortest premise acceptable for a correct representation of the text, premise (3∗) is closest to the text and both (3) and (3∗) lead to the same result concerning the proof.

Instead of premise (2) and (3) we can also use premise (2′). This is justified by the observation that the second occurrence of the Latin word “est” in the Latin quotation above can be interpreted in a twofold way: as meaning an implication and as meaning an equivalence. Both interpretations are possible. The interpretation with the equivalence makes the enthymemetic second premise dispensable.

There is a further interesting point to be observed. Premise (3) is logically equivalent – by Classical First Order Predicate Logic – with \( \forall x (QM(x) \rightarrow (QMM(x) \rightarrow QMR(x))) \). But this equivalence is not valid intuitionistically. And it is a serious question whether the above symbolic sentence could be justified by Anselm’s text. It says: “It holds for all things \( x \): if \( x \) is a \( QM \), then, if \( x \) is a \( QM \) existing in the mind then \( x \) is a \( QM \) existing in reality”. But the text speaks of a \( QM \) which exists in the mind and not in reality. And a proof for the consistency of the existence of God which uses this sentence as the third premise, would run quite differently. It would not be an indirect proof and certainly not a representation of Anselm’s argument. This shows an important aspect of using intuitionistic means or more generally of using a weaker logic: that
there are non-trivial differences when a philosophical text (in this case Anselm’s) is interpreted with the help of intuitionistic logic instead of an interpretation with the help of classical logic. In general it is known that translations of every day language texts or of scientific texts into symbolic language of (classical) First Order Predicate Logic (PL1) are not invariant w.r.t. transformations with the help of logical equivalence of PL1.\footnote{For details see Schurz-Weingartner [13], Schurz [12], and Weingartner [18] and [19].}

**Premise (4):** This is the assumption of the contrary for the indirect proof. *Modus ponens, modus tollens* and the move from (4) to (7) are intuitionistically valid.

As it has been pointed out, this conclusion is weaker than the one derived with the help of classical logic, since the double negation which means non-absurdity or consistency of existence cannot be reduced to existence (as it is the case in classical logic). On the other hand it holds according to a theorem by Glivenko for all propositions $p$: if $p$ is classically provable, then the non-absurdity of $p$ (i.e. $\neg\neg p$) is intuitionistically provable. Hence the result is here: The *id quo maius cogitari non potest which exists in reality* is consistent or not absurd.

It should be mentioned, however, that this proof can always be completed classically such that we end up with the existence (in reality) of the *id quo maius cogitari non potest* (see, step (16) of the proof). This shows that this proof can also be used as an interpretation of Anselm’s argument with the help of classical logic. But the important advantage of this formulation of the proof is that the empirical premise at the beginning (premise (1)) is really needed for the proof; i.e. Anselm’s beginning of the text is not a superfluous introduction, but is taken seriously and is incorporated into the proof. A consequence of this is that Anselm’s argument cannot be called a purely a priori argument, since it essentially contains an empirical premise.

### 3.4 A Difficulty Concerning the Premises

A serious difficulty concerning the premises is the doubt about the ambiguous use of the term *id quo maius cogitari non potest* (*QM*). In premise (2) it has to be understood as not involving real existence. Otherwise the whole argument is a *petitio principii*. But in premise (3) it has to be understood as involving real existence, otherwise premise (3) would not be true. If *QM* would not involve real existence, then there could be an $x$...
which is a QMM and not a QMR and is at the same time a QM. That is, in the understanding of premise (2), the id quo maius cogitari non potest does not involve its real existence (the fool does not accept it). Anselm could say it appears not to be involving the real existence, but in a hidden way it still does and this hidden way is made explicit in premise (3). So far so good, but that still means that the expression “QM” is used in premise (2) in a different sense than in premise (3). And this kind of equivocation can hardly be avoided in a precise representation of the argument it seems.

4 Are the Premises of Anselm’s Argument Self-evident in the sense of Aquinas?

4.1 Self-evident according to Thomas Aquinas

“...A proposition is self-evident because the predicate is included in the essence of the subject, as ‘Man is an animal’, for animal is contained in the essence of man.” This definition of self-evident occurs in the answer to the first preliminary question before Thomas Aquinas offers his Five Ways to prove the existence of God. This preliminary question is whether the existence of God is self-evident. Thomas Aquinas answers the question in the negative. In the second objection of this article, Thomas Aquinas says that God’s existence can be self-evident in the sense of proving it by Anselm’s argument. In his answer he then says that the first empirical premise is not self-evident, since then everyone would understand by God an id quo maius cogitari non potest. But this is not the case as has been pointed out in section 2.1 above, although the first premise can be true for a single instance as formulated in the first premise. It will subsequently be shown that premise (2), (3) and (2’) are also not self-evident according to the definition of self-evident given above by Thomas Aquinas. In order to show this, we have to first give an interpretation of Aquinas’s definition with the help of modern logic.

4.2 Interpretation of “the predicate is included in the essence of the subject”

It is easy to interpret the opposite form “the subject is included in the predicate” if we think of subject-terms and predicate-terms as they are used in syllogistic. Since in this case the expression “included” merely means

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6Thomas Aquinas [2], I, 2, 1
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the usual set-theoretical inclusion (class-inclusion): $S \subseteq P$. Observe that syllogistic is a logic of general terms, i.e., it does not have individual variables.\(^7\)

Furthermore it is not difficult to make sense of the expression “animal is contained in the essence of man”. One only needs to know the theory of the “real definition” according to Aristotle, which was accepted by Aquinas. A real definition describes the essence of (usually a) species in its definiens. Thus in the definiens of man, i.e. rational animal, the term “animal” is contained. This is a kind of syntactical interpretation of the expressions “contained in” or “included in”. But can we also give a semantic interpretation? This should be an interpretation which stands in close connection with the opposite inclusion $S' \subseteq P$, which we may call extensional inclusion, if it is defined in the usual way with the help of elementhood:

$$S \subseteq P \overset{\text{df}}{=} \forall x (x \in S \rightarrow x \in P)$$

Already Aristotle distinguishes two kinds of opposite inclusion: “... in this sense the genus is called part of a species, though in another sense a species is part of its genus.”\(^8\) De Morgan interprets Aristotle in quite a clear way: “the genus is said to be part of the species; but in another point of view the species is part of the genus. ‘All animal’ is in ‘man’, notion in notion: ‘all man’ is in ‘animal’, class in class. In the first, all the notion ‘animal’ part of the notion ‘man’: in the second, all the class ‘man’ part of the class ‘animal’.”\(^9\)

Now it is less difficult to make sense of De Morgan’s reading of Aristotle: “all the notion animal part of the notion man”. We might just say: all men are included in all animals iff all the properties of animals are included in the properties of men. Or: the class man is included in the class animal iff all the superclasses of animal (i.e. those in which the class animal

\(^7\)Aristotle’s Syllogistics is part of the logic of classes, more accurately: part of the logic of virtual classes. See, Hilbert-Ackermann [6], §3 or Bocheński-Menne [4], §27. The stronger existential import of the Aristotelian A-sentence can be handled as an additional premise in the four syllogistic modes darapti, felapton, bamalip and fesapo: The expression, virtual theory of classes has been used by Quine to denote that rudimentary theory of classes which can be developed in First Order Predicate Logic without using any axiom of Set Theory (in the sense of Zermelo-Fraenkel, Neumann-Bernays-Gödel or Quine).

\(^8\)Aristotle [3], V, 25; 1023b17 and 22 - 25.

\(^9\)De Morgan [5], p. 201, note 2.
Paul Weingartner

is contained) are included in all superclasses of man. Applied to subject
\((S)\) and predicate \((P)\) – interpreted syllogistically, i.e. within the theory of
virtual classes – this leads to the following inclusion: the class representing
the subject term is included in the class representing the predicate term
iff all the superclasses of the class representing the predicate term are
included in all the superclasses of the class representing the subject term.
This kind of inclusion between subject and predicate will be denoted by
\(P \leq S\) in contradistinction to \(S \subseteq P\). It can be defined thus:

\[
P \leq S \overset{\text{df}}{=} \forall Z(P \subseteq Z \rightarrow S \subseteq Z)
\]

where the definiens (right part) uses the usual class-inclusion. The kind
of inclusion \(P \leq S\) I have called intensional inclusion. It is easy to prove
the equivalence between \(S \subseteq P\) and \(P \leq S\), i.e., \(S \subseteq P \iff P \leq S\) This
expresses the traditional principle of the equivalence between extensional
and intensional inclusion. If one additionally defines intensional intersec-
tion, one gets a dual theory which can be developed as a dual to the theory
of virtual classes.\(^{10}\)

5 Applying the definition of self-evident to the premises

It should be mentioned first that Thomas Aquinas distinguishes between
self-evident in itself and not to us and self-evident in itself and to us. If
the essence of subject and predicate is known to all, the proposition is self-
evident to all. If this is not the case, the proposition might be self-evident
in itself, but not to us. Subsequently we are dealing with the notion of
self-evident to us.

**Premise (1):** This premise does not have a syllogistical form. But we
might consider some generalisation of premise (1) in general terms. For
example: the human conception of God \((HCG)\) is a \(QM\) in the human
mind \((QMM)\); or: all human conceptions of God are \(QM\) in the human

\(^{10}\)For details and proofs, see [17], II, 1, p. 158ff. and [15]. For a short discussion,
see [20] p. 9ff. For the occurrence of this kind of intension in the history of logic, see
[16]. Observe that this kind of intension is the historical one initiated by Aristotle
and further developed by Thomas Aquinas, Leibniz and De Morgan. It is not Fregean
and non-modal.
that some have understood God as a body. Applying our interpretation of Aquinas’s definition of self-evident leads to the following inclusions:

\[ HCG \subseteq QMM \quad \text{or equivalently} \quad QMM \leq HCG \]

Thus the statement \( HCG \subseteq QMM \) (the human conception of God is a \( QM \) in the human mind, i.e. a \( QMM \)) is self-evident iff \( QMM \) is included in \( HCG \), that is iff all the properties (characteristics) of a \( QM \) existing in the human mind are contained in the properties (characteristics) of the human conception of God. But this is hardly defensible as a general statement since an understanding of God as creator or as the first cause would not satisfy it. The result is this: Although Anselm’s assumption of premise (1) (a certain \( a \) is a \( QMM \) in some concrete individual human person) is (contingently) true and even subjectively evident for the particular person, it cannot be called self-evident according to the definition by Thomas Aquinas.

**Premise (2):** This premise can also be written as the following class inclusion:

\[ QMM \subseteq QM \quad \text{or equivalently} \quad QM \leq QMM \]

Are these inclusions self-evident to us? In order to be so, the properties (or the characteristics) of \( QM \) should be contained in the properties (or in the characteristics) of \( QMM \). At first sight the answer seems to be yes; but on a closer look it is doubtful whether this is the right answer. The answer decisively depends on the question whether the properties of \( QM \) already contain the existence in reality (the actual existence) or not. If they do not, we may say that the inclusion is self-evident to us, too. But as soon as the notion (the properties) of \( QM \) leave it open whether the \( id \ quo \ maius \ cogitare \ non \ potest \) exists only mentally or also actually, we cannot say that the notion (properties) of \( QM \) is (are) contained in the notion of \( QMM \). In this case, these inclusions are not self-evident to us. This problem will become more apparent in the consideration of the third premise.

**Premise (3):** Is this premise self-evident? The third premise is the decisive premise of Anselm’s Ontological Argument. In the version we have chosen above, it says: every \( id \ quo \ maius \ cogitare \ non \ potest \ which \ exists \ only \ mentally, \ but \ not \ actually \ (not \ in \ reality) \ is \ not \ a \ (genuine) \ id \ quo \ maius \ cogitare \ non \ potest \). Because the notion (properties, characteristics) \( id \ quo \ maius \ cogitare \ non \ potest \) requires the inclusion of the
notions (properties, characteristics) of the greatest or the maximum; and something which exists only mentally, but not in reality (actually) cannot be the greatest or the maximum, since what also exists actually would be greater. Explained in this way, premise (3) seems to be highly plausible and perhaps evident to us. For a closer look, let us try to put premise (3) into the appropriate inclusions in order to apply Thomas’s definition. This is not that easy, because of the negations involved. However if we translate $\neg QM(x)$ as the complement $QM$ of $QM$ and similarly with $\neg QMR(x)$: $QMR$, then the inclusions are as follows:

$$QMM \cap QMR \subseteq QM \text{ or equivalently } QM \leq QMM \cap QMR$$

The second inclusion says: The characteristics (properties) of things not satisfying $QM$ (say of imperfect things) are included in the characteristics of things satisfying $QMM$ but not $QMR$, i.e., of most perfect things which exist only in the mind. One can see immediately that this is not true.

We might also try to see this from another angle, namely from an equivalence translation of premise (3) with the help of classical logic, which drops the negations. Premise (3) is classically logically equivalent to the following negation-free proposition:

$$\forall x(QMM(x) \land QM(x) \rightarrow QMR(x))$$

This proposition is self-evident if and only if the notion (characteristics, properties) of $QMR$ is (are) contained in the notions (characteristics, properties) of $QMM$ and $QM$. Now it is easy to see that the notion of $QMR$ cannot be contained in the notion of $QMM$, since the actual existence is contained in the notion of $QMR$, but not in the notion of $QMM$. Therefore the above proposition can only be self-evident if the notion of $QMR$ is contained in the notion of $QM$. But in this case the consequent of the second premise already contains the notion of $QMR$, i.e., it already contains the actual existence. And then the whole argument seems to in fact rest on a petitio principii: the notion of $QM$ (something which the fool understands and has in his mind) already contains the actual existence of the $id quo maius cogitari non potest$.

Therefore, if we save the argument from a petitio principii, it follows that premise (3) is not self-evident, neither self-evident in itself nor self-evident to us. This is also Thomas Aquinas’s answer to the respective objection in which the Ontological Argument by Anselm is used to show that God’s existence would be self-evident.

The respective inclusions of the negation-free proposition are:
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\((QMM \cap QM) \subseteq QMR\) or equivalently \(QMR \leq (QMM \cap QM)\)

Also from the right inclusion one can see that the property of real existence is not included in the properties of the intersection of \(QMM\) and \(QM\); provided that the properties of \(QM\) do not already contain that of real existence. But in the latter case we have a clear *petitio principii*.

**Premise (4):** The fourth premise is an assumption which serves the Indirect Proof. Thus it is an assumption to the contrary. And it serves to derive a contradiction (step (12)) in the argument.

Finally we may summarise Thomas Aquinas’s Commentary on the Ontological Argument of Anselm of Canterbury as follows: The premises of the argument are not self-evident to us. Therefore the conclusion, i.e., the proposition that God exists, is also not self-evident to us.

6 Are the Premises of Anselm’s Argument Analytic in the sense of Kant?

6.1 Analytic in the sense of Kant

It is interesting that Kant uses the same definiens for defining analytic (analytisches Urteil) as Thomas Aquinas uses for defining self-evident (proposition): the predicate is contained in the subject.\(^{11}\) Therefore, for a first interpretation of analytic we might apply our representation of subject-predicate propositions (judgements or Urteile) by the two kinds of inclusions. The result concerning the analyticity of the premises of Anselm’s argument is then analogous to that concerning their self-evidence in the sense of Thomas Aquinas: The premises of Anselm’s argument are not analytic in this sense of analyticity.

\(^{11}\)In German: “das Prädikat \(B\) gehört zum Subjekt \(A\) als etwas, was in diesem Begriff \(A\) (inwerseckter Weise) enthalten ist,” , [11], A6, B10. Or in [10], §2: “Analytic judgements say nothing in the predicate that was not already thought in the concept of the subject, though not so clearly and with the same consciousness.” It is quite likely that Kant knew the definition of self-evident proposition by Thomas Aquinas, since question 2 of the Five Ways was commented on by many scholars in the time of the Enlightenment as well as later on. To cite a forerunner was often obsolete in the time of Enlightenment, especially to cite somebody from the Medieval Times. There are almost no citations in Kant’s writings, although he certainly knew philosophers of the Antiquity and of the 17th century.
There are however other senses of analyticity. Jaakko Hintikka has offered a detailed study on analyticity in four essays which distinguish four main types of analyticity and many subtypes. Many of these types of analyticity are more or less connected to ideas of Kant, which he expresses in a rather loose and imprecise way. Therefore it cannot be the task of this section to investigate all these different senses in reference to the premises of Anselm’s argument. However we shall select some important versions of analyticity and will apply it to the question of the analyticity of the premises in Anselm’s argument.

6.2 Different senses of analyticity

(1) Analytic truths (AT) are true in the sole virtue of the meanings of the terms they contain.\textsuperscript{13}

(a) AT’s are based on the definitions of terms.
(b) AT’s are based on the definitions of terms and their logical consequences.
(c) AT’s can be proved by means of logical laws and definitions. (Frege)
(d) AT’s are truths of logic and all truths which can be reduced to them by substituting synonyms for synonyms. (Quine)

Let us apply these senses of analyticity to the premises of Anselm’s argument.

We immediately see that no premise in Anselm’s argument is based on definitions of the terms. This is so first of all, because Anselm gives us no definitions. Furthermore, premise (1) is contingent and is not based on a definition. Premise (3) (and (2\textsuperscript{'})) cannot be based on definitions either. This also holds for (3\textsuperscript{*}). The only premise of which one could think that it is based on definitions is premise (2). It could be based on a definition like the following: something \( x \) is an id quo maius cogitari non potest existing in the mind (QMM) iff \( x \) is an id quo maius cogitari non potest (QM) and \( x \) exists in the mind. But one has to observe what has been said concerning premise (2) in section 4.3 above: if QM already includes the existence in reality, the above definition would not be correct. If we call an argument analytic, if all of its premises are analytic, then Anselm’s

\textsuperscript{12}Hintikka [7]. Cf. also Hintikka [8] and [9].
\textsuperscript{13}Cf. the list of Hintikka [7], p. 214.
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argument is not analytic according to (1a) since premise (1) and (3) (also (2’) and (3*)) are not analytic in that sense.

The same holds for sense (1b).

What about sense (1c)? Can we prove all or some of the premises with the help of logical laws (plus definitions)? Certainly premise (1) is not a logical law nor can it be derived from a logical law.

If premise (2) has the form “if an A is a B then it is an A” then this seems to be a logical law. Thus also “if a QM exists in the mind, i.e., is a QMM then it is also a QM” seems to be a logical law in this sense. However it has already been pointed out that the concept of a QM is somewhat ambiguous; it leaves open whether it includes the real existence or not (see above). And as long as “QM” is ambiguous in this sense, a proposition in which “QM” occurs cannot be a logical law. It would be a tautology, however, if “QM” meant exactly the same as “QMM”. But this would not fit in Anselm’s text. Thus we cannot unconditionally say that premise (2) is analytic in the sense of (1c).

Does premise (3) follow from a law of logic? Premise (3) logically follows from \( \forall x(QM(x) \rightarrow QMR(x)) \); i.e., it follows according to classical logic from the assumption that the id quo maius cogitari non potest exists in reality. But this assumption would clearly be a petitio principii. It is connected to the ambiguity of the term “QM”. If we do not make this assumption, then premise (3) is not a law of logic and does not follow from one, which consequently means that premise (3) is not analytic in the sense of (1c). Premise (3*) delivers a similar result: Under the assumption that QM already includes the real existence, i.e., if \( \forall x(QM(x) \rightarrow QMR(x)) \) is added to the antecedent of (3*), then the resulting proposition becomes a logical truth. That means that under this assumption, (3*) becomes analytic in the sense of (1c). But as it has been said above, this assumption is a petitio principii. Also premise (2’) cannot be turned into a logical truth under this assumption. Summarising, we may say that without committing a petitio principii, none of the premises are analytic in the sense of (1c).

This result also answers the question of analyticity in Quine’s version (1d): None of the premises of Anselm’s argument seem to be analytic in the sense of (1d).

(2) Analyticity of arguments

(a) An analytic argument cannot lead from the existence of an individual to the existence of a different individual. (Kant)
An argument step is analytic, if it does not introduce new individuals into the discussion.

An argument step is analytic iff it does not increase the number of individuals considered in their relation to each other.

Let us now apply these senses of analyticity to Anselm’s argument and to the argument steps involved.

We shall begin with sense (2a) and ask whether Anselm’s whole argument is analytic in sense (2a). The answer is quite interesting: Anselm’s argument in the intuitionistic version is analytic, but Anselm’s argument in the classical version is not. In the intuitionistic version the argument does not lead from the existence of an individual (say from the particular existence of a \(QM\) in someone’s mind as postulated by the first premise \(QMM(a))\) to the existence of a different individual (say the real existence of \(QM\) as God). But it only leads to the non-absurdity or consistency of the latter. However in the classical version, it in fact leads to the new individual, therefore the argument is not analytic w.r.t. the classical version.

Now we apply senses (2b) and (2c) to the argument steps. Again, under the intuitionistic interpretation of negation, we do not find any step (or any move from one step to the next) which would introduce a new individual or which would increase the number of individuals. Thus also in this case, the steps of the argument are analytic in the sense of (2b) and (2c). However the step from (15) to (16) clearly introduces the (real) existence of an individual, namely of the \(id\ quo\ maius\ cogitari\ non\ potest\). And thus the classical version or the classical completion of Anselm’s argument is a non-analytic step in the sense of (2b) and (2c).

Bibliography


THE PREMISES OF ANSELM’S ARGUMENT


Part VII

Debate
Maydole-Oppy
Maydole on Ontological Arguments

Graham Oppy

Maydole in [1] writes:

“Ontological arguments are captivating. They convince some people but not others. Our purpose here was not to convince, but simply to show that some ontological arguments are sound, do not beg the question, and are insulated from extant parodies. Yet good logic does convince sometimes. Other times, something else is needed.” (p. 586)

On the way I understand these matters, any argument that is both sound and non-question-begging ought to be considered successful (and no argument that is subject to successful parody is sound). Be that as it may, what I propose to do here is to critically examine the ontological arguments of Maydole [1]. I claim that close examination reveals that these arguments are both question-begging and subject to successful parodies – and hence that nothing else is needed to permit non-believers to dismiss these arguments with good conscience.

1 Anselm’s Ontological Argument

Maydole ‘reconstructs’ Anselm’s argument in the following way. We begin with definition of primitive vocabulary:

U(x):  x is understood,
S(x):  the concept of x exists in the understanding,
E(x):  x exists in reality,
G(x,y): x is greater than y,
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F(x,y):  
x refers to y,

D(x):  
x is a definite description,

d:  

\(\text{‘}(\iota x)\neg\exists yG(y, x)\text{’}\),

g:  

\(\text{‘}(\iota x)\neg\exists yG(y, x)\text{’}\),

P(X):  

X is a great-making property,

\(\Box\):  

it is conceivable that.

In standard form, the argument then runs as follows:

(1)  
\(D(d) \land U(d)\)

(2)  
\(F(d, g)\)

(3)  
\(\forall x\forall y(D(x) \land F(x, y) \land U(x) \rightarrow S(y))\)

(4)  
\(\forall x\forall y(P(Y) \land \neg Y(x) \land \Box Y(x) \rightarrow \Box\exists yG(y, x))\)

(5)  
\(P(E)\)

(6)  
\(\forall x(S(x) \rightarrow \Box E(x))\)

(7)  
\(\neg\Box\exists yG(y, g)\)

(8)  
(Hence) \(\neg E(g)\)

It is easy to give a derivation of the conclusion from the premises. I think that Maydole makes slightly heavy weather of this. What we do is show that the hypothesis that \(\neg E(g)\) leads to a contradiction:

(1)  
\(\neg E(g)\)  \hspace{1cm} \text{hypothesis for reductio}

(2)  
\(D(d) \land U(d)\)  \hspace{1cm} \text{premise}

(3)  
\(F(d, g)\)  \hspace{1cm} \text{premise}

(4)  
\(\forall x\forall y(D(x) \land F(x, y) \land U(x) \rightarrow S(y))\)  \hspace{1cm} \text{premise}

(5)  
\(\forall x\forall y(P(Y) \land \neg Y(x) \land \Box Y(x) \rightarrow \Box\exists yG(y, x))\)  \hspace{1cm} \text{premise}

(6)  
\(P(E)\)  \hspace{1cm} \text{premise}

(7)  
\(\forall x(S(x) \rightarrow \Box E(x))\)  \hspace{1cm} \text{premise}

(8)  
\(\neg\Box\exists yG(y, g)\)  \hspace{1cm} \text{premise}

(9)  
\(D(d) \land F(d, g) \land U(d)\)  \hspace{1cm} (2), (3), \text{Conj}
In words, Maydole gives the following standard form to the argument:

1. The definite description ‘that than which it is not conceivable for something to be greater’ is understood.
2. ‘That than which it is not conceivable for something to be greater’ refers to that than which it is not conceivable for something to be greater.
3. The concept of whatever a definite description that is understood refers to has existence in the understanding.
4. It is conceivable that something is greater than anything that lacks a great-making property that it conceivably has.
5. Existence in reality is a great making property.
6. Anything the concept of which has existence in the understanding conceivably has existence in reality.
7. It is not conceivable that something is greater than that than which it is not conceivable for something to be greater.
8. (Hence) That than which it is not conceivable for something to be greater exists in reality.

Maydole explains how he understands the distinction between existence in the understanding and existence in reality.

“Notions, concepts, ideas, thoughts, beliefs, and so on are the kinds of things that do or might have existence in the understanding. Tables, persons, angels, numbers, forces, and so on, and God, are the kinds of things that do or might have existence in reality. We can identify the
realm of things that have existence in the understanding with the totality of mental things that actually exist in minds; and we can identify the realm of things that have existence in reality with the totality of non-mental things that actually exist in the world. It is inconceivable that one and the same thing could have both existence in reality and existence in the understanding.” (pp. 554 - 555).

As Maydole acknowledges, this account of the distinction between existence in the understanding and existence in reality is multiply inconsistent with the text of Proslogion II. Maydole says: “Either Anselm has been mistranslated or he misspoke” (p. 555). I don’t think that we should accept this. Rather, we should suppose that Maydole is offering an argument that he was inspired to put forward by his reading of Proslogion II, even though it is absurd to suppose that this argument is actually developed in that text.

Maydole also tells us that “referring singular terms and definite descriptions are free of existential import, and quantifiers range over possibilia” (p. 555). That is: when we suppose that singular terms and definite descriptions are understood, we do not necessarily suppose that there are things that exist in reality to which those terms and descriptions refer – but we do necessarily suppose that there are things in the domains of some possible worlds to which those terms and descriptions refer.

It is worth reflecting on the consequences of this point. $g$ is defined to be the thing than which it is not conceivable that there is a greater thing. Given Premise 4, it follows from the definition that $g$ has all great making properties that it is conceivable that $g$ possesses. While Maydole admits that he cannot give a theory of greatness and great making properties, it seems plausible to guess that Maydole supposes that it follows from the definition of $g$ and the correct account of greatness that perfect wisdom, perfect power, perfect goodness, completeness, existence in reality, sole creator of the universe ex nihilo, and necessary existence will all be on the list of great making properties that $g$ possesses (since, in each case, he supposes that it is conceivable that $g$ possesses that great making property).

Consider, now, the situation of a Fool who is committed to $S5$, and who is contemplating $g$. By the Fool’s lights, can it be true both that he understands $g$ and that, if he understands $g$, then $g$ refers to a thing in the domain of at least one possible world? Plainly not, if we are supposing that $g$ is perfectly wise, perfectly powerful, perfectly good, necessarily existent and sole creator of the world ex nihilo – for the Fool denies that God exists.
and so, *ipso facto*, denies that it is even possible that there is a perfectly wise, perfectly powerful, perfectly good, necessarily existent, sole creator of the world *ex nihilo*.

The general points to be made are these. For an arbitrary description \( d \), whether one should grant both (i) that one understands \( d \), and (ii) that, if one understands \( d \), then \( d \) refers to a thing in the domain of at least one possible world, cannot be determined while there are features of the interpretation of \( d \) that remain to be nailed down. Moreover, for an arbitrary description \( d \), whether one should grant both (i) that one understands \( d \), and (ii) that, if one understands \( d \), then \( d \) refers to a thing in the domain of at least one possible world, is not a straightforward matter if \( d \) is taken to have modal content. Both of these points bear on Maydole’s \( g \). Unless we have a definite proposal about greatness in mind, it is not clear what we are being asked to countenance when we are asked to countenance ‘*the thing than which it is not conceivable that there is a greater thing*’. And if we are being asked to suppose that necessary existence is a great making property, then no answer to the question whether \( g \) refers can float free of considerations about what one supposes exists in reality. Consider ‘*the actually existent tallest Martian*’. Perhaps there is some sense in which this expression is understood: at the very least, it is surely in no worse standing than \( g \). But, if no Martians exist in reality, then it cannot be that there is a thing in the domain of at least one possible world that is picked out by this description. Or consider ‘*the necessarily existent, necessarily uncaused, necessarily initial, essentially natural state of the universe*’. Again, it seems that there is some sense in which this description is understood; and, again, it seems obvious that it is in no worse standing than \( g \). But, if our world did not begin with an uncaused initial natural state, then it cannot be that there is a thing in the domain of at least one possible world that is picked out by this description.

A natural conclusion to draw at this point is that the framework that Maydole erects for his Anselmian argument is ill-suited to the task of supporting a successful proof of the existence of God. This natural conclusion receives considerable further support when we turn our attention to parallel derivations based on descriptions such as ‘*that island than which it is not conceivable for some island to be greater*’ or ‘*that essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater*’.

Maydole makes heavy weather of his discussion of parodies of Anselm’s argument. The major problem is that he fails to consider the most obvious
version of Gaunilo’s perfect island objection. As before, we begin with
definition of primitive vocabulary:

I(x): \( x \) is an island,

U(x): \( x \) is understood,

S(x): the concept of \( x \) exists in the understanding,

E(x): \( x \) exists in reality,

G(x,y): \( x \) is greater island than \( y \),

F(x,y): \( x \) refers to \( y \),

D(x): \( x \) is a definite description,

d: \( 'I(x) \land \neg \exists y(I(x) \land G(y, x))' \),

g: \( I(x) \land \neg \exists y(I(x) \land G(y, x)) \),

P(X): \( X \) is a great-making property for islands,

\( \Box \): it is conceivable that.

In standard form, the argument then runs as follows:

(1) \( D(j) \land U(j) \)

(2) \( F(j, i) \)

(3) \( \forall x \forall y(D(x) \land F(x, y) \land U(x) \rightarrow S(y)) \)

(4) \( \forall x(I(x) \rightarrow \forall Y(P(Y) \land \neg Y(x) \land \Box Y(x) \rightarrow \Box \exists y(I(y) \land G(y, x)))) \)

(5) \( P(E) \)

(6) \( \forall x(S(x) \rightarrow \Box E(x)) \)

(7) \( \neg \Box \exists y(I(y \land G(y, i)) \)

(8) (Hence) \( \neg E(i) \)

It is a trivial matter to show that the conclusion can be derived from
the premises. Certainly, it is no more difficult than in the case that is
being parodied. We show that the hypothesis that \( \neg E(i) \) leads to contra-
diction. To simply the derivation, we leave out the derivation of \( \text{ii} \) from
the definition of \( i \). Showing this is routine, but tiresome.

(1) \( \neg E(i) \)

\textit{hypothesis for reductio}
(2) \( I(i) \)

(3) \( D(j) \land U(j) \)

(4) \( F(j, i) \)

(5) \( \forall x \forall y (D(x) \land F(x, y) \land U(x) \rightarrow S(y)) \)

(6) \( \forall x (I(x) \rightarrow \forall Y (P(Y) \land \neg Y(x) \land \circ Y(x) \rightarrow \circ \exists y (I(y) \land G(y, x)))) \)

(7) \( P(E) \)

(8) \( \forall x (S(x) \rightarrow \circ E(x)) \)

(9) \( \neg c \circ \exists x (I(x) \land G(x, i)) \)

(10) \( D(d) \land F(d, g) \land U(d) \)

(11) \( D(d) \land F(d, g) \land U(d) \rightarrow S(i) \)

(12) \( S(i) \)

(13) \( S(i) \rightarrow \circ E(i) \)

(14) \( \circ E(i) \)

(15) \( P(E) \land \neg E(i) \land \circ E(i) \)

(16) \( I(i) \rightarrow \forall Y (P(Y) \land \neg Y(i) \land \circ Y(i) \rightarrow \circ \exists y (I(y) \land G(y, i))) \)

(17) \( \forall Y (P(Y) \land \neg Y(g) \land \circ Y(g) \rightarrow \circ \exists y (I(y) \land G(y, i))) \)

(18) \( P(E) \land \neg E(g) \land \circ E(g) \rightarrow \circ \exists y (I(y) \land G(y, i)) \)

(19) \( \circ \exists y (I(y) \land G(y, i)) \)

(20) \( \text{contradiction} \)

In words, we could give the following standard form to the argument:

(1) The definite description ‘that than which it is not conceivable for some island to be greater’ is understood.

(2) ‘That island than which it is not conceivable for some island to be greater’ refers to that island than which it is not conceivable for some island to be greater.

(3) The concept of whatever a definite description that is understood refers to has existence in the understanding.

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(4)  *It is conceivable that some island is greater than any island that lacks a great-making property for islands that it conceivably has.*

(5)  *Existence in reality is a great making property for islands.*

(6)  *Anything the concept of which has existence in the understanding conceivably has existence in reality.*

(7)  *It is not conceivable that some island is greater than that island than which it is not conceivable for some island to be greater.*

(8)  *(Hence) That island than which it is not conceivable for some island to be greater exists in reality.*

Since the argument of the parody is plainly valid if the original argument is valid, all that remains is to consider the premises. As Maydole observes, what needs to be argued is that the premises of the parody are just as plausible (or implausible) as the premises of the original argument.

First, there is no evident reason why we should suppose that the description ‘*that than which it is not conceivable for something to be greater*’ is better understood than the description ‘*that island than which it is not conceivable for some island to be greater*’. Perhaps it might be said that all islands are equal: it makes no sense to suppose that one island is greater than another. However, even if this point were granted, it wouldn’t matter: after all, for the purposes of the parody of the original argument, there is nothing special about islands. We can run exactly the same kind of argument in connection with cities, or horses, or poems, or human beings, or essentially supernaturally unaccompanied beings, or physically embodied creatures, or anything else for which we suppose that there is gradation on the scale of greater and lesser. The last case that I mentioned seems particularly apt. Anselm himself – along with any other medieval proponents of the great chain of being – clearly supposed that some physically embodied creatures are greater than others. But surely he would not have supposed that there is a physically embodied creature than which it is not conceivable for some physically embodied creature to be greater. Not only do physically embodied beings immeasurably greater than us exist out there in the stars, but those aliens are so great that they could not be conceivably greater – and we can demonstrate their existence and superiority purely *a priori*!

Second, there is no evident reason why we should suppose that the description ‘*that than which it is not conceivable for something to be greater*’ refers to that than which it is not conceivable for something to be greater,
even though the description ‘that island than which it is not conceivable for some island to be greater’ fails to refer to that island than which it is not conceivable for some island to be greater. (Hereafter, for ease of exposition, I shall proceed as if it is agreed that some islands are greater than other islands. We have already seen that there is no risk involved in proceeding in this way.)

Third, there is no evident reason why we should suppose that it is conceivable that something is greater than anything that lacks a great-making property that it conceivably has, and yet not conceivable that some island is greater than any island that lacks a great-making property for islands that it conceivably has.

Fourth, there is no evident reason why we should suppose that existence is a great making property (for things in general) and yet existence is not a great making property for islands. How could that be? If, all else being equal, a thing that exists in reality is a greater thing than a merely possible thing, how could it not be that, all else being equal, an island that exists in reality is a greater island than a merely possible island?

Fifth, there is no evident reason why it is not conceivable that something is greater than that than which it is not conceivable for something to be greater, and yet it is conceivable that some island is greater than that island than which it is not conceivable for some island to be greater. Again, how could that be?

Since all of the remaining premises are in common between the two arguments, and since we have failed to find anything that speaks in favour of the original argument over the parody, and since the conclusion of the parody is absurd, we can quite properly conclude that the original argument is no good.

As I noted earlier, Maydole argues at some length that his version of Anselm’s argument is not question begging. I think that he is wrong about this; moreover, I think that I can show that he is wrong about this. Consider the description ‘that essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater’. (A being is ‘supernaturally unaccompanied’ in world w iff there are no supernatural beings – no gods, no spooks, no spirits, etc. – in world w. A being is ‘essentially supernaturally unaccompanied’ iff there are no supernatural beings in any world in which that being exists.)

Consider the following argument (I omit symbolisation and derivation, since these are entirely straightforward):
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1. The definite description ‘that essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater’ is understood.

2. ‘That essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater’ refers to that essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater.

3. The concept of whatever a definite description that is understood refers to has existence in the understanding.

4. It is conceivable that some essentially supernaturally unaccompanied being is greater than any essentially supernaturally unaccompanied being that lacks a great-making property for essentially supernaturally unaccompanied beings that it conceivably has.

5. Existence in reality is a great making property for essentially supernaturally unaccompanied beings.

6. Anything the concept of which has existence in the understanding conceivably has existence in reality.

7. It is not conceivable that some essentially supernaturally unaccompanied being is greater than that essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater.

8. (Hence) That essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater exists in reality.

There are naturalists – myself among them – who hold that naturalism is necessary: there are no possible worlds in which there are supernatural entities. Such naturalists do not deny that it is conceivable that there are supernatural entities: but we deny that conceivability entails alethic or ontic possibility. (Conceivability is more properly considered to be something like doxastic possibility: it is conceivable that \( p \) iff it is alethically possible that there is a reasonable person who believes that \( p \). Of course, reasonable people can have misleading evidence – e.g. false testimony from trusted authorities that leads them to believe claims that are alethically impossible.)
Some theists – e.g. Maydole – suppose that there is a (supernatural) being which lacks spatiotemporal location but which causes the existence of the entire spatiotemporal realm. These theists suppose that this supernatural being exists of necessity, and that, necessarily, any spatiotemporal realm is created by this being.

Some naturalists – e.g. I – suppose that there are no supernatural beings: no causal agents that lack spatiotemporal location. Moreover, these naturalists suppose that this is a matter of necessity: there could not be causal agents that lack spatiotemporal location.

Maydole’s version of ‘Anselm’s argument’ and the above ‘Naturalistic argument’ have contradictory conclusions: it cannot be that they are both sound. However, it is obvious that any member of the class of naturalists described above will prefer the ‘Naturalistic argument’ to ‘Anselm’s argument’. For, while they may not believe that there is an essentially supernaturally unaccompanied being than which it is not conceivable for some essentially supernaturally unaccompanied being to be greater exists in reality, they may also be agnostic about the existence of such a being – whereas they will positively disbelieve that there is a (supernatural) being than which it is not conceivable that a greater (supernatural) being exists in reality.

I take it that to insist – in the absence of any further argument – that ‘Anselm’s argument’ is successful whereas the ‘Naturalistic argument’ is not, or to insist – in the absence of further argument – that the ‘Naturalistic argument’ is successful whereas ‘Anselm’s argument’ is not, would be to engage in blatantly question – begging behaviour. Moreover, and for similar reasons, I take it that to insist in the absence of any further argument – that ‘Anselm’s argument’ is not question-begging whereas the ‘Naturalistic argument’ is question-begging, or to insist – in the absence of further argument – that the ‘Naturalistic argument’ is not question-begging whereas ‘Anselm’s argument’ is question-begging, would equally be to engage in blatantly question-begging behaviour. But this latter is precisely what Maydole commits himself to when he claims that ‘Anselm’s argument’ is sound and not question-begging.

Maydole criticizes Rowe for saying that “the assumption that a greatest possible being possibly exists in reality is virtually equivalent to the proposition that it actually exists in reality” (p. 562). Rowe’s claim is made as a criticism of the following argument:

(1) *Some possible object exemplifies the concept of God.*
(2) No object that fails to exist in reality could exemplify the concept of God. (Because God is defined as a being than which none greater is possible, and it is assumed that the property of existence in reality is great making.)

(3) Every possible object either exists in reality or it does not.

(4) (Therefore) God exemplifies the property of existence in reality.

I think that Maydole misses Rowe’s point. Given that the argument (1), (2) and (3) therefore (4) is valid, the argument not-(4), (2) and (3) therefore not-(1) is also valid. If we grant (2) and (3) for the sake of further discussion, then we need to have a further reason to prefer one of these arguments to the other. In the absence of any further reason, it is simply question-begging to assert that one of them is successful. Of course, as the pair of arguments makes clear, any reasonable person who accepts (2) and (3) and who supposes that some possible object exemplifies the concept of God will also suppose that God exemplifies the property of existence in reality; and any reasonable person who accepts (2) and (3) and who supposes that God does not exemplify the property of existence in reality will also suppose that there is no possible object that exemplifies the concept of God. But it is a commonplace that those who do not believe in a necessarily existent God do not suppose that such a God is even possible (at least given the unstated background acceptance of $S5$). Given $S5$ and (2) and (3), the proposition that a greatest possible being possibly exists in reality swings rationally with the proposition that a greatest possible being actually exists in reality (this being what Rowe had in mind with his talk of ‘virtual equivalence’); but that’s not enough to justify the claim that the argument that Rowe criticises is a successful ontological argument.

2 The Descartes-Leibniz Ontological Argument

According to Maydole, ‘the Descartes-Leibniz ontological argument’ runs as follows:

**Lemma 1:**

(1) If any two perfections are compatible, then all perfections are compatible.

(2) If any two perfections are incompatible, then they are necessarily incompatible.

(3) If any two perfections are necessarily incompatible, then it is either self-evident that they are incompatible or it can be demonstrated that they are incompatible.
(4) It is not self-evident that any two perfections are incompatible.

(5) If it can be demonstrated that any two perfections are incompatible, then either one is the negation of the other, or some part of the one is incompatible with the other.

(6) If one perfection is the negation of the other, then one of them is not positive.

(7) Perfections are simple positive qualities.

(8) If some part of a perfection is incompatible with another perfection, then one of them is not simple.

(9) (Therefore) All perfections are compatible.

Lemma 2:

(1) All perfections are compatible.

(2) Every essential property of a supremely perfect being is a perfection.

(3) If something’s essential properties are perfections and all perfections are compatible, then its essential properties are compatible.

(4) If the essential properties of something are compatible, then it is possible that it exists.

(5) (Therefore) It is possible that a supremely perfect being exists.

Theorem:

(1) For every X and Y, if the property of being a Y is contained in the concept or essence of being an X, then necessarily everything that is an X is a Y.

(2) The property of necessarily existing if existing at all is contained in the concept or essence of a supremely perfect being.

(3) It is possible that a supremely perfect being exists.

(4) Necessarily, supremely perfect beings are necessarily supremely perfect.

(5) (Therefore) A supremely perfect being exists.

Maydole claims that this argument is valid, non question-begging, not susceptible of parody, and not refuted by the antian objection that it treats existence as a real predicate. Moreover, he indicates that he believes that this argument is sound.

I think that ‘the Descartes-Leibniz ontological argument’ is susceptible of parody – though not the kinds of parodies that Maydole considers – and that these parodies suffice to show that ‘the Descartes-Leibniz ontological argument’ is unsuccessful.
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‘The Descartes-Leibniz ontological argument’ is implicitly committed to the claim that perfections form a totality (over which it is possible to quantify). For ease of exposition, I shall suppose that this totality is a set – but nothing would be lost if it were supposed that the totality is instead a proper class, or some other less set-like entity.

Call the set of perfections $P$. Suppose that $\{p_i\}$ is a set of logically independent members of $P$ that collectively entail $P$. Let $\{p_1\}$ be the property of existing necessarily if existing at all.

Let $\{q_i\}$ be a proper subset of $\{p_i\}$ that includes $p_1$. Call by $Q$ the set entailed by $\{q_i\}$. Maydole tells us that perfections entail only perfections. If that is correct, then $Q$ is a proper subset of $P$.

Call the members of $Q$, $Q$-perfections. Say that $x$ is supremely $Q$-perfect iff $x$ has all and only $Q$-perfections essentially.

We can then argue as follows:

**Lemma 1:**

1. If any two $Q$-perfections are compatible, then all $Q$-perfections are compatible.
2. If any two $Q$-perfections are incompatible, then they are necessarily incompatible.
3. If any two $Q$-perfections are necessarily incompatible, then it is either self-evident that they are incompatible or it can be demonstrated that they are incompatible.
4. It is not self-evident that any two $Q$-perfections are incompatible.
5. If it can be demonstrated that any two $Q$-perfections are incompatible, then either one is the negation of the other, or some part of the one is incompatible with the other.
6. If one $Q$-perfection is the negation of the other, then one of them is not positive.
7. $Q$-perfections are simple positive qualities.
8. If some part of a $Q$-perfection is incompatible with another $Q$-perfection, then one of them is not simple.
9. (Therefore) All $Q$-perfections are compatible.

**Lemma 2:**

1. All $Q$-perfections are compatible.
2. All $Q$-perfections are compatible.
3. If something’s essential properties are $Q$-perfections and all $Q$-perfections are compatible, then its essential properties are compatible.
(4) If the essential properties of something are compatible, then it is possible that it exists.

(5) (Therefore) It is possible that a supremely Q-perfect being exists.

Theorem:

(1) For every X and Y, if the property of being a Y is contained in the concept or essence of being an X, then necessarily everything that is an X is a Y.

(2) The property of necessarily existing if existing at all is contained in the concept or essence of a supremely Q-perfect being.

(3) It is possible that a supremely Q-perfect being exists.

(4) Necessarily, supremely Q-perfect beings are necessarily supremely Q-perfect.

(5) (Therefore) A supremely Q-perfect being exists.

It is, I think, obvious to inspection that this argument for the existence of a supremely Q-perfect being is valid just in case ‘the Descartes-Leibniz ontological argument’ for the existence of a supremely perfect being is valid. Moreover, it is also, I think, obvious to inspection, that there is no premise in the argument for the existence of a supremely Q-perfect being that is less acceptable than the corresponding premise in the argument for the existence of a supremely perfect being. (It is obvious that, if all perfections are compatible, then all Q-perfections are compatible, since all Q-perfections are perfections. It follows immediately from the definition of supreme Q-perfection that every essential property of a supremely Q-perfect being is a Q-perfection. Premises (3) and (4) in Lemma 2 evidently stand or fall with the corresponding premises in ‘the Descartes-Leibniz ontological argument’. And premises (2) and (4) in the Theorem evidently stand or fall with the corresponding premises in ‘the Descartes-Leibniz ontological argument’.)

Suppose that you insist that ‘the Descartes-Leibniz ontological argument’ is successful. Suppose, further, that there are N + 1 members of P. Then, by the above considerations, there are 2^N distinct arguments, each of which is successful, and each of which demonstrates the existence of a distinct necessarily existent being that possesses essentially all and only some distinct subset of the properties possessed essentially by a supremely perfect being. If you suppose that, say, omnipotence, omniscience, perfect goodness, and perfect freedom are logically independent perfections, then this result is clearly heterodox (to say the least). But if you suppose that being the sole creator of everything else is also a perfection, then the result is simply logical incoherence.

Perhaps we might think to avoid these problems by insisting that there are only one or two logically distinct perfections. (Either there is just the perfection of existing
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necessarily if existing at all, or there is this perfection and one more that is logically
distinct from it.) I’m inclined to think that this would be grasping at straws. At
the very least, it would be passing strange to suppose that one needs to prove that
perfections are compatible if one also supposes that there is only one perfection (and
hardly less strange to suppose that something like the ‘Leibniz’ Lemma (Lemma 1) is
needed if there are only two perfections)!

Perhaps we might think to avoid the problems raised two paragraphs back by insist-
ing on further assumptions about perfections that play no role in ‘the Descartes-Leibniz
ontological argument’, but that would undermine the construction of the argument
from \( Q \)-perfections. Suppose, for example, that it is insisted that the property of hav-
ing all perfections is itself a perfection. Since the property of having all perfections
cannot be a \( Q \)-perfection, it is clear that some amendment to the above account will
be required. However, I do not think that the required amendment is hard to find.

We start in the same fashion as before. Call the set of perfections \( P \). Suppose that
\( \{p_i\} \) is a set of logically independent members of \( P \) that collectively entail \( P \). Let
\( \{p_1\} \) be the property of existing necessarily if existing at all.

However, at this point, we need to draw a distinction between two different kinds
of properties that belong to \( P \). On the one hand, there are basic properties: prop-
erties that are properly defined without making any mention of perfection. On the
other hand, there are non-basic properties that are properly defined only in terms of
perfection – e.g. the property of possessing all of the perfections.

We now define \( \{q_i\} \) in the following way. First, \( \{q_i\} \) contains \( p_1 \). Second, \( \{q_i\} \)
contains a proper subset of the basic properties that are contained in \( \{p_i\} \). Third, for each
non-basic property in \( \{p_i\} \), \( \{q_i\} \) contains a corresponding proper definition in which ‘perfection’ is everywhere replaced with ‘\( Q \)-perfection’.
(So, for example, \( \{q_i\} \) contains the property of possessing all of the \( Q \)-perfections.)

Given that there are \( N + 1 \) basic members of \( P \), there will be \( 2^N \) distinct arguments,
each of which purports to demonstrate the existence of a distinct necessarily exis-
tent being that possesses essentially all and only some distinct subset of the basic
properties possessed essentially by a supremely perfect being.

Perhaps it might be said that there is something questionable about the impredica-
tivity of the construction of the non-basic \( Q \)-perfections. But, of course, this impred-
icativity was introduced in the suggestion that the property of having all perfections
is itself a perfection. If impredicativity is objectionable, then the original version of
the parody stands; if impredicativity is not objectionable, then the revised version of
the parody cannot be faulted on these grounds. And, I think, it is very hard to see
any other grounds on which the revised version of the parody could be faulted.
3 The Modal Perfection Argument

‘The Modal Perfection Argument’ may be presented in the following way (I have made a few minor alterations to the presentation that Maydole actually gives):

We begin with some definitions. We say that \( x \) is supreme just in case, necessarily, \( x \) is greater than anything that is distinct from \( x \): \( \Box \forall y (x \neq y \rightarrow G(x, y)) \). We say that \( F \) is a perfection just in case, necessarily, \( F \) is a property that it is better to have than not to have. We say that a property \( F \) entails a property \( G \) just in case, necessarily, \( anything \ that \ is \ F \ is \ G \): \( \Box \forall x (F(x) \rightarrow G(x)) \). (We could also say that a set of properties \( \Delta \) entails a property \( F \) just in case, necessarily, \( anything \ that \ possesses \ all \ of \ the \ properties \ in \ \Delta \ is \ F \). But we won’t need this extension for our derivation.)

We rely on the following simple lemma:

**S-Lemma:** If it is not possible that \( F \) is instantiated – i.e. if it is not possible that something is \( F \) – then \( F \) entails every property (including, in particular, \( \neg F \)).

‘The Modal Perfection Argument’ has the following three premises:

1. A property is a perfection only if its negation is not a perfection.
2. Perfections entail only perfections.
3. The property of being supreme is a perfection.

We can formalize the derivation of the argument as follows:

1. \( \neg \Diamond \exists x S(x) \) hypothesis for reductio
2. \( \forall F (P(F) \rightarrow \neg P(\neg F)) \) premise
3. \( \forall F (P(F) \land E(F, G) \rightarrow P(G)) \) premise
4. \( P(S) \) premise
5. \( E(S, \neg S) \) \( (1), S\text{-Lemma} \)
6. \( \forall F (P(F) \land E(S, \neg S) \rightarrow P(\neg S)) \) \( (3), UI \)
7. \( P(S) \land E(S, \neg S) \) \( (4), (5), Conj \)
8. \( P(\neg S) \) \( (6), (7), MP \)
9. \( P(S) \rightarrow \neg P(\neg S) \) \( (2), UI \)
10. \( \neg P(\neg S) \) \( (4), (9), MP \)
11. \( contradiction \) \( (8), (10), Conj \)
12. \( \Diamond \exists x S(x) \) \( (1), (11) \)
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(13) $\diamond \exists x S(x) \rightarrow \exists x \diamond S(x)$  \hspace{1cm} \text{Barcan Formula}

(14) $\exists x \diamond S(x)$ \hspace{1cm} (12), (13), MP

(15) $\exists x \square \forall y (x \neq y \rightarrow G(x,y))$ \hspace{1cm} (14), \text{definition of } S

(16) $\exists x \square \forall y (x \neq y \rightarrow G(x,y))$ \hspace{1cm} (15), \text{rules for S5}

(17) $\exists x S(x)$ \hspace{1cm} (16), \text{definition of } S

Maydole notes that the Barcan Formula is controversial. However, he goes on to claim that the argument is valid, possessed of plausible premises, resistant to ‘sundry salient parodies’ and not question-begging. In particular, Maydole says that “we can easily show that an Oppy-Style parody based on the idea of being almost supreme does not refute the Modal Perfection Argument” (p. 581).

I think that Maydole’s ‘The Modal Perfection Argument’ is susceptible to a Supreme Island parody. The simplest way to develop the argument is to interpret the above formalisation of the argument over the domain of possible islands, reading ‘$G(x,y)$’ as ‘$x$ is a greater island than $y$’, ‘$S(x)$’ as ‘necessarily, $x$ is a greater island than any island distinct from $x$’ (‘$x$ is island-supreme’), and ‘$P(F)$’ as ‘necessarily, $F$ is a property that it is better for an island to have than not to have’ (‘$F$ is an island-perfection’).

The premises of the Supreme Island parody are then as follows:

(1) A property is an island-perfection only if its negation is not an island-perfection.

(2) Island-perfections entail only island-perfections.

(3) The property of being island-supreme is an island-perfection.

Following Maydole, we might think to justify these premises in the following terms:

(1) is true because it is better for an island to have a property than not only if it is not better for that island to not have that property than not; (2) is true because it is always better for an island to have that which is a necessary condition for whatever it is better for that island to have than not; and (3) is true because it is reasonable to assume that an island is island-supreme iff it is necessarily greater than every other island solely by virtue of having some set of island-perfections, making the extension of the property of being island-supreme identical with the intersection of the extensions of those island-perfections. (Cf., p. 581.)

Clearly, we should not accept the conclusion of the Supreme Island parody. However, the notion of island-perfection seems innocuous. (As before, if you think that there are no island-perfections, then we can just change the subject, and talk about cities, or horses, or poems, or human beings, or
essentially supernaturally unaccompanied beings, or physically embodied creatures, or anything else for which we suppose that there is gradation on the scale of better and worse, but for which we also suppose that it is evident that there is no supreme exemplar.) And if the notion of island-perfection is innocuous, then the first two premises seem unproblematic. So, I think, the problem must lie with the third premise: the claim that the property of being island-supreme is an island-perfection.

We might reflect in the following way. Given the definition of ‘island-supremacy’, if there is nothing that has that property, then it is impossible for there to be anything that has that property. But if it is impossible for anything to have that property, then that property entails all other properties. And yet our first two premises guarantee that no property that is an island-perfection entails all other properties. So whether or not we deem the third premise acceptable clearly depends upon whether or not we suppose that there is some island that possesses the property of being island-supreme. Given that we have good reason to suppose that there is no island that possesses the property of being island-supreme, we also have good reason to deny that third premise (its superficial attractiveness notwithstanding).

But now return to Maydole’s Modal Perfection Argument. By the same kind of reasoning as that set out in the previous paragraph, whether or not we deem the third premise acceptable clearly depends upon whether or not we suppose that there is something that possesses the property of being supreme. If we properly take ourselves to have good reasons to deny that there is something that is supreme, then we shall also properly take ourselves to have good reasons to deny that the property of being supreme is a perfection. People who see things this way could argue as follows:

(1) A property is a perfection only if its negation is not a perfection.

(2) Perfections entail only perfections.

(3) There is no supreme being.

(4) (Hence) Supremacy is not a perfection.

Is there a non-question-begging reason to prefer Maydole’s Modal Perfection Argument to this alternative argument? I don’t think so. Certainly, Maydole has not provided us with one. Look closely at his attempt to justify the claim that supremacy is a perfection. He says that ‘it is reasonable to suppose that something is supreme iff it is necessarily greater than everything else solely by virtue of having some set of perfections, making
the extension of the property of being supreme identical with the intersection of the extensions of those perfections’. While this might sound innocuous, the key point is that the way in which the word ‘necessarily’ is interpreted in Maydole’s claim builds in the requirement that necessary existence is one of the requirements of supremacy. (Something is supreme only if, in each possible world, that thing is greater than every other thing in that world – hence, only if that thing exists in every possible world.) A genuinely innocuous interpretation of Maydole’s claim would build in no more than a cross-world comparison: \( x \) is supreme in world \( w \) iff, for every possible world \( w' \), for every possible thing \( x' \), if \( x \) in \( w \) is distinct from \( x' \) in \( w' \), then \( x \) in \( w \) is greater than \( x' \) in \( w' \). But this innocuous claim is simply insufficient to yield Maydole’s desired conclusion.

For all that Maydole says – in the article presently under examination and elsewhere – he has provided no non-question-begging reason to suppose that supremacy is a perfection. Indeed, for all that Maydole says – in the article presently under examination and elsewhere – he has provided no non-question-begging reason to prefer the hypothesis that supremacy is a perfection to the hypothesis that island-supremacy is an island-perfection. He may think it obvious that it is not the case that island supremacy is an island-perfection; but many of us suppose that it is no less obvious that it is not the case that supremacy is a perfection (given that supremacy is taken to entail necessary existence).

4 The Temporal-Contingency Argument

Maydole claims that ‘the Temporal-Contingency Argument’ is “a quasi-ontological argument that is arguably sound” (p. 586). In Maydole’s eyes, ‘There is ... no evidence to indicate that it begs the question. And it seems that it would be particularly resistant to being parodied, given its dependence on sundry logically contingent facts about a possible world, and the historical absence of any parodies against Third Way arguments.” (p. 586).

Maydole gives the following verbal formulation of the argument. We begin by numbering a list of propositions as follows:

1. Something presently exists.
2. Only finitely many things have existed to date.
3. Every temporally contingent being begins to exist at some time and ceases to exist at some time.
Everything that begins to exist at some time and ceases to exist at some time exists for a finite period of time.

If everything exists for only a finite period of time, and there have been only finitely many beings to date, then there was a time when nothing existed.

If there was a time when nothing existed, then nothing presently exists.

A being is temporally necessary iff it is not temporally contingent.

Everything has a sufficient reason for its existence.

Anything that has a sufficient reason for its existence also has a sufficient reason for its existence than is a sufficient reason for its own existence.

No temporally contingent being is a sufficient reason for the existence of a temporally necessary being.

Every temporally necessary being that is a sufficient reason for its own existence is a being without any limitations.

A being without any limitations is necessarily greater than any other being.

It is not possible for anything to be greater than itself.

It is necessarily the case that ‘greater than’ is asymmetric.

We then argue:

Possibly ((1) and (2) and ... and (13) and (14)).

Hence There exists a supreme being.

Obviously, there are three parts to the argument. The first stage of the argument claims that the conjunction of (1) - (14) entails: There exists a supreme being. The second stage of the argument consists in the observation that it then follows from Possibly (the conjunction of (1) - (14) that Possibly (There exists a supreme being). And the third stage of the argument consists in the observation that Possibly (There exists a supreme being) entails There exists a supreme being.

In order to think about this argument, let us contrast two different theories, which I shall call ‘Theism’ and ‘Naturalism’. Of course, there are many theories that might properly be called ‘Theism’, and there are many theories that might properly be called ‘Naturalism’. I don’t pretend that
the theories that are offered are the best deservers for these names; however, they are theories that are good deservers of these names.

Theism says: All alethically (ontically) possible worlds have an initial causal state that involves the same necessarily existent entity: God. It is controversial whether all possible worlds have the same initial causal state, because it is controversial whether there is variation in the properties possessed by God in that initial state across possible worlds. If all possible worlds have the same initial state, then, if there is more than one possible world, this is because chance plays a role in the evolution of state. If not all possible worlds have the same initial state, this is because there are brute-inexplicable-differences in the properties possessed by God in the initial state (e.g. brute differences in God’s creative intentions, or God’s preferences, or the like). According to Theism, God is the necessarily existent, essentially omnipotent, essentially omniscient, essentially perfectly good sole creator ex nihilo of the natural universe.

Naturalism says: All alethically (ontically) possible worlds have an initial causal state that involves the same necessarily existent natural entity: ‘the initial singularity’. It is controversial whether all possible worlds have the same initial causal state, because it is controversial whether there is variation in the properties possessed by the initial singularity in that initial state across possible worlds. If all possible worlds have the same initial state, then, if there is more than one possible world, this is because chance plays a role in the evolution of state. If not all possible worlds have the same initial state, this is because there are brute-inexplicable-differences in the properties possessed by the initial singularity in the initial state (e.g. brute differences in the values of natural constants, or natural boundary conditions, or the like). According to Naturalism, the initial singularity is the necessarily existent, natural causal origin of the natural universe (and there is nothing supernatural in any possible world).

According to Naturalism, there are some propositions on Maydole’s list that are not alethically possible. In particular, given that time commences with the initial state of the natural universe, it is not even possibly true that, if everything exists for only a finite period of time, and there have been only finitely many beings to date, then there was a time when nothing existed. Moreover, given the plausible claim that it is necessary that the physical universe is not a being that is without limitations, it is not even possibly true that every temporally necessary being that is a sufficient
reason for its own existence is a being without any limitations. (For this second point, I assume that anything whose existence is alethically necessary is a sufficient reason for its own existence.) Given these points, it follows that, according to Naturalism, the premise of Maydole’s argument is false.

I claim that the considerations just rehearsed establish that Maydole’s argument is question-begging. It cannot be that both Theism and Naturalism are true. However, to insist that the premise of Maydole’s argument is true is, *ipso facto*, to insist that Naturalism is false. But no successful argument for Theism (against Naturalism) requires the assumption that Naturalism is false: any argument that requires the assumption that Naturalism is false simply begs the question against Naturalism.

Of course, there are many naturalists who do not accept Naturalism. However, naturalists who do not accept Naturalism will reject other assumptions that are common to both Theism and Naturalism. Perhaps, for example, a naturalist might suppose that causal reality has a contingent origin. Or perhaps a naturalist might suppose that causal reality involves an infinite regress. Or perhaps a naturalist might be sceptical about causality, or modality, or sufficient reason. And so forth. The significant point upon which I want to insist here is just this: a naturalist who grants as much as possible to Theism can still reasonably prefer Naturalism to Theism – or, at any rate, Maydole’s argument does nothing at all towards establishing that this is not the case.

Perhaps it is worth observing that it is highly contentious to describe ‘the Temporal-Contingency Argument’ as ‘quasi-ontological’. The considerations that are relevant to its assessment seem to me to be vastly different from the considerations that are relevant to the assessment of ‘Anselm’s Ontological Argument’, ‘the Ontological Argument of Descartes and Leibniz’, ‘Ontological Arguments of the Twentieth Century’, ‘Gödel’s Ontological Argument’, and ‘the Modal Perfection Argument’. True enough, the final of the three stages of the argument – considered on its own – is a reasonably good deserver of the label ‘ontological argument’. But it seems to me that the first two stages of the argument are something altogether different.

5 Conclusion

I agree with Maydole that ontological arguments are interesting. But that is pretty much where our agreement ends. I think that ontological argu-
ments convince almost no one – and quite properly so: extant ontological
arguments should not convince anyone. I have not yet seen an ontolo-
gical argument that is (1) valid; (2) resistant to successful parody; and
(3) non-question-begging. Certainly, the argument set out in Maydole [1],
while satisfying (1), all fail to satisfy both (2) and (3). Of course – as
the preceding discussion shows – it is very important to match ontological
arguments to appropriate parodies: there are different kinds of parodies
that are appropriate to different kinds of ontological arguments. It is an
interesting feature of the discussion in Maydole [1] that – at least by my
lights – in almost every case he fails to consider the kind of parody that
is most threatening to whichever ontological argument is the immediate
focus of his discussion.

Bibliography

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Ontological Arguments Redux

ROBERT E. MAYDOLE

Oppy claims in “Maydole on Ontological Arguments” [3] that four of the ontological arguments that I support in “The Ontological Argument” [2] “are question-begging and subject to successful parodies” (p. 445). Even though he is technically correct about my rendition of Anselm’s Ontological Argument (AOA), it is easily repairable. He is completely wrong about my Descartes-Leibniz Argument (DLA), my Modal Perfection Argument (MPA), and my Temporal Contingency Argument (TCA).

1 Anselm’s Argument

AOA is:

(1) *The definite description ‘that being than which it is not conceivable for some being to be greater’ is understood.*

(2) ‘That being than which it is not conceivable for some being to be greater’ refers to that being than which it is not conceivable for some being to be greater.

(3) *The concept of whatever a definite description that is understood refers to has existence in the understanding.*

(4) *It is conceivable that some being is greater than any being that lacks a great-making property that it conceivably has.*

(5) *It is conceivable that some being is greater than any being that lacks a great-making property that it conceivably has.*
Ontological Arguments Redux

(6) Any being the concept of which has existence in the understanding conceivably has existence in reality.

(7) It is not conceivable that some being is greater than that than which it is not conceivable for some being to be greater.

Therefore,

That being than which it is not conceivable for some being to be greater exists in reality.

Oppy intimates that AOA begs the question because I allow the quantifiers to range over possibilia (all things that either might or do exist). That means, according to Oppy, that Anselm’s Fool would have to grant the existence of God if she/he accepted both modal logic $S5$ and the belief that God must have necessary existence. And it also means, he says, that if necessary existence is a great making property, then God cannot even refer or be understood (if understanding the term “God” implies that it refers) unless God exists in reality (pp. 448 - 449).

Reply. First, there is no reason to think that AOA involves or presupposes $S5$; and it is a straw man to charge that AOA begs the question on that score. Second, to say that a thing has the property of necessary existence could be construed to mean either that it necessarily has the property of existence ($\Box E$), or that it has the conditional property of existing necessarily if at all ($E \rightarrow \Box E$). We would, of course, beg the question of God’s existence were we to include ‘$\Box E$’ in our definition of the term ‘God’. But we would not beg that question were we to include only ‘$E \rightarrow \Box E$’ in the definition and then go on to prove that God exists. Actually, neither ‘$\Box E$’ nor ‘$E \rightarrow \Box E$’ is included in the meaning of the term ‘God’ as it is used in AOA. ‘God’ there means only ‘that than which nothing greater can be conceived’. Anything else that we justifiably say about God must be inferred, including the proposition that God necessarily exists in reality.

Parodies of AOA

A parody refutes a valid argument just in case it has the same valid structure, an absurdly false conclusion, and premises that are at least as justifiable as those of argument parodied, insuring thereby that the argument parodied has an unjustifiable premise and cannot be sound. Oppy constructs two parodies that successfully refute AOA. The first is close in spirit and content to Gaunilo’s parody of Anselm’s original ontological
argument. I’ll call it the ‘Perfect Island Parody’ (‘PIP’). PIP results from substituting ‘island’ for ‘being’ and ‘property for islands’ for ‘property’ in AOA. The second parody, the ‘Essentially Supernaturally Unaccompanied Parody’ (‘SUP’) is just like the PIP except for having ‘essentially supernaturally unaccompanied being’ in place of ‘island’. As Oppy defines it “a being is essentially supernaturally unaccompanied if and only if there are no supernatural beings in any [possible] world in which that being exists” (p. 453).

I wrongly stated in “The Ontological Argument” [2] that the fourth premise of AOA is intuitively obvious. I failed to account there for the fact that non-great making essential properties can counteract the effects of great making essential properties. If all of the essential properties of $\Phi$ and $\Theta$ are great making and the essential properties of $\Phi$ is a proper subset of $\Theta$, then $\Theta$ is greater than $\Phi$. But if some of the essential properties of $\Theta$ are not great making, then $\Theta$ might not be greater than $\Phi$.

Let the argument $\text{AOA}^*$ be obtained from AOA by substituting $4^*$ for its fourth premise, and adding $8^*$ as an additional premise:

(4*) It is conceivable that some being is greater than any being that lacks a great-making property that it conceivably has, and that only has great making properties essentially.

(8*) All essential properties of that than which nothing greater can be conceived are great making properties.

And let the argument $\text{PIP}^\sharp$ be obtained from the PIP by substituting $4^\sharp$ and $5^\sharp$ for its fourth and fifth premises, respectively, and adding $8^\sharp$ as an additional premise:

(4$^\sharp$) It is conceivable that some island is greater than any island that lacks a great-making property for islands that it conceivably has, and that only has great making properties for islands essentially.

(5$^\sharp$) Existence in reality is a great making property for islands.

(8$^\sharp$) All essential properties of that island than which nothing greater can be conceived are great making properties for islands.

Likewise, let the argument $\text{SUP}^\sharp$ be obtained from the SUP by substituting $4^\sharp$ and $5^\sharp$ for its fourth and fifth premises, respectively, and adding $8^\sharp$ as an additional premise:
(4²) It is conceivable that some essentially supernaturally unaccompanied being is greater than any essentially supernaturally unaccompanied being that lacks a great-making property for essentially supernaturally unaccompanied beings that it conceivably has, and that only has great making properties for essentially supernaturally unaccompanied beings essentially.

(5²) Existence in reality is a great making property for essentially supernaturally unaccompanied beings.

(8²) All essential properties of that essentially supernaturally unaccompanied being than which nothing greater can be conceived are great making properties for essentially supernaturally unaccompanied beings.

AOA⁺, the PIP⁺, and the SUP⁺ are valid. Moreover, I think that 4⁺, 4⁺, and 4⁺ are true; and, at the very least, that they stand or fall together. The same holds for 5⁺, 5⁺, and 5⁺. The additional premise, 8⁺ of AOA⁺ is also true. But the additional premises, 8⁺ and 8⁺ of PIP⁺ and the SUP⁺, respectively, are false; for, all islands and all essentially supernaturally unaccompanied beings, howsoever great, have the property of being material, which is hardly a great making property for islands or essentially supernaturally unaccompanied beings, respectively. Islands and essentially supernaturally unaccompanied beings are essentially limited by their very nature of being islands or essentially supernaturally unaccompanied beings, respectively, and they thereby have some properties essentially that could not even be construed as great making properties for islands or essentially supernaturally unaccompanied beings, respectively. We can conclude that neither the PIP⁺ nor the SUP⁺ refute AOA⁺.

2 The Descartes-Leibniz Argument

The DLA is:

(1) For every X and Y, if the property of being a Y is contained in the concept or essence of being an X, then necessarily everything that is an X is a Y.

(2) The property of necessarily existing if existing at all is contained in the concept or essence of a supremely perfect being.

(3) It is possible that a supremely perfect being exists.

(4) Necessarily, supremely perfect beings are necessarily supremely perfect.
Therefore,

*A supremely perfect being exists.*

While Oppy agrees that this argument is valid, he attempts to refute it with a parody argument about supremely Q-perfect beings. According to Oppy, a *Q-perfection* is any member of a set of properties that are entailed by the members of a proper subset of the set of all logically independent perfections that collectively entail all perfections, and includes the property of existing necessarily if at all (N) (p. 460). So Q-perfections are properties of properties. Suppose, for example, that there were only three perfections plus N: omniscience (O₁), omnipotence (O₂) and omnibenevolence (O₃). Then the respective entailments of the following sets would be extensions of different Q-perfections: {N}, {N, O₁}, {N, O₂}, {N, O₃}, {N, O₁, O₂}, {N, O₁, O₃}, {N, O₂, O₃}. But the entailments of {N, O₁, O₂, O₃} would not be the extension of a Q-perfection.

Oppy defines a *supremely Q-perfect* being as one that has all and only Q-perfections essentially. For example, if Q₁ is the property with an extension which is the entailment of {N, O₁, O₂}, then a being is supremely Q₁-perfect if and only if it has all and only the members of the entailment of {N, O₁, O₂}, essentially, i.e., □N, □O₁, □O₂, □(O₁∧O₂), etc. He then constructs the following parody argument of the DLA:

1. *For every X and Y, if the property of being a Y is contained in the concept or essence of being an X, then necessarily everything that is an X is a Y.*

2. *The property of necessarily existing if existing at all is contained in the concept or essence of a supremely Q-perfect being.*

3. *It is possible that a supremely Q-perfect being exists.*

4. *Necessarily, supremely Q-perfect beings are necessarily supremely Q-perfect.*

Therefore,

*A supremely Q-perfect being exists* (p. 459).

This parody argument fails to refute the DLA. I agree with Oppy that both arguments are valid, and the first three premises of each stand or fall together. Not the corresponding fourth premises. Consider first the following sound argument that proves that this parody argument has a false/absurd conclusion and cannot therefore be sound:
Ontological Arguments Redux

(1) Every essential property of a supremely Q-perfect being is a Q-perfection.

(2) Some perfection is not a Q-perfection.

(3) Every Q-perfection is a perfection.

(4) The negation of the necessitation of a perfection is not a perfection.

Therefore,

Nothing is supremely Q-perfect.

Let ‘R∗’ name the property of being supremely Q-perfect, and let ‘Q’ name the property of being a Q-perfection.

The following deduction shows that this argument is valid:

(1) ∀x∀Y (R∗(x) ∧ □Y(x) → Q(Y))          premise
(2) ∃Y (P(Y) ∧ ¬Q(Y))                   premise
(3) ∀Y (Q(Y) → P(Y))                    premise
(4) ∀Y (P(Y) → ¬P(¬□Y))                premise
(5) ∃x R∗(x)                           Assume for IP
(6) R∗(µ)                               (5), EI
(7) P(Γ) ∧ ¬Q(Γ)                        (2), EI
(8) R∗(µ) → ∀Y (□Y(µ) → Q(Y))          (1), UI, exp, UG
(9) ¬Q(Γ) ∧ ∀Y (□Y(µ) → Q(Y)) → ¬□Γ(µ)    logical truth
(10) R∗(µ) → ¬□Γ(µ)                   (7), (8), (9), simp, MT
(11) ¬□Γ(µ) → □¬□Γ(µ)                  S5 logical truth
(12) R∗(µ) → □¬□Γ(µ)                   (10), (11), HS
(13) R∗(µ) ∧ □¬□Γ(µ) → Q(¬□Γ)          (1), UI
(14) P(Γ) → ¬P(¬□Γ)                   (4), UI
(15) ¬P(¬□Γ)                           (7), (14), simp, MP
(16) Q(¬□Γ) → P(¬□Γ)                  (3), UI
(17) R∗(µ) → Q(¬□Γ)                   (12), (13), exp, HS, idem
(18) R∗(µ) → P(¬□Γ)                   (16), (17), HS
The premises of this argument for ‘nothing is supremely Q-perfect’ are true. Premise (1) follows from Oppy’s definition of ‘supremely Q-perfect’. Premises (2) and (3) comes from his definition of ‘Q-perfect’. Premise (4) seems to be contained in the notion of a perfection; and it is implied by the conjunction of ‘∀Y(P(Y) → P(□Y))’ and ‘∀Y(P(□Y) → ¬P(¬□Y))’, both of which seem intuitively obvious and intrinsic to what it means to be a perfection.

Since Oppy’s parody against the DLA is valid and its conclusion is false, it must have at least one false premise. Given that the corresponding first three premises of the DLA and the parody stand or fall together, it follows that the parody refutes the DLA only if the fourth premise of the parody is more plausible than the fourth premise of the DLA. We can show just the opposite: the fourth premise of the parody is arguably false, and the fourth premise of the DLA is clearly true.

There are two ways to show that the fourth premise of this parody is false. The first is by implication from the truth of the first three premises of the DLA and their parity with the first three premises of the parody. The second would be to assume that the property of being essentially supremely Q-perfect is itself a Q-perfection. Oppy never makes that assumption, but there are two possible reasons why he should. First, it make sense to believe that the property of being essentially supreme is itself a perfection, and Oppy’s putative parody of the DLA clearly plays on establishing a parallel between the two. Second, since the Q-perfections are closed under entailment, and everything that is supremely Q-perfect must have every Q-perfection essentially, the property of being supremely Q-perfect attributes every Q-perfect property essentially to whatever is supremely Q-perfect. So the property of being essentially supremely Q-perfect should itself be a Q-perfection. We could then argue thus:

Assume that □R* is a Q-perfection. Then □R* must be a perfection. If □R* is a perfection then R* is a perfection. Also, anything that has R* has all and only Q-perfections essentially. By definition, for every Q-perfection, there is some perfection Y that is not a Q-perfection. So there is some perfection Y such that everything that is an R* fails to essentially have Y. In other words, for some perfection Y, everything that is an R* has the property of not essentially having Y. Now the property of not essentially having perfection Y cannot be a perfection,
for the reasons stated two paragraphs back. Thus, everything that is an \( R^* \) must have some non-perfection as a property simply by virtue of being an \( R^* \). That is, the property of being an \( R^* \) entails the property of not essentially having some perfection. Since perfections are closed under entailment, \( R^* \) cannot then be perfection. Everything that is an \( R^* \) is limited. So, \( \Box R^* \) cannot be a Q-perfection. Hence, the fourth premise of Oppy’s parody of the DLA is arguably false.

The fourth premise of the DLA is clearly true, however. A supremely perfect being, unlike a supremely Q-perfect being, is conceived as being fully unlimited, and distinguished by having every perfection essentially. As John Findlay has argued in [1], a supremely perfect being is by nature worthy of worship and absolutely perfect in every possible respect; it cannot “possess its various excellences in some adventitious or contingent manner” (p. 95 - 96). The upshot is that Oppy’s parody argument against the DLA does not refute the DLA. To the contrary, since the first three premises of the DLA are also true, as I show in “The Ontological Argument” [2], both the parody is unsuccessful and the DLA is sound.

3 The Modal Perfection Argument

The MPA is:

(1) A property is a perfection only if its negation is not a perfection.
(2) Perfections entail only perfections.
(3) The property of being supreme is a perfection.

Therefore,

A supreme being exists.

Oppy claims that the MPA is refuted by what he calls ‘The Supreme Island Parody’ (SIP). He also argues that the MPA begs the question. Let us consider the SIP first.

Oppy defines a being as island-supreme if and only if it is necessarily greater than any island from which it is distinct. And he defines a property as an island-perfection just in case it is necessarily better for an island to have it than not. The SIP then runs thus:

(1) A property is an island-perfection only if its negation is not an island-perfection.
(2) Island-perfections entail only island-perfections.

(3) The property of being island-supreme is an island-perfection.

Therefore,

A supreme island exists.

Oppy agrees with me that the MPA is valid in the quantified modal logic $S5$ that includes the Barcan Formula, ‘$\Box \exists x Y(x) \rightarrow \exists x \Box Y(x)$’. He then claims that (1). The SIP is also valid, because the two arguments have the same logical form; and (2). The first two premises of both the SIP and the MPA are equally plausible; and (3). There can be no question begging reason to believe that the third premise of the MPA is any more plausible than the corresponding third premise of the SIP. I shall argue, however, that SIP is not valid and does not have the same logical form as the MPA. Yet all the premises of the MPA are arguably true.

It will be useful for the following discussion to continue to use the characters ‘$G$', ‘$S$', and ‘$P$' to stand only for my MPA phrases ‘is greater than’, ‘necessarily, is greater than anything else’ (‘is supreme’), and ‘is a perfection’, respectively, but to use different characters to stand for the corresponding phrases of the SIP, contrary to Oppy’s convention to use my MPA characters to stand for both the MPA phrases and the corresponding phrases of SIP, respectively. (Oppy innocuously conflates the two presumably for the purpose of showing that the MPA and the SIP have the same logical form.) To that end, let us simply flag those phrases of the MPA with an ‘$i$’ to get the corresponding phrases of the SIP thus: ‘$Gi$', ‘$Si$', and ‘$Pi$’ stand for the SIP phrases ‘is a greater island than’, ‘necessarily, is a greater island than any other island’ (‘is island-supreme’), and ‘is an island-perfection’, respectively. (There is no need to define ‘perfection’ and ‘island-perfection’ in terms of ‘being better to have (for an island) than not’ at the validity checking stages of the MPA and the SIP, respectively, although those definitions are relevant for checking the truth of the argument/parody premises.) Finally, let ‘$[\hat{a}X]$’ represent the property of being an X.

A preliminary logical regimentation of the MPA is this:

(M1) $\forall X (P([\hat{a}X]) \rightarrow \neg P([\hat{a}\neg X]))$.

(M2) $\forall Y (P(Y) \rightarrow \forall Z (\Box \forall x (Y(x) \rightarrow Z(x)) \rightarrow P(Z)))$.

(M3) $\forall X (P([\hat{a}S])$.

Therefore,
Ontological Arguments Redux

$$\exists x S(x).$$

Similarly, a preliminary logical regimentation of the SIP is:

$$(O_1) \forall X (P^i([\hat{a}X]) \rightarrow \neg P^i([\hat{a}\neg X])).$$

$$(O_2) \forall Y (P^i(Y) \rightarrow \forall Z (\Box \forall X (Y(x) \rightarrow Z(x)) \rightarrow P^i(Z))).$$

$$(O_3) \forall X (P([\hat{a}S^i])).$$

Therefore,

$$\exists x S^i(x).$$

These preliminary regimentations suggest that the MPA and the SIP do have the same logical form. Moreover, even if we substitute the definiens ‘$$\forall y (x \neq y \rightarrow G(x, y))’ for ‘$$S(x)’, and ‘$$\Box \forall y (x \neq y \rightarrow G^i(x, y))’ for ‘$$S^i(x)’, it also appears that MPA and SIP have the same logical form. But appearances can be misleading. A deeper regimentation shows that the SIP does not have the same logical form as the MPA, and that the SIP is not valid.

Consider the notion of being island-supreme. Oppy defines the propositional function ‘$$S^i(x)’ as ‘necessarily, x is a greater island than any island distinct from x’ (p. 462). If we assume with Oppy that the domain of discourse is limited to possible islands, then ‘$$S^i(x)’ could be understood as ‘$$\Box \forall y (x \neq y \rightarrow G^i(x, y))’. But if we are interested in the validity of an argument, and there are possibilia other than islands, which seems obvious, then we cannot arbitrarily limit the domain of discourse to possible islands. An argument is valid just in case there is no possible world where the premises are true and the conclusion false, not that there is no possible island world where the premises are true and the conclusion false. We should allow our domain of discourse to be as wide as possible; and if we want to restrict our discussion to things of a certain kind, then we should build the restrictions into predicate letters that refer to just that kind of thing. For example, if we want to have sentences about islands, then we should use the predicate ‘is an island’, and not arbitrarily restrict the domain of discourse only to islands, especially when one of our tasks is to check for validity.

Accordingly, when we regiment ‘$$S^i(x)’ we should use the propositional function ‘$$I(x)’ (is an island) in some way. But, what way? Well, it really depends on what we mean when we say that something x is island-supreme. I think that what we mean when we say that x is island-supreme is that x is a supreme island. And what we mean when we say that x is a supreme
island is that $x$ is an island that necessarily is greater than every other island. In other words, $x$ is an island, and, necessarily, if any island $y$ is distinct from $x$ then $x$ is greater than $y$. Symbolically, \( 'S(x)' \) should be regimented as \( 'I(x) \land \Box \forall y (I(y) \land x \neq y \rightarrow G(x, y))' \).

But this deeper regimentation of \( 'S(x)' \) proves disastrous for the validity of SIP. Indeed, since \( 'I(x) \land \Box \forall y (I(y) \land x \neq y \rightarrow G(x, y))' \) is not fully modalized, the existence of a supreme island cannot be inferred from its possibility using the rules for modal logic \( S5 \).

I imagine again that someone might reply that what we mean when we say that $x$ is island-supreme is that necessarily $x$ is a supreme island, to wit: \( \Box (I(x) \land \forall y (I(y) \land x \neq y \rightarrow G(x, y))) \). But that would be odd. For \( \Box (I(x) \land \forall y (I(y) \land x \neq y \rightarrow G(x, y))) \) entails \( \Box I(x) \). So something could be island-supreme in some possible world only if it were an island in every possible world. Now some properties might indeed be necessitative in the sense of being instantiated in some possible world only if they are instantiated in all, such as the property of being valid, or the property of being a logical truth, or the property of being supreme. Yet the property of being an island is hardly one of them. Islands are contingent things, because they exist in some worlds and not others. (Moreover, the property of being an island is surely not a perfection.) Thus, the property of being an island cannot be instantiated in every possible world. So either the property of being island-supreme cannot be defined by \( \Box (I(x) \land \forall y (I(y) \land x \neq y \rightarrow G(x, y))) \) or there cannot be anything that is island-supreme in that sense.

I can also imagine that someone might then say that if the property of being island-supreme should be defined by \( 'I(x) \lor \Box \forall y (I(y) \land x \neq y \rightarrow G(x, y))' \) then the property of being supreme should be defined similarly by \( 'B(x) \lor \Box \forall y (B(y) \land x \neq y \rightarrow G(x, y))' \), where \( 'B' \) means \( 'is a being' \), thereby making MPA invalid too. This is redundant complexity, however. \( 'B(x)' \) and \( 'B(y)' \) are logically true for all values of $x$ and $y$. So \( 'B(x) \lor \Box \forall y (B(y) \land x \neq y \rightarrow G(x, y))' \) reduces to \( \Box \forall y (x \neq y \rightarrow G(x, y)) \). Hence, the MPA remains valid.

**On Begging the Question**

Oppy (pp. 463 - 464) claims there is no non-question-begging reason to prefer the MPA to the following atheistic inversion of the MPA:

\[
(1) \quad A \text{ property is a perfection only if its negation is not a perfection.}
\]

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(2) Perfections entail only perfections.
(3) There is no supreme being.

Therefore,

Supremacy is not a perfection.

Call this the Atheistic Modal Perfection Argument (AMPA).

The AMPA is clearly valid, as is every inversion argument that results from exchanging the negation of the conclusion of a valid argument with the negation of a premise.

To show that there is no non-question-begging reason to prefer the MPA to the AMPA, Oppy would have to prove a negative, a well nigh impossible task. I am led to believe, then, that he asserts the proposition ‘There is no non-question-begging reason to prefer the MPA to the AMPA’ as a rhetorical ploy instead of a philosophical truth. Indeed, he actually defends only the much weaker proposition that I have not provided a non-question-begging reason to prefer the MPA to the AMPA.

To show that a particular reason or argument \( \lambda \) is a non-question-begging reason to prefer the MPA to the AMPA two things must be shown. First, \( \lambda \) must not beg the question. Second, \( \lambda \) must make a stronger case for ‘Supremity is a perfection’ than all or some other cases that are or might be made for ‘There is no supreme being’. Now it is also well nigh impossible to satisfy this second requirement, especially in the abstract or in the absence of any proposed actual or possible arguments for ‘There is no supreme being’. The best that one could hope to accomplish would be to show that \( \lambda \) makes a stronger case for ‘Supremity is a perfection’ than some specific reason or argument provides for asserting ‘There is no supreme being’. Since Oppy does not provide any such reason or argument, I can only be reasonably expected to provide a “plausible” non-question-begging reason or argument for why supremity is a perfection, which I have already given in my “The Ontological Argument” [2].

It is a common criticism of many flawed ontological arguments that they “define God into existence”. Oppy strongly suggests that my MPA is similarly flawed when he says that my claim that something is supreme by virtue of its perfections “builds in the requirement that necessary existence is one of the requirements of supremacy” and implies that something is supreme “only if that thing exists in every possible world” (p. 464). I was somewhat surprised by this accusation, since I have always tried to avoid the logical fallacy of trying to define things into existence, and I
believe that I have been quite explicit about this in my formulations of ontological arguments that I think might have merit. Let me be clear about this once again: neither the categorical concept of the property of existence nor the categorical concept of the property of necessary existence should ever be included in the concept of anything, on pain of begging the question of its existence! What might and may be included in the concepts of some things is the conditional concept of the property of existing necessarily if existing at all. And, contrary to what Oppy intimates, this inclusion is perfectly innocuous: it neither defines a being into existence, nor underwrites a begging the question fallacy. Yet not even the concept of this conditional property is included in the definition of ‘supreme’ in the MPA. Nor does the logic of MPA depend on it. The MPA merely defines a supreme being as a greatest possible being. Its modal logic then insures, as well, that a supreme being exists if it possibly exists. The MPA proves rather than assumes that a supreme being exists in every possible world. To propose that a thing is supreme if and only if it is necessarily greater than everything else solely by virtue of some set of perfections is merely to propose that certain perfections, and nothing else, are what make something supreme, whatever those perfections might be. The logic of the MPA is independent of this proposal.

4 The Temporal Contingency Argument

Something is generated just in case there is a time when it exists and an earlier time when it does not; and something is corrupted just in case there is a time when it exists and a later time when it does not. Define something as temporally-necessary if and only if it is necessarily not generated and necessarily not corrupted. And define something as temporally-contingent if and only if it is possibly generated or possibly corrupted. Call the conjunction of the following fourteen propositions ‘Ω’:

(1) *Something presently exists.*

(2) *Only finitely many things have existed to date.*

(3) *Every temporally-contingent being begins to exist at some time and ceases to exist at some time.*

(4) *Everything that begins to exist at some time and ceases to exist at some time exists for a finite period of time.*
Ontological Arguments Redux

(5) If everything exists for only a finite period of time, and there have been only finitely many beings to date, then there was a time when nothing existed.

(6) If there was a time when nothing existed, then nothing presently exists.

(7) A being is temporally-necessary if and only if it is not temporally-contingent.

(8) Everything has a sufficient reason for its existence.

(9) Anything that has a sufficient reason for its existence also has a sufficient reason for its existence that is a sufficient reason for its own existence.

(10) No temporally-contingent being is a sufficient reason for the existence of a temporally necessary being.

(11) Every temporally-necessary being that is a sufficient reason for its own existence is a being without any limitations.

(12) A being without any limitations is necessarily greater than any other being.

(13) It is not possible for anything to be greater than itself.

(14) It is necessarily the case that ‘greater than’ is asymmetric.

Let ‘Σ’ name the proposition that a greatest possible being exists. The TCA is the argument: ◊Ω : Σ. The TCA is sound and does not beg the question. I show that it is valid in “The Ontological Argument” [2]. ‘◊Ω’ is arguably true, since it is a fairly modest modal claim that says only that ‘Ω’ is possibly true, and there is no apparent reason to think that it is ‘Ω’ incoherent or contradictory.

Oppy does not evaluate the logic of the TCA directly. Rather, he constructs two particular metaphysical theories of the world, one that he calls ‘Theism’ and the other ‘Naturalism’. According to Oppy, Theism says that all possible worlds have an initial causal state that involves the same necessarily existent supernatural entity, God, who is the essentially omniscient, essentially omnipotent, essentially perfectly good sole creator ex nihilo of the natural universe. Naturalism says that all possible worlds have an initial causal state that involves the same necessarily existent natural entity, the initial singularity, which is the natural causal origin of the natural universe, and there is nothing supernatural in any possible world. He then claims that the TCA is incompatible with Naturalism, and that the TCA
cannot establish the truth of Theism without begging the question against Naturalism.

**Comments**

1. While Theism and Naturalism are incompatible, Theism could be true and the natural universe might still have had an initial singularity as an immediate cause and have God as the creator ex nihilo of that singularity.

2. Naturalism is false, because it postulates that the initial singularity of the natural universe necessarily exists. While it is true that science currently favors the idea that the natural universe/time began with an initial singularity, credible scientists have recently advocated other models of the universe, such as a “Steady State” model, an Oscillating Universe model, a Big Bang model, or a Hawking no-boundary, imaginary time model (no beginning of the universe, and time is finite and unbounded). If any of these alternatives are logically possible, then the postulated initial singularity of the natural universe cannot exist necessarily, and Naturalism is false. Indeed, we seem to have no reason to think that these other models are logically impossible. Rather, if they are not favored by some scientists, it is because those alternative models do not satisfactorily explain and predict observational facts, or square with other empirical theories of the world that do posit the existence of an initial singularity. Science can only warrant contingencies.

3. Oppy assumes that time commences with the initial singularity and then argues that the fifth and eleventh conjuncts of the premise of the TCA are false if the initial singularity necessarily exists. And from this he infers that the TCA begs the question against Naturalism. But this puts the cart before the horse.

The fifth conjunct of the premise of the TCA does require that it is not necessarily true that time “commences” with the initial state of the natural universe, and ipso facto that not every possible world has an initial singularity as an initial causal state. The TCA neither assumes nor denies that the natural universe of the actual world and time in it “begins” with a singularity. It merely denies that all possible worlds are that way; and that is because, as we argued above, it is an empirical or theoretical fact that there is an initial singularity, not a logical fact.

Oppy also claims that the eleventh conjunct of the premise of the TCA begs the question against Naturalism by assuming that every temporally-necessary being that is a sufficient reason for its own existence is a being
without limitations. He reasons that if the natural universe begins with an initial singularity that exists necessarily, then it is logically impossible that every temporally-necessary being that is a sufficient reason for its own existence is a being without limitations, because the physical universe exists necessarily and is necessarily limited, and everything that exists necessarily is a sufficient reason for its own existence. But the TCA does not deny the consequent of this conditional in order to be able to deny its antecedent. As we have argued, that antecedent is independently false. It is logically possible that every temporally-necessary being that is a sufficient reason for its own existence is a being without limitations because it is simply not contradictory to assume otherwise, not because it begs the question against the assumption that the initial singularity exists necessarily.

4. The TCA is not Theism per se. Nor does the TCA try to prove that God caused the natural universe. Like Theism, it does not deny that an initial singularity was a cause of the natural universe of the actual world, even though it is incompatible the necessary existence of such an entity. Or an advocate of the TCA might believe that physical time is infinite. Or God might be outside of physical time. One could even consistently embrace the TCA and naturalism (small case ‘n’) and hold that all explanations of natural phenomena should be scientific. The TCA does not augur for causation or creation of the natural universe of the actual world. Whether or not God caused or created the natural universe could be a matter of faith, not logic.

5 Conclusion

Oppy concludes his astute critique with the opinion that “ontological arguments convince almost no one – and quite properly so: extant ontological arguments should not convince anyone” (p. 467). I have parried his critique by showing that my arguments do not beg the question, and are either immune to his parodies or mendable with a minor fix. I doubt, however, that logic alone will be persuasive. Yet our philosophical discussion will hopefully shed some light on these important issues, and so inch us closer to the truth.
Bibliography


Response to Maydole

Graham Oppy

At the conclusion to his response to Oppy [3], Maydole [2] claims: “I have parried [Oppy’s] critique by showing that my arguments do not beg the question, and are either immune to his parodies or mendable with a minor fix”. Perhaps unsurprisingly, I disagree with this assessment of the state of play. On the one hand, I do not think that he has successfully parried the critique set out in Oppy [3]; on the other hand, I think that there are places where he misrepresents the nature of that critique. I shall take up these points in turn.

1 Anselm’s Ontological Argument

In response to the criticisms in Oppy [3], Maydole [2] amends his rendition of ‘Anselm’s Ontological Argument’ so that it reads as follows:

(1) The definite description ‘that than which it is not conceivable for something to be greater’ is understood.

(2) ‘That than which it is not conceivable for something to be greater’ refers to that than which it is not conceivable for something to be greater.

(3) The concept of whatever a definite description that is understood refers to has existence in the understanding.

(4) It is conceivable that something is greater than anything that lacks a great-making property that it conceivably has and that only has great making properties essentially.

(5) Existence in reality is a great making property.
Response to Maydole

6) Anything the concept of which has existence in the understanding conceivably has existence in reality.

7) It is not conceivable that something is greater than that than which it is not conceivable for something to be greater.

8) All essential properties of that than which it is not conceivable for something to be greater are great making properties.

9) (Hence) That than which it is not conceivable for something to be greater exists in reality.

As he notes – subject to minor correction on my part – the corresponding island-parody for this argument then runs as follows:

1) The definite description ‘that island than which it is not conceivable for some island to be greater’ is understood.

2) ‘That island than which it is not conceivable for some island to be greater’ refers to that island than which it is not conceivable for some island to be greater.

3) The concept of whatever a definite description that is understood refers to has existence in the understanding.

4) It is conceivable that some island is greater than any island that lacks a great-making property for islands that it conceivably has and that only has great making properties for islands essentially.

5) Existence in reality is a great making property for islands.

6) Anything the concept of which has existence in the understanding conceivably has existence in reality.

7) It is not conceivable that some island is greater than that island than which it is not conceivable for some island to be greater.

8) All essential properties of that island than which it is not conceivable for some island to be greater are great making properties for islands.

9) (Hence) That island than which it is not conceivable for some island to be greater exists in reality.

Maydole [2] claims that premise (8) in this parody is false:

“All islands ... however great ... have the property of being material, which is hardly a great making property for islands ... Islands are ... essentially
limited by their very nature of being islands ... and they thereby have some properties essentially that could not even be construed as great making properties for islands.” (p. 472)

It seems to me that Maydole is just wrong about this. A great making property for islands is a property that contributes to making something a great island. A fortiori, any property that is essential to islands is a great making property for islands: something that lacks a property that is essential to islands is not an island and therefore certainly not a great island. Hence, in particular, contra Maydole, being material is a great making property for islands.

Since Maydole has so far only advanced this one argument, Maydole has not yet produced any considerations that speak more strongly in favour of premise (8) in the target argument than in favour of premise (8) in the parody.

2 The Descartes-Leibniz Ontological Argument

In response to the criticism of Oppy [3], Maydole [2] offers the following argument in favour of the falsity of the fourth premise of the proffered parody of ‘The Descartes-Leibniz Argument’:

(1) Every essential property of a supremely Q-perfect being is a Q-perfection.
(2) Some perfection is not a Q-perfection.
(3) Every Q-perfection is a perfection.
(4) The negation of the necessitation of a perfection is not a perfection.
(5) (Therefore) Nothing is supremely Q-perfect.

The key premise in this argument is (4). About it, Maydole [2] says:

“It seems to be contained in the notion of a perfection; and it is implied by the conjunction of ‘∀Y(P(Y) → P(□Y))’ and ‘∀Y(P(□Y) → ¬P(¬□Y))’, both of which seem intuitively obvious and intrinsic to what it means to be a perfection.” (p. 475)

While I am happy with the claim that ∀Y(P(□Y) → ¬P(¬□Y)), it is not at all obvious to me that ∀Y(P(Y) → P(□Y)). I think that libertarians about freedom will says that, while always freely choosing the good
Response to Maydole

is a perfection, necessarily always freely choosing the good is impossible – and so, *a fortiori*, not a perfection. I am also tempted by the thought that, while washing your hands before dinner is a perfection – cleanliness is next to godliness – necessarily washing your hands before dinner is a psychological pathology! Even if this latter point is just a trick of ordinary language, it may nonetheless make some contribution to the thought that it is not simply intuitively obvious and intrinsic to what it means to be a perfection that $\forall Y (P(Y) \rightarrow P(\Box Y))$.

My conclusion here is somewhat circumspect: I think that the jury is out on the question whether $\forall Y (P(Y) \rightarrow P(\Box Y))$. If $\forall Y (P(Y) \rightarrow P(\Box Y))$, then, I think, Maydole does have a satisfactory response to the $Q$-perfection parody of the ‘The Descartes-Leibniz Argument’. In case that turns out to be so, let me offer an alternative parody for the ‘The Descartes-Leibniz Argument’:

1. For every $X$ and $Y$, if the property of being a $Y$ is contained in the concept or essence of being an $X$, then necessarily everything that is an $X$ is a $Y$.

2. The property of necessarily obtaining if obtaining at all is contained in the concept or essence of an initial entirely natural state that obtains necessarily if it obtains at all.

3. It is possible that an initial entirely natural state that obtains necessarily if it obtains at all obtains.

4. Necessarily, initial entirely natural states that obtain necessarily if they obtain at all are necessarily initial entirely natural states that obtain necessarily if they obtain at all.

5. (Therefore) An initial entirely natural state that obtains necessarily if it obtains at all obtains.

This parody is plainly valid just in case the ‘Descartes-Leibniz Argument’ is valid. The first premise is the same in each argument. If anything, the second premise is more obvious in the parody than it is in the original argument. It is hard to see any reason to prefer the fourth premise of the original argument to the fourth premise of the parody. And – though I think that Maydole will disagree – it is hard to see any non-question-begging reason to prefer the third premise of the original argument to the third premise of the parody.
3 The Modal Perfection Argument

In response to the criticism of the ‘Modal Perfection Argument’ in Oppy [3], Maydole [2] makes very heavy weather of the regimentation of the expression ‘$x$ is island-supreme’. I doubt that anyone will be surprised to learn that I intended it to be regimented as: $\Box (I(x) \land \forall y(I(y) \land x \neq y \rightarrow G^i(x, y)))$. Maydole [2] objects that this regimentation would be ‘odd’:

“[S]omething could be island-supreme in some possible world only if it were an island in every possible world. Now some properties might indeed be necessitative ... such as the property of being valid, or the property of being a logical truth, or the property of being supreme. Yet the property of being an island is hardly one of them. Islands are contingent things, because they exist in some worlds and not others. ... So either the property of being island-supreme cannot be defined by $\Box (I(x) \land \forall y(I(y) \land x \neq y \rightarrow G^i(x, y)))$ or there cannot be anything that is island-supreme in that sense.” (p. 479)

But this is no objection at all to the claim that the ‘Supreme Island Parody’ succeeds. On the one hand, we can define terms however we like: I insist that ‘$x$ is island-supreme’ $\overset{\text{df}}{=} \Box (I(x) \land \forall y(I(y) \land x \neq y \rightarrow G^i(x, y)))$. On the other hand, I agree that nothing is island-supreme – whence it follows that nothing can be island-supreme; this, after all, is what makes us confident that the ‘Supreme Island Parody’ is not a good argument! (Perhaps I should add here that I would not be so quick to dismiss the trivialising theory of modality that says that there is only one possible world: in my opinion, that theory is not unattractive if there are no objective chances.)

Since Maydole grants that the ‘Supreme Island Parody’ is valid – on my intended regimentation of ‘$x$ is island-supreme’ – iff the ‘Modal Perfection Argument’ is valid, the only live question is whether the Fool has more reason to accept one of the premises of the ‘Modal Perfection Argument’ than he has to accept the corresponding premise of the ‘Supreme Island Parody’.

I do not think that Maydole denies that the first premise of the ‘Supreme Island Parody’ – a property is an island-perfection only if its negation is not an island-perfection – is no less acceptable to the Fool than the first premise of the ‘Modal Perfection Argument’ – a property is a perfection only if its negation is not a perfection.
Response to Maydole

I do not think that Maydole denies that the second premise of the ‘Supreme Island Parody’ – island-perfections entail only island-perfections – is no less acceptable to the Fool than the second premise of the ‘Modal Perfection Argument’ – perfections entail only perfections.

And I do not think that Maydole denies that the third premise of the ‘Supreme Island Parody’ – the property of being island-supreme is an island-perfection – is no less acceptable to the Fool than the third premise of the ‘Modal Perfection Argument’ – the property of being supreme is a perfection.

Remember: the Fool denies that there is either a supreme being or a supreme island. Consequently, the Fool denies that there is a set of properties – the perfections – that is non-trivially closed under entailment and that includes the property of being supreme; and the Fool also denies that there is a set of properties – the island-perfections – that is non-trivially closed under entailment and that includes the property of being island-supreme. What the Fool requires – and what Maydole’s ‘Modal Perfection Argument’ manifestly does not deliver – is a non-question-begging argument for the revision of his opinion concerning only one of these alleged sets of properties.

4 The Temporal-Contingency Argument

In response to the criticism of the ‘Temporal-Contingency Argument’ in Oppy [3], Maydole [2] makes a series of numbered points. I think that these numbered points are all vitiated by misunderstanding of the nature of the criticism that I made of his argument. So let me try again.

Suppose that the Fool is a Naturalist. Ex hypothesi, the Fool supposes that there are none but natural causes. So, the Fool supposes, there are two live hypotheses about global causal reality: either there is an infinite regress of natural causes, or there is an uncaused initial natural cause. In the latter case, there are two live sub-hypotheses: either the uncaused initial natural cause is necessary, or it is contingent. In the latter case, again, there are two live sub-hypotheses: either the uncaused initial natural cause involves something that exists of necessity, or it does not.

There is a range of live hypotheses that we can frame about the beliefs that the Fool has about the extent of alethic possibility. At one extreme, the Fool may suppose that alethic possibility is limited only by contradiction – and, in this case, the Fool will suppose that (consistent) conceivability is a proper guide to alethic possibility. At the other extreme,
the Fool may suppose that alethic possibility is so tightly constrained that all possible worlds evolve according to the same laws, share some initial history, and differ only because of the out-workings of objective chance.

It is evident that, if we suppose that alethic possibility is sufficiently tightly constrained, then we must also suppose that live hypotheses need not be alethic possibilities: given that we are imagining a dispute that may involve defence and denial of the existence of allegedly necessary beings, we cannot coherently suppose that both sides of the dispute are alethic possibilities. However, if we are then to make sense of the dispute – from the standpoint of disputants on either side – we have to suppose that we can assign some content to the views of those on either side. So, I think, we are led to the idea that there are doxastic possibilities that are not alethic possibilities: there are alethic impossibilities that nonetheless can be consistently defended.

A full treatment of responses that the Fool might make to the ‘Temporal-Contingency Argument’ requires the examination of all cases. In the discussion in Oppy [3], I considered only one. I claimed that, if the Fool supposes that there is a necessary uncaused initial natural cause, and if the Fool adopts the most tightly constrained view of alethic possibility, then the Fool will plainly deny that the conjunction of the fourteen premises in the ‘Temporal-Contingency Argument’ is even possibly true. I thought – and still think – that this is enough to show that the Fool can reasonably resist the ‘Temporal-Contingency Argument’. However, I also think that there are other options open to the Fool that also yield reasonable lines of resistance. For instance, the Fool might adopt the most tightly constrained view of alethic possibility while also supposing that there is an infinite regress of natural causes: in this case, too, the Fool will plainly deny the conjunction of the premises in the ‘Temporal-Contingency Argument’ is even possibly true. Etc.

Without pretending to offer a full account, let me add the following observations:

1. If the Fool adopts the most tightly constrained view of alethic possibility while also supposing that there is an infinite regress of natural causes, then the Fool will deny that the conjunction of the second and third premises is so much as possible.

2. If the Fool adopts the most tightly constrained view of alethic possibility while also supposing that there is a necessary uncaused initial natural cause, then – on the assumption that there have only been finitely many
natural things to date – the Fool will deny that the fifth premise is possible (since it is necessary that time begins with the necessary uncaused initial natural cause). On the other hand, if we suppose that there have been infinitely many natural things to date – because there were infinitely many natural things in the initial natural causal state – then the Fool will deny that the second premise is possible (since it is necessary that there are infinitely many natural things in the initial natural causal state).

3. If the Fool adopts the most tightly constrained view of alethic possibility while also supposing that there is a necessary uncaused initial natural cause, then the Fool will surely deny the possibility of the conjunction of the eighth and eleventh premises. If the Fool accepts that whatever exists of necessity ipso facto has a sufficient reason for its own existence in its necessity, then the Fool will plainly deny that it is possible that whatever has a sufficient reason for its own existence is unlimited – because the Fool will plainly deny that it is possible that the uncaused initial natural causal state is unlimited – and so will deny the possibility of the eleventh premise. On the other hand, if the Fool does not accept that whatever exists of necessity ipso facto has a sufficient reason for its own existence in its necessity, then the Fool can surely deny that it is possible that everything has a sufficient reason for its existence – because the Fool can then deny that it is possible that the uncaused initial natural causal state has a sufficient reason for its existence – and so can deny the possibility of the eighth premise.

4. If the Fool adopts something like the view that alethic possibility is limited only by contradiction, then the Fool will need to follow Maydole in making bold claims about the presence and absence of contradictions. Maydole [2] asserts: “It is logically possible that every temporally-necessary being that is a sufficient reason for its own existence is a being without limitations because it is simply not contradictory to assume otherwise”. Suppose that the Fool asserts: ‘It is logically possible that every being is a natural being, because it is simply not contradictory to assume otherwise’. (Surely this is a ‘fairly modest modal claim’, since ‘there is no apparent reason to think that it is incoherent or contradictory to suppose that every being is a natural being’.) We have a disagreement: Maydole supposes that the assertion of the possibility of the conjunction of his fourteen claims is ‘modest’; and the Fool supposes that the assertion of the possibility of there being none but natural beings is ‘modest’. But at least one of these ‘modest’ claims is mistaken (since Maydole’s claim entails that it is...
possible that there is a supreme being, and the claim that it is possible that there is a supreme being (plausibly) entails that it is not possible that there are none but natural beings). To resolve this disagreement by argument, further considerations must be introduced; it would be plainly question-begging to insist at this point, in the absence of the introduction of further considerations, that argument has come down on one side or the other. And yet this is precisely the kind of insistence that Maydole makes.

5 Further Comments

1. In his discussion of ‘Anselm’s Ontological Argument’, Maydole [2] writes:

   “Oppy intimates that AOA begs the question because I allow the quantifiers to range over possibilia ... This means, according to Oppy, that Anselm’s Fool would have to grant the existence of God if she accepted both modal logic $S5$ and the belief that God must have necessary existence. ... There is no reason to think that AOA involves or presupposes $S5$; and it is a straw man to charge that AOA begs the question on that account. ... Actually, neither ‘ $\square E$ ’ nor ‘ $E \rightarrow \square E$ ’ is included in the meaning of the term ‘God’ as it is used in AOA. ‘God’ there means only ‘that than which nothing greater can be conceived’ ” (p. 470)

   Oppy [3] ‘intimates’ no such thing. The discussion of ‘Anselm’s Ontological Argument’ in Oppy [3] has three parts: (i) a discussion of difficulties that confront the theoretical framework upon which ‘Anselm’s Ontological Argument’ is established; (ii) the presentation of a parody of Maydole’s proof; and (iii) an argument for the conclusion that ‘Anselm’s Ontological Argument’ is question-begging. The material to which Maydole refers in the above comments belongs to (i), and not to (iii).

   Here is a different way of getting at some of the theoretical difficulties that were the focus of the first part of my discussion of ‘Anselm’s Ontological Argument’. Consider the description ‘the merely possible tallest Martian’. Surely this description is understood; perhaps one might even venture that it is at least as well understood as the description ‘that being than which it is not conceivable for some being to be greater’. But, on Maydole’s account, given that it is understood, it refers to the merely possible tallest Martian. Further, on Maydole’s account, this entails that the concept of the merely possible tallest Martian exists in the understanding. But it is a premise in ‘Anselm’s Ontological Argument’ that anything that
concept of which has existence in the understanding conceivably has existence in reality. So, on Maydole’s account, it follows that it is conceivable that the merely possible tallest Martian exists in reality. But that is inconceivable, because absurd: no merely possible thing can exist in reality! (Perhaps it is also worth observing that the final remark that Maydole makes in the passage cited above is entirely gratuitous, since the word ‘God’ appears nowhere in ‘Anselm’s Ontological Argument’.)

2. Maydole [2] wavers on what exactly is required for successful parody. At the beginning of his discussion of ‘Anselm’s Ontological Argument’ he gets it right: “A parody refutes a valid argument just in case it has the same valid structure, an absurdly false conclusion, and premises that are at least as justifiable as those of the argument parodied” (p. 470) (my underline). However, in his discussion of the ‘Descartes-Leibniz Argument’, there is a departure from grace: “Given that the corresponding first three premises of the DLA and the parody stand or fall together, it follows that the parody refutes the DLA only if the fourth premise of the parody is more plausible than the fourth premise of the DLA” (p. 475) (again, my underline).

Maydole [2] makes other more serious missteps in his discussion of ‘begging the question’ in connection with the ‘Modal Perfection Argument’:

“Oppy claims that there is no non-question-begging reason to prefer the MPA to [an ] atheistic inversion. ... To show that there is no non-question-begging reason to prefer the MPA to the AMPA, Oppy would have to prove a negative, a well nigh impossible task. I am led to believe, then, that he asserts the proposition ‘There is no non-question-begging reason to prefer the MPA to the AMPA’ as a rhetorical ploy instead of a philosophical truth.” (pp. 479 - 480)

Here is the relevant part of what I wrote in [3]:

“Is there a non-question-begging reason to prefer Maydole’s Modal Perfection Argument to this alternative argument? I don’t think so. Certainly, Maydole has not provided us with one. ... For all that Maydole says – in the article presently under examination and elsewhere – he has provided no non-question-begging reason to prefer the hypothesis that supremacy is a perfection to the hypothesis that island-supremacy is an island-perfection. He may think it obvious that it is not the case that island-supremacy is an island-perfection, but many of us suppose that it is no less obvious that it is not the case that supremacy is a perfection (given that supremacy is taken to entail necessary existence).” (pp. 463 - 464)
I’m not sure that I have here asserted that there is no non-question-begging reason to prefer the MPA to the AMPA; indeed, one might be given to think that the suggestion that I have asserted this should be deemed a rhetorical ploy on Maydole’s part! More importantly, I think that Maydole has lost sight of the larger picture. If the MPA is to be preferred to the AMPA, then it must be that the MPA gives the Fool greater reason to accept the claim that there is a supreme being than the AMPA gives a believer in the existence of a supreme being reason to accept the claim that there is no supreme being. (Equivalently, if the MPA is to be preferred to the AMPA, then the MPA must give someone who is completely undecided between the claim that there is a supreme being and the claim that there is no supreme being more reason to accept the claim that there is a supreme being than the AMPA gives that one reason to accept the claim that there is no supreme being.) To repeat what I said above, I see no reason to think that this condition is satisfied.

Maydole [2] says:

“To show that a particular reason or argument $\lambda$ is a non-question-begging reason to prefer the MPA to the AMPA two things must be shown. First, $\lambda$ must not beg the question. Second, $\lambda$ must make a stronger case for ‘Supremity is a perfection’ than all or some other cases that are or might be made for ‘There is no supreme being’. ... The best that one could hope to accomplish would be to show that $\lambda$ makes a stronger case for ‘Supremity is a perfection’ than some specific reason or argument provides for asserting ‘There is no supreme being’. Since Oppy does not provide any such reason or argument, I can only be reasonably expected to provide a ‘plausible’ non-question-begging reason or argument for why supremity is a perfection, which I have already given.” (p. 480)

This is, I think, confused. MPA and AMPA are arguments. When we ask whether there is reason to prefer one to another, we are asking whether there is reason to suppose that one is a better argument than the other. We should not confuse this question with the further question whether there are better arguments for the premises of one or other of MPA and AMPA. At the very least, it seems reasonable to insist that, if you think that there are better arguments for the premises of MPA than for the premises of AMPA, then you should explicitly set out the expanded version of MPA with those premises included, so that an assessment can be made of the merits of the expanded argument.

“It is a common criticism of many flawed ontological arguments that they ‘define God into existence’. Oppy strongly suggests that my MPA is similarly flawed when he says that my claim that something is supreme by virtue of its perfections ‘builds in the requirement that necessary existence is one of the requirements of supremacy’ and implies that something is supreme ‘only if that thing exists in every possible world’. ... What might and may be included in the concepts of some things is the conditional concept of the property of existing necessarily if at all. And, contrary to what Oppy intimates, this inclusion is perfectly innocuous: it neither defines a being into existence, nor underwrites a begging the question fallacy.” (pp. 480 - 481)

These remarks, too, seem to me to be based on misunderstanding. Maydole [1] says the following:

“M3 (‘the property of being supreme is a perfection’) is true because it is reasonable to assume that a thing is supreme iff it is necessarily greater than everything else solely by virtue of having some set of perfections.” (p. 581)

In Oppy [3], I objected to this justification for M3 on the grounds that, whereas it might be unobjectionable to suppose that being the greatest possible being is a perfection (which would arguably justify the claim that $x$ is supreme in world $w$ just in case for every possible world $w'$, for every possible thing $x'$, if $x$ in $w$ is distinct from $x'$ in $w'$, then $x$ in $w$ is greater than $x'$ in $w'$), it is not unobjectionable to suppose that being the greatest being in every possible world is a perfection (which is what would be needed to justify the claim that $x$ is supreme in $w$ just in case for every possible world $w'$ and any $x'$ distinct from $x$ in $w'$, $x$ in $w'$ is greater than $x'$ in $w'$).

It is important to note that this is not an objection to MPA; but it is an objection to the claim that Maydole has provided a non-question-begging reason to believe the key premise of MPA, or to prefer the key premise of MPA to the key premise of AMPA. The claim is not that Maydole is defining God into existence, or somehow formulating improper definitions; rather, the claim is that Maydole has provide no support at all for the claim that the Fool ought to be moved to amend his views upon consideration of Maydole’s MPA (and hence no support at all for the claim that Maydole’s MPA establishes that there is some inconsistency or incoherence in the beliefs of the Fool).
Bibliography


Reply to Oppy’s Response to “Ontological Redux”

ROBERT E. MAYDOLE

This paper is my counter-reply to Graham Oppy’s reply in “Response to Maydole” [6] to my “Ontological Arguments Redux” [4], which is my reply to Oppy’s reply “Maydole on Ontological Arguments” [5] to my “The Ontological Argument” [3].

1 Anselm’s Ontological Argument

In [4] I amended my rendition of Anselm’s Ontological Argument to the ontological argument AOA by adding an eighth premise:

“All essential properties of that than which nothing greater can be conceived are great making properties.”

I also argued there that a corresponding parody, the Perfect Island Parody (PIP), that results by substituting ‘island’ for ‘being’ and ‘property for islands’ for ‘property’ in the fourth, fifth, and eighth premises of AOA fails to refute AOA. The eighth premise of AOA is justifiably true and the corresponding eighth premise of PIP is arguably false: a greatest possible being cannot be limited, but islands are. Islands have essential properties that are limiting, such as the property of being material and the property of being composite; and limiting properties are not great making, not even for islands. Oppy objects in [6]:

“A great making property for islands is a property that contributes to making something a great island. A fortiori, any property that is essential
to islands is a great making property for islands: something that lacks a property that is essential to islands is not an island and therefore certainly not a great island ... contra Maydole, being material is a great making property for islands.” (p. 489)

Let us consider the strength of Oppy’s objection. Oppy uses an enthymeme to argue for the proposition that every property essential to islands is a great making property for islands. His stated premise is the true proposition that every property that contributes to making something a great making property is a great making property for islands. One can only guess about what the missing premise(s) might be. An obvious choice for making his objection valid would be the proposition that every property essential to islands is a property that contributes to making something a great island. But surely, this would be missing premise cannot be true. Otherwise we could also conclude that every property essential to islands is a property that contributes to making something a puny island, or conclude that every property essential to islands is a property that contributes to making something any kind of island. We can also note that some essential properties of great islands might simply contribute to making certain landmasses islands, such as the property of being totally surrounded by water, and other properties might contribute to making those same landmasses great.

Now I am not about to infer from reasoning based upon a guess that Oppy’s objection to my argument that PIP® fails to refute AOA*. However, since Oppy offers nothing other than this enthymeme in support of the eighth premise of PIP®, which is false because some essential properties of islands are limiting, I can hereby only reaffirm my amended rendition of Anselm’s Ontological Argument (AOA*) against the corresponding Perfect Island Parody (PIP®).

2 The Descartes-Leibniz Ontological Argument

In [6], p. 490, Oppy offers a new parody to my rendition of the Descartes-Leibniz Argument (DLA):

(1) For every X and Y, if the property of being a Y is contained in the concept or essence of being an X, then necessarily everything that is an X is a Y.

(2) The property of necessarily obtaining if obtaining at all is contained in the concept or essence of an initial entirely natural state that obtains necessarily if it obtains at all.
It is possible that an initial entirely natural state that obtains necessarily if it obtains at all obtains.

Necessarily, initial entirely natural states that obtain necessarily if they obtain at all are necessarily initial entirely natural states that obtain necessarily if they obtain at all.

Therefore, an initial entirely natural state that obtains necessarily if it obtains at all obtains.

Like DLA, this new parody is valid, and its first three premises are justifiably true, notwithstanding Oppy’s passing observation that “the second premise is more obvious in the parody than it is in the original argument” ([6], p. 490). As for the fourth premise, Oppy says only “It is hard to see any reason to prefer the fourth premise of the original argument to the fourth premise of the parody” ([6], p. 490). Now I have argued in [3] that the fourth premise of DLA is true, which Oppy never disputes. I shall now show that the fourth premise of this new parody is false:

An instance of the fourth premise might have a true antecedent and a false consequent. This is because there seems to no reason why some state of every possible world cannot be initial and entirely natural in one possible world, and either not initial or not entirely natural in another. The densest state of the universe, for example, might be an initial entirely natural state of the universe in one possible world and its second state in another. Or it might be initial in every possible world, and partly non-natural in one. The important thing about this idea is that there be markers that enable us to identify states across possible worlds. I see no a priori reason to think that the property of being initial and entirely natural is necessarily the only such marker. The property of being the densest state of the universe serves such a purpose in this example. Hence, the fourth premise of the new parody is false.

Since all the premises of the DLA are arguably true, this new parody fails to refute the DLA.

3 The Modal Perfection Argument

In [4] I argued that Oppy’s Supreme Island Parody (SIP) is invalid if ‘x is island-supreme’ is construed as ‘I(x) ∧ □∀y(I(y) ∧ x ≠ y → G(x, y))’; and I argued that the conclusion of the SIP is false if ‘x is island-supreme’ is
construed as ‘\(\Box(I(x) \land \forall y (I(y) \land x \neq y \rightarrow G(x,y)))\)’, because the existence of an island supreme being would then entail that islands are necessary beings, which seems philosophically odd or extreme. Oppy, however, is unmoved by this oddity, intent as he is to establish that the SIP is a parody that refutes the MPA. So he “insists” in [6] that ‘x is island-supreme’ should be defined as ‘\(\Box(I(x) \land \forall y (I(y) \land x \neq y \rightarrow G(x,y)))\)’ and then acknowledges, “that nothing can be island-supreme” and that the SIP is a bad argument, from which he concludes that the Modal Perfection Argument (MPA) must also be a bad argument (p. 491). Given this stipulative definition, the SIP does indeed have the same relevant logical form as the MPA and is valid. But it also logically follows that at least one of the premises of the SIP must be false. And it follows as well that the SIP fails to refute the MPA if there is more reason to accept at least one premise of the MPA than there is to accept the corresponding false premise of the SIP, given that the corresponding remaining premises of each are equally supportably plausible. Since we know that at least one premise of the SIP is false, if the corresponding premise if the MPA arguably true, the SIP fails to refute the MPA, given that the corresponding remaining premises of each are equally supportably plausible.

I grant that the first two premises of the SIP are just as plausible as the corresponding first two premises of the MPA. I argue below and elsewhere, that the third premise of the MPA – supremity is a perfection – is true. But the third premise of SIP – island-supremity is an island-perfection – is arguably false. Here’s why:

\[
\text{Suppose that island-supremity is an island-perfection. Then by the second premise of the SIP, and the fact that the property of being island-supreme entails the property of being an island, the property of being an island must be an island-perfection. That is, it must be better for an island to be an island than not. But surely islands are just islands. Islands as such are not praiseworthy. Some island might be praiseworthy because it is beautiful, abundant in natural resources, habitable, a good place for a vacation, rich in history, inspirational, etc., but not simply because it is an island. The property of being an island is neutral. It is not better for islands to be islands than not. Hence, by reductio ad absurdum, island-supremity is not an island-perfection.}
\]

The upshot is that the SIP fails to refute the MPA.
4 The Temporal-Contingency Argument

Let ‘Σ’ name the proposition that a greatest possible being exists. And let ‘Ω’ name the conjunction of the fourteen propositions listed at the beginning of the section on the Temporal Contingency Argument (TCA) in [4]. Then the TCA is the valid argument: \(\Diamond \Omega \vdash \Sigma\).

Oppy claims that the proposition ‘Possibly every being is a natural being’ is fairly modest if the proposition ‘\(\Diamond \Omega\)’ is fairly modest. I concur. Yet ‘\(\Diamond \Omega\)’ and ‘Possibly every being is a natural being’ are incompatible. How do we choose between them? Oppy answers in [6]:

“To resolve this disagreement by argument, further considerations must be introduced; it would be plainly question-begging to insist at this point, in the absence of the introduction of further considerations, that argument has come down on one side or the other. And yet this is precisely the kind of insistence that Maydole makes.” (p. 495)

I insist no such thing. To the contrary, I specifically show in both [1] and [3] that the fourteen conjuncts of Ω are co-possible. Of the fourteen conjuncts of Ω, the first, second, third, sixth, eighth, ninth, tenth and eleventh are logically contingent facts about some possible world \(\omega\), the twelfth is an S5 necessary metaphysical truth, and the other five are analytic truths. The possible contingent facts and analytic truths are obviously so. My argument for the metaphysical conjunct was and still is this:

Assume \(x\) is a being without limitations in \(\omega\). Then \(x\) possesses every great making property in \(\omega\). In particular, \(x\) possesses the property in \(\omega\) of not being limited in world \(\omega_1\) by anything. In other words, if \(x\) is a being without any limitations in \(\omega\), then \(x\) possesses every great making property in \(\omega\). But the property of not being limited in \(\omega_1\) is a great making property of \(\omega\). So it is true in \(\omega\) that it is true in \(\omega_1\) that \(x\) is unlimited. But in S5, for any statement \(p\), if it is true in world \(\alpha\) that \(p\) is true in world \(\beta\), then \(p\) is true in world \(\beta\). Hence, \(x\) is unlimited in world \(\omega_1\). Now if \(x\) is unlimited in \(\omega_1\), then in \(\omega_1\) \(x\) is greater than any other being in \(\omega_1\); otherwise \(x\) would be limited by not possessing a great making property possessed by something else. Hence it is true in \(\omega_1\) that \(x\) is greater than every other being. Since \(\omega_1\) is an arbitrarily selected possible world, it follows that it is true in every possible world that \(x\) is greater than every other being. So, it is necessarily the case that \(x\) is greater than every other being. Therefore, a being without any limitations is necessarily than any other being.
Reply to Oppy’s Response to “Ontological Redux”

It follows that there is a possible world where each of the fourteen conjuncts of ‘Ω’ are true. Consequently, ‘◊Ω’ is true.

I would have thought that this line of reasoning would satisfy everyone except modal skeptics. Yet Oppy reminds us that there are modal conservatives, some of his Fools, who will also demur because they have a much more tightly constrained notion of alethic possibility than those of us who take consistency and conceivability as guides to alethic possibility. For these modal conservative Fools, “alethic possibility is so tightly constrained that all possible worlds evolve according to the same laws, share some initial history, and differ only because of the out-workings of objective chance” ([6], p. 493). They then can deny, Oppy contends, that the second, fifth, eighth, and eleventh conjuncts of ‘Ω’ are alethic possibilities, and that ‘◊Ω’ is true (cf., [6], pp. 493 - 495).

Does this mean that my case in behalf of the TCA begs the question against the modal conservative Fool? Not at all. Oppy’s argument that it does boils down to the following:

(1) *The modal conservative Fool’s reasons for believing that some conjuncts of ‘Ω’ are not possible are just as good as Maydole’s reasons for believing that all conjuncts of ‘Ω’ are possible.*

(2) *Nevertheless, Maydole says that all conjuncts of ‘Ω’ are co-possible.*

(3) *Therefore, Maydole begs the question against the modal conservative Fool.*

Now in this argument the modal conservative Fool seems uses the modal term ‘possibility’ to mean roughly what most of us mean when we say that something is physically possible or consistent with the laws of nature, and I use the modal term ‘possibility’ to mean what most of us mean when we say that something is logically possible. So Oppy’s argument that I beg the question against the modal conservative Fools is an equivocation on the term ‘possibility’.

5 Further Comments

Comment 1. I do not believe that there is a tallest Martian. I do believe that it is possible that there is a tallest Martian. So I also believe two other things, and well I logically should: (1). It is merely possible that there is a tallest Martian, and (2). It is conceivable that it is merely possible that there is a tallest Martian. In short, there is no tallest Martian in the actual world, but there is one in some merely possible world. Oppy
says in [6], p. 425, “But that is inconceivable, because absurd: no merely possible thing can exist in reality!” Either Oppy means something other than what I mean by the phrase ‘merely possible’ or he is wrong!

**Comment 2.** First, Oppy chides me for incorrectly saying in [4], p. 475, that his parody of the DLA refutes the DLA only if the fourth premise of the parody is more plausible than the fourth premise of the DLA, instead of correctly saying that DLA refutes the DLA only if the fourth premise of the parody is at least as justifiable as the fourth premise of the DLA ([6], p. 490). Surprisingly, he fails to also note that I effectively corrected myself in the very next sentence, “We can show just the opposite: the fourth premise of the parody is arguably false, and the fourth premise of the DLA is clearly true”, and then again in the subsequent three paragraphs. So the fourth premise of the parody is really less justified and less plausible than the fourth premise of the DLA. Small mistake, no worry!

Second, Oppy charges that I make “more serious missteps” in my “discussion of ‘begging the question’ in connection with the ‘Modal Perfection Argument’ ” ([6], p. 496). He repeats in [6], p. 496, what he says in [5] when discussing the Supreme Island Parody (SIP), to wit:

“For all that Maydole says – in the article presently under examination and elsewhere – he has provided no non-question-begging reason to prefer the hypothesis that supremacy is a perfection to the hypothesis that island-supremacy is an island-perfection.”

True enough, I did not argue in [4] that there are better reasons to prefer the hypothesis that supremacy is a perfection to the hypothesis that island-supremacy is an island-perfection. I was content there to show that SIP is either invalid, or valid and unsound (because SIP has a false conclusion), depending on how the notion of being island-supreme is construed. But, I show in the above section on the MPA of the present paper that island-supremity is not an island-perfection. And, since I show in [2] and [3] that supremacy is perfection, I can confidently reaffirm that the MPA does not beg the question against the SIP, and that the SIP does not refute the MPA.

Third, Oppy charges that I am confused about evaluating the MPA and the AMPA with respect to their respective premises. He says,

“MPA and AMPA are arguments. When we ask whether there is reason to prefer one to another, we are asking whether there is reason to suppose that one is a better argument than the other. We should not confuse this
question with the further question whether there are better arguments for
the premises of one or other of MPA and AMPA.” ([6], p. 497)

Both the MPA and the AMPA are valid arguments. But they will not
be good arguments if any of their premises are not true. Moreover, the
premises of arguments for premises of an argument are ultimately includ-
able in an expanded argument for its conclusion. So Oppy’s distinction
between the evaluation the MPA and the AMPA qua arguments and the
evaluation of the premises of the MPA and the AMPA is distinction with-
out a difference. Since two of the three premises of MPA and AMPA are
the same, and the third premise of MPA is the proposition that supremity
is a perfection, while the third premise of the AMPA is the proposition
that there is no supreme being, the respective strengths of the MPA and
the AMPA are indeed a function of the cases that are made in behalf of
their respective third premises. Since Oppy does not refute my argument
for the proposition that supremity is a perfection, and he fails to argue
for the proposition that there is no supreme being, he fails to sustain the
charge that MPA begs the question against AMPA.

Comment 3. Oppy writes in [6]:

“[W]hereas it might be unobjectionable to suppose that being the greatest
possible being is a perfection (which would arguably justify the claim that
x is supreme in world w just in case for every possible world w', for every
possible thing x', if x in w is distinct from x' in w', then x in w is greater
than x' in w'); it is not unobjectionable to suppose that being the greatest
being in every possible world is a perfection (which is what would be needed
to justify the claim that x is supreme in w just in case for every possible
world w' and any x' distinct from x in w', x in w' is greater than x' in
w') .” (p. 498)

The comparison of the greatness distinct of things across different worlds
is not relevant to the validity of MPA or to the truth of its third premise.
The comparison of the greatness of distinct things in the same world is rel-
vent to both, but only because something is supreme by definition if and
only if it is impossible for anything to be greater and impossible for there to
be something else than which it is not greater: ¬◊∃yG(y, x) ∧ ¬◊∃y(x ≠
y ∧ ¬G(x, y)). There is nothing about this or any other stipulative defini-
tion that requires justification. Neither does its possible worlds consequent
that for every, x is supreme in world w just in case in every possible world
w' and any x' distinct from x in w', x is greater than x' in w'. And never
would I – or anyone, I would think – even presume to justify this definition or its consequent by supposing that being the greatest being in every possible world is a perfection. To what then is Oppy objecting in the quotation seven sentences back?

Oppy does clearly say in the final paragraph of [6] that I have not provided a non-question begging reason to believe that the property of being supreme is a perfection. The truth is that I give two such arguments in [3], (p. 581), one in standard logical form, and the other obliquely. The standard form argument runs thus:

1. For every Z, all of the non-tautological essential properties entailed by Z are perfections if and only if the property of being a Z is a perfection.
2. Every non-tautological essential property entailed by the property of being supreme is a perfection.
3. Therefore, the property of being supreme is a perfection.

And I obliquely argued thus:

M3 is true because it is reasonable to assume that a thing is supreme if and only if it is necessarily greater than everything else solely by virtue of having some set of perfections, making the extension of the property of being supreme identical with the intersection of the extensions of those perfections.

This passage suggests that there is a connection between perfections and great making; and it signals an argument for why the property of being necessarily greater than everything else (the property of being supreme) is a perfection: anything that is necessarily greater than everything else is that way by virtue of some particular set of great making properties that make it the greatest possible being, and great making properties are perfections. Less compacted:

1. Anything that has the property of being necessarily greater than everything else (the property of being supreme) attributes all and only great making properties.
2. Great making properties are perfections.
3. Perfections are better to have than not.
4. If all and only the properties that a property attributes are properties that it is better to have than not, then it is better to have that property than not.
Therefore, the property of being supreme is a perfection.

I am unaware of any analysis by Oppy that shows that these two arguments beg the question by assuming either that the property of being supreme is a perfection or that there is no supreme being.

6 Conclusion

I have once again tried to show that Oppy has not refuted various ontological arguments that I have championed as sound and non-question begging. He will surely disagree. Yet, even though our analyses, conclusions, and worldviews diverge, philosophers aim for truth, howsoever elusive. Criticism helps. I am profoundly grateful to Graham Oppy.

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